Orthogonalized VARs

A. Recursively orthogonalized VAR
B. Variance decomposition
C. Historical decomposition
D. Structural interpretation
E. Generalized IRFs
A. Recursively orthogonalized VARs

Nonorthogonal IRF:

\[ \Psi_s = \frac{\partial E(y_{t+s} | y_t, y_{t-1}, \ldots, y_{t-p})}{\partial y_t'} \]  

Column 1 = \[ \frac{\partial E(y_{t+s} | y_{1t}, y_{2t}, \ldots, y_{nt}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}} \]

e.g., already have data on \( y_{2t}, \ldots, y_{nt} \) and ask how a 1-unit change in \( y_{1t} \) affects forecast.
Could instead ask \[ \frac{\partial E(y_{t+s} | y_{1t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}} \]
e.g., don’t have any data from period \( t \) except for \( y_{1t} \) and ask how 1-unit change in \( y_{1t} \) affects forecast.
Knowing \( y_{1t} \) gives us information about \( y_{2t}, \ldots, y_{nt} \) if VAR forecast errors are correlated.
How calculate $h_{s1} = \frac{\partial E(y_{t+s} | y_{t+1}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}}$?

Method 1: local projection
Estimate by $n$ OLS equations

$y_{t+s} = c_s + h_{s1}y_{1t} + H_{s2}y_{t-1} + \cdots + H_{sp}y_{t-p+1} + u_{t+s}$
Method 2: calculate answer implied by VAR

\[
y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'
\]

\((n \times 1)\)

\[
x_t = (1, y'_{t-1}, y'_{t-2}, \ldots, y'_{t-p})'
\]

\((k \times 1)\)

\[k = np + 1\]

\[
y_t = \Gamma' x_t + \varepsilon_t
\]

\[E(\varepsilon_t \varepsilon_t') = \Omega\]
Given parameters, observation of $y_{1t}, y_{t-1}, \ldots, y_{t-p}$ allows us to observe

$$\varepsilon_{1t} = y_{1t} - \gamma'_{1} x_{t}$$
Can calculate optimal forecast of $\varepsilon_{it}$ given $\varepsilon_{1t}$ as

$$E(\varepsilon_{it}|\varepsilon_{1t}) = \frac{\sigma_{i1}}{\sigma_{11}} \varepsilon_{1t}$$

$$E(\varepsilon_t|\varepsilon_{1t}) = \begin{bmatrix} 1 \\ \sigma_{21}/\sigma_{11} \\ \vdots \\ \sigma_{n1}/\sigma_{11} \end{bmatrix} \varepsilon_{1t} = a_1 \varepsilon_{1t}$$
\[
\frac{\partial E(y_t|y_{1t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}} = a_1
\]

\[
\frac{\partial E(y_{t+s}|y_{1t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}} = \frac{\partial E(y_{t+s}|y_{t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_t'} \frac{\partial E(y_t|y_{1t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{1t}}
\]

\[
= \Psi_s a_1
\]
\[ \hat{\Gamma}' = \left( \sum_{t=1}^{T} y_t x_t' \right) \left( \sum_{t=1}^{T} x_t x_t' \right)^{-1} \Rightarrow \hat{\Psi}_s \]

\[ \hat{\varepsilon}_t = y_t - \hat{\Gamma}' x_t \]

\[ \hat{\Omega} = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t' \Rightarrow \hat{a}_1 = \begin{bmatrix} 1 \\
\hat{\sigma}_{21}/\hat{\sigma}_{11} \\
\vdots \\
\hat{\sigma}_{n1}/\hat{\sigma}_{11} \end{bmatrix} \]
Could also do this using Cholesky factor:

\[ \hat{\Omega} = \hat{P}\hat{P}' \]  (\( \hat{P} \) lower triangular)

\[ \hat{a}_1 = \hat{p}_1 / \hat{p}_{11} \]

\[ \hat{p}_1 = \text{column 1 of } \hat{P} \]

\[ \hat{p}_{11} = \text{row 1 col 1 element of } \hat{P} \]
\( \Psi_s a_1 = \frac{\partial E(y_{t+s} | y_{1t}, y_{t-1}, ..., y_{t-p})}{\partial y_{1t}} \)

is effect of one-unit increase in \( y_{1t} \) or \( \varepsilon_{1t} \) on forecast

\( \Psi_s p_1 \) is effect of one-standard-deviation increase in \( \varepsilon_{1t} \) on forecast.
$\Psi_s a_1$ shows IRF to shock in observed units
$\Psi_s p_1$ shows IRF to shock of typical size
Plots will look identical just with different units on vertical axis $\Psi_s p_1 = \Psi_s a_1 p_{11}$
Recursively orthogonalized VAR estimated 1954-2007

Response of GDP to GDP alone (VAR)

Response of inflation to GDP alone (VAR)

Response of fed funds to GDP alone (VAR)
Recursively orthogonalized local projection estimated 1954-2007

Response of GDP to GDP alone (local projection)

Response of inflation to GDP alone (local projection)

Response of fed funds to GDP alone (local projection)
Suppose next that we’ve observed $y_{1t}$ and $y_{2t}$ but not $y_{3t}, y_{4t}, \ldots, y_{nt}$.

