Econ 172A - Class Exercise 1

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Formulation Problem 1

A bus company demands $b_i$ drivers for each of the next five years. $b = (b_1, \ldots, b_5) = (60, 70, 50, 65, 75)$.

At the beginning of each year, the bus company must decide how many drivers to hire or fire. It costs $20000 to hire a driver and $10000 to fire a driver. A driver’s salary is $50,000 per year. At the beginning of year 1, the company has 50 drivers. A driver hired at the beginning of a year can be used to meet that year’s requirements and is paid full salary for the current year. Any driver hired remains with the company (and able to meet demand) for all subsequent years unless he or she is fired.

Formulate a linear programming problem that can be used to determine a hiring plan that minimizes the total hiring/firing/salary costs of the firm while meeting (or exceeding) the demand in each year.
1. What are the variables?
   Number of drivers hired at the beginning of year $i$: $h_i$ and number of drivers fired at the beginning of year $i$: $f_i$ (total ten variables)
   Useful quantity: $w_i$ number of workers available in year $i$.
   Defined for $i = 1, \ldots, 5$.

   \[ w_{i+1} = w_i + h_{i+1} - f_{i+1} \text{ and } w_0 = 50 \quad (1) \]
   \[(i = 0, \ldots, 4)\]

2. What is the objective function?
   \[
   \min \sum_{i=1}^{5} 20000h_i + 10000f_i + 50000w_i
   \]

3. What are the constraints?
   Nonnegativity of all variables; (1); and $w_i \geq b_i$. 