Practice Midterm Exam Questions

DIRECTIONS: No books, notes, or calculators of any kind are allowed.

1.) Suppose that \( x_t \) follows an MA(1) process

\[
x_t = \mu + \varepsilon_t + \theta \varepsilon_{t-1}, \quad 0 < \theta < 1
\]

\[
E(\varepsilon_t) = 0
\]

\[
E(\varepsilon_t^2) = \sigma^2
\]

\[
E(\varepsilon_t \varepsilon_{t-j}) = 0 \quad \text{for} \quad j \neq 0,
\]

and we then calculate a new series \( y_t = (1 - \rho L)x_t \) for \( L \) the lag operator and \( \rho \) some real number.

a.) Calculate the population linear projection of \( y_t \) on \( (1, \varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2})' \).

b.) For what values of \( \rho \) is \( y_t \) covariance stationary? Calculate the mean and population spectrum of \( y_t \) when those objects exist.

c.) Let \( \bar{y} = T^{-1} \sum_{t=1}^{T} y_t \) denote the sample mean from a sample of size \( T \) and consider the variance of the sample mean \( E[\bar{y} - E(y_t)]^2 \) provided \( E(y_t) \) exists. Calculate

\[
\lim_{T \to \infty, \rho \to 1} T E[\bar{y} - E(y_t)]^2
\]

d.) Suppose that \( \rho = 2 \). Can you find a moving-average representation for the process for \( y_t \) of the form

\[
y_t = c + v_t + \psi_1 v_{t-1} + \psi_2 v_{t-2} + \cdots
\]

where \( v_t \) are the fundamental innovations for \( y_t \), that is,

\[
v_t = y_t - \hat{E}(y_t|y_{t-1}, y_{t-2}, \ldots)
\]

If so, find the values for \( \psi_j \) and for \( E(v_t^2) \).

2.) Suppose an \((n \times 1)\) vector \( y_t \) can be characterized by

\[
y_t = c + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t
\]

where \( \varepsilon_t \) is i.i.d. \( N(0, \Omega) \).

a.) What additional assumptions (if any) would you need to make in order to conclude that \( y_t \) is covariance stationary?
b.) Give the formulas for the maximum likelihood estimates of $c, \Phi_1, \Phi_2, \Omega$. Note: you do not need to derive these, just state the formulas for the estimates (or accurately describing them in words is also ok).

c.) What additional assumptions (if any) would you need in order to be able to interpret the usual OLS $t$ or $F$ tests using the standard $t$ and $F$ tables?

d.) Calculate the value of the nonorthogonalized impulse-response function for horizon $s = 2$:

$$\frac{\partial y_{t+2}}{\partial \varepsilon_t'}$$

e.) A researcher estimates a just-identified structural VAR of the form

$$B_0 y_t = \lambda + B_1 y_{t-1} + u_t$$

$$u_t = \Lambda u_{t-1} + v_t$$

for $v_t \sim \text{i.i.d. } N(0, I_n)$. What is the relationship between the maximum likelihood estimates $\{\hat{\lambda}, \hat{B}_0, \hat{B}_1, \hat{\Lambda}\}$ and the values $\{\hat{c}, \hat{\Phi}_1, \hat{\Phi}_2, \hat{\Omega}\}$ that you specified above?

f.) Give an expression for an estimate of the effect of a structural disturbance $v_t$ on the value of $y_{t+s}$. 