What is effect on forecast of changing $y_{2t}$?

$$\frac{\partial E(y_t|y_{1t}, y_{2t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{2t}} = a_2 = p_2/p_{22}$$

$p_2 = \text{column 2 of } P$

$p_{22} = \text{row 2 col 2 element of } P$

first element of $a_2$ is zero
\[
\frac{\partial E(y_{t+s}|y_{1t}, y_{2t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{2t}} = \frac{\partial E(y_{t+s}|y_{t}, y'_{t-1}, \ldots, y_{t-p})}{\partial y'_{t}} - \frac{\partial E(y_{t}|y_{1t}, y_{2t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{2t}} = \Psi_s a_2
\]
(n × n) matrix of recursively orthogonalized shocks:

\[ \Psi_s P \text{ or } \Psi_s A \]

\[
A = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & 1
\end{bmatrix}
= P[\text{diag}(P)]^{-1}
\]

Note last col of \( \Psi_s A \) is identical to last col of \( \Psi_s \)
We have broken down the news arriving in period $t$ into $n$ separate uncorrelated components

$\varepsilon_{1t} = y_{1t} - \gamma_1^t x_t = \text{news about } y_{1t}$

$u_{2t} = \varepsilon_{2t} - a_{21} \varepsilon_{1t} = \text{news about } y_{2t} \text{ not already revealed by } y_{1t}$

$\vdots$

$u_{nt} = \varepsilon_{nt} - E(\varepsilon_{nt}|\varepsilon_{1t}, \ldots, \varepsilon_{n-1,t})$

$= \text{news about } y_{nt} \text{ not already revealed by } y_{1t}, \ldots, y_{n-1,t}$
Simple way to summarize these components:

\[
\mathbf{v}_t = \mathbf{P}^{-1} \mathbf{\epsilon}_t = \begin{bmatrix}
    p^{11} & 0 & \cdots & 0 \\
    p^{21} & p^{22} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    p^{n1} & p^{n2} & \cdots & p^{nn}
\end{bmatrix} \begin{bmatrix}
    \mathbf{\epsilon}_{1t} \\
    \mathbf{\epsilon}_{2t} \\
    \vdots \\
    \mathbf{\epsilon}_{nt}
\end{bmatrix}
\]

\[
E(\mathbf{v}_t \mathbf{v}_t') = \mathbf{P}^{-1} E(\mathbf{\epsilon}_t \mathbf{\epsilon}_t') \mathbf{P}'^{-1} = \mathbf{P}^{-1} \Omega \mathbf{P}'^{-1} \\
= \mathbf{P}^{-1} \mathbf{P} \mathbf{P}' \mathbf{P}'^{-1} = \mathbf{I}_n
\]
\( v_t \) is a linear combination of \( \varepsilon_t \) whose elements are uncorrelated with each other

\[
v_{1t} = p^{11} \varepsilon_{1t} \\
= \text{rescaled error forecasting } y_{1t}
\]

\[
v_{2t} = p^{21} \varepsilon_{1t} + p^{22} \varepsilon_{2t} \\
= \text{rescaled error forecasting } \varepsilon_{2t} \text{ from } \varepsilon_{1t}
\]

\[
v_{nt} = p^{n1} \varepsilon_{1t} + p^{n2} \varepsilon_{2t} + \cdots + p^{nn} \varepsilon_{nt} \\
= \text{rescaled error forecasting } \varepsilon_{nt} \text{ from } \varepsilon_{1t}, \ldots, \varepsilon_{n-1,t}
\]
B. Variance decomposition

\[ y_{t+s} = \hat{y}_{t+s|t} + \Psi_0 \varepsilon_{t+s} + \Psi_1 \varepsilon_{t+s-1} + \Psi_2 \varepsilon_{t+s-2} + \cdots + \Psi_{s-1} \varepsilon_{t+1} \]

\[ E(y_{t+s} - \hat{y}_{t+s|t})(y_{t+s} - \hat{y}_{t+s|t}) = \sum_{m=0}^{s-1} \Psi_m \Omega \Psi_m' \]

\[ \varepsilon_t = P \nu_t = p_1 \nu_{1t} + p_2 \nu_{2t} + \cdots + p_n \nu_{nt} \]

Contribution of \( \nu_{i,t+1}, \nu_{i,t+2}, \ldots, \nu_{i,t+s} \) to forecast error:

\[ \Psi_0 p_i \nu_{i,t+s} + \Psi_1 p_i \nu_{i,t+s-1} + \cdots + \Psi_{s-1} p_i \nu_{i,t+1} \]
\[ E(y_{t+s} - \hat{y}_{t+s|t})(y_{t+s} - \hat{y}_{t+s|t}) = \sum_{m=0}^{s-1} \Psi_m p_1 p_1' \Psi_m' + \cdots + \sum_{m=0}^{s-1} \Psi_m p_n p_n' \Psi_m' \]

First term: amount by which could reduce MSE if we knew the values of \( \epsilon_{1,t+1}, \ldots, \epsilon_{1,t+s} \)

Second term: amount by which we could reduce MSE if we knew the values of \( u_{2,t+1}, \ldots, u_{2,t+s} \)
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C. Historical decomposition

\[ y_{t+s} = \hat{y}_{t+s|t} + \sum_{m=0}^{s-1} \Psi_m \epsilon_{t+s-m} \]

\[ = \hat{y}_{t+s|t} + \sum_{m=0}^{s-1} \Psi_m [p_1 \nu_{1,t+s-m} + \cdots + p_n \nu_{n,t+s-m}] \]

Can decompose the observed value for any variable at any date into component that could have been predicted as of some earlier date plus innovations in individual \( \nu_{i,t+m} \) since then.
D. Structural interpretation

Suppose we hypothesized the following structural model for the behavior of the Fed:

\[ i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + b_{31}' y_{t-1} + \cdots + b_{3p}' y_{t-p} + u_{3t} \]

- \( i_t \) = fed funds rate
- \( y_t \) = GDP growth rate
- \( \pi_t \) = inflation rate
- \( \psi_y, \psi_\pi \) = coefficients in Taylor Rule
- \( b_{3m} \) allow for inertia in monetary policy
- \( u_{3t} \) = serially uncorrelated shock to monetary policy
  - = deviation from Fed’s usual rule, uncorrelated with \( y_{t-1}, \ldots, y_{t-p} \) by definition

Would like to know \( \partial y_{t+s} / \partial u_{3t} \)
Suppose I also thought there was a Phillips Curve of the form

\[ \pi_t = \lambda_2 + \alpha y_t + b_{21}' y_{t-1} + \cdots + b_{2p}' y_{t-p} + u_{2t} \]

\( \alpha \) = slope of Phillips Curve

\( b_{2m} \) allow for inertia in PC

\( u_{2t} \) = unpredictable shock to PC

\( u_{2t} \) uncorrelated with \( y_{t-1}, \ldots, y_{t-p} \) by definition

\( u_{2t} \) also assumed to be uncorrelated with \( u_{3t} \)

(assumption that monetary policy shocks take more than one period to affect inflation)
Model equilibrium output as
\[ y_t = \lambda_1 + b'_{11} y_{t-1} + \cdots + b'_{1p} y_{t-p} + u_{1t} \]

\[ u_{1t} = \text{error forecasting GDP one period ahead} \]

\[ u_{1t} \text{ uncorrelated with } y_{t-1}, \ldots, y_{t-p} \text{ by definition} \]

\[ u_{1t} \text{ also assumed to be uncorrelated with } u_{2t}, u_{3t} \]

(assumption that PC and monetary shocks take more than one period to affect output)
\[ i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + b_{31}^t y_{t-1} + \cdots + b_{3p}^t y_{t-p} + u_{3t} \]

Above assumptions mean \( u_{3t} \) uncorrelated with \( y_t \) and \( \pi_t \).

\( \Rightarrow \) could estimate by OLS

\( \hat{\psi}_y \) and \( \hat{\psi}_\pi \) are same as step 0 Jordá projection

\( \hat{\psi}_y \) and \( \hat{\psi}_\pi \) are same as \( \hat{\alpha}_{31} \) and \( \hat{\alpha}_{32} \)
\[ \pi_t = \lambda_2 + \alpha y_t + b'_{21} y_{t-1} + \cdots + b'_{2p} y_{t-p} + u_{2t} \]

Above assumptions mean \( u_{2t} \) uncorrelated with \( y_t \).

\Rightarrow could estimate by OLS

\( \hat{\alpha} \) is same as step 0 Jordá projection

\( \hat{\alpha} \) is same as \( \hat{\alpha}_{21} \)
Conclusion: under above assumptions with

$$A = P[\text{diag}(P)]^{-1}$$

$$u_t = A^{-1} \varepsilon_t$$

$$u_{1t} = \varepsilon_{1t}$$

The error I make forecasting $y_{1t}$ given $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ is the shock to equilibrium output.
The error I make forecasting $y_{2t}$ given $y_{1t}, y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ is the shock to PC.

The error I make forecasting $y_{3t}$ given $y_{1t}, y_{2t}, y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ is the shock to monetary policy.
Recursively orthogonalized VAR gives the dynamic effects of monetary policy.

\[
\frac{\partial E(y_{t+s} \mid y_{1t}, y_{2t}, y_{3t}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{3t}} = \frac{\partial y_{t+s}}{\partial u_{3t}}
\]
Orthogonal Cholesky IRF with 95% confidence intervals (54-07)
• A monetary contraction (higher fed funds rate) is followed by slower GDP growth 2-3 quarters later
• But unanticipated monetary policy shocks account for only 10% of variance of output
• Most of variation in fed funds rate comes from predictable response of monetary policy to output and inflation
• A monetary contraction is followed by higher inflation (known as “price puzzle”)
• Assumption-free statement of price puzzle:
  – if you tell me that fed funds rate is higher than I would have predicted given current output, inflation, and lags, then I will revise my expectation of future inflation up.

• Natural interpretation:
  – Fed raised funds rate because it anticipated future inflation.
  – Our 3-variable equation is too simplistic a description of Fed
• Popular “fix” for price puzzle:
  – Add other variables that better capture information about future inflation (such as commodity prices) to Fed policy equation
Christiano, Eichenbaum, Evans (1996)

\[ y_{1t} = \text{log of real GDP} \]
\[ y_{2t} = \text{log of GDP deflator} \]
\[ y_{3t} = \text{index of sensitive commodity prices} \]
\[ y_{4t} = \text{fed funds rate} \]
\[ y_{5t} = \text{nonborrowed reserves} \]
\[ y_{6t} = \text{total reserves} \]
\[ y_{7t} = \text{one of a set of macro variables} \]
Structural model:

\[ B_0 y_t = B x_t + u_t \]

\[ x_t = (1, y_{t-1}', y_{t-2}', \ldots, y_{t-p}')' \]

\[ u_t = \text{vector of structural shocks} \]

\[ E(u_t u_t') = D \ (\text{diagonal}) \]
$\mathbf{B}_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\times & 1 & 0 & 0 & 0 & 0 & 0 \\
\times & \times & 1 & 0 & 0 & 0 & 0 \\
\times & \times & \times & 1 & 0 & 0 & 0 \\
\times & \times & \times & \times & 1 & 0 & 0 \\
\times & \times & \times & \times & \times & 1 & 0 \\
\times & \times & \times & \times & \times & \times & 1 \\
\end{bmatrix}$

Variable 4 is fed funds rate, equation 4 is monetary policy equation.
Note that

\[
\frac{\partial E(\mathbf{y}_{t:s}|y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{t-1}, y_{t-2}, \ldots, y_{t-p})}{\partial y_{4t}} = \frac{\partial E(\mathbf{y}_{t:s}|y_{2t}, y_{1t}, y_{3t}, y_{4t}, y_{t-1}, y_{t-2}, \ldots, y_{t-p})}{\partial y_{4t}}
\]

Will have the identical answer for effect of variable 4 any way we order variables 1-3 and any way we order variables 5-7. Jordá estimate identical if reorder (keeping 4 in place).
If all we care about is effect of monetary policy, we only need to assume block-recursive

$$B_0 = \begin{bmatrix}
  x & x & x & 0 & 0 & 0 & 0 \\
  x & x & x & 0 & 0 & 0 & 0 \\
  x & x & x & 0 & 0 & 0 & 0 \\
  x & x & x & 0 & 0 & 0 & 0 \\
  x & x & x & 1 & 0 & 0 & 0 \\
  x & x & x & x & x & x & x \\
  x & x & x & x & x & x & x \\
  x & x & x & x & x & x & x \\
  x & x & x & x & x & x & x \\
\end{bmatrix}$$
67% confidence bands
E. Generalized IRFs

• If we put fed funds fourth, estimated effect of monetary policy does not depend on how we order variables 1-3.

• But if we switch fed funds from 4 to 3, results could change
  – Put variable #1 first to find effect of variable 1
  – Put variable #2 first to find effect of variable 2
  – Put variable #n first to find effect of variable n
GIRF: for every $i$, calculate

$$\frac{\partial E(y_{t+s}|y_{it}, y_{t-1}, \ldots, y_{t-p})}{\partial y_{it}}$$
• Conclusion: any IRF or GIRF is giving answer to a forecasting question.
• Best practice: describe forecasting question explicitly and explain the reason that question is interesting.