Test 2
Economics 136 – Human Resources
Spring 2012
Prof. Julian Betts

November 15, 2012

Name: ____________________

Student ID: ________________

There are 4 written problems in this test, worth a total of 50 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the graders.

If you use pencil, the exam cannot be regraded. If you do submit your test for regrading, you must do within the time and other guidelines listed in the syllabus.

SHOW ALL YOUR WORK!

You have 80 minutes. Good luck.

For the graders:

1. _____/6
2. _____/12
3. _____/14
4. _____/18
SUM _____/50
1. (6 points) Firms generally do not want to experience a lot of worker turnover because of the costs of recruiting and training new workers. But there are three reasons we covered in class for why worker turnover can be a good thing from the point of view of the firm. List those reasons and explain each in a sentence or two.

1) **Organizational Change**
   - Firms develop new plans that require different types of workers.

2) **Firms at which differences in talent matter**
   - These firms use turnover to open up slots for promotions (probation, up-or-out contracts, etc.).

3) **Firms with hierarchy that narrows a lot at the top.**
   - Firms rely on departures to open up spots to promote from lower ranks.

2. (12 points) In this problem assume that both workers and firms discount future earnings or profits at a rate of 25% (that is, \( r = 0.25 \)).

Suppose a small firm is suffering from a recession and realizes that one of its workers, who is 63 and who plans to retire in two years, at age 65, is likely to have value marginal product of $60,000 and $64,000 this year and next year, but based on a union contract the worker’s salary is going to be $63,000 in both of those years.

a) What is the present value of profits from keeping this worker for the next two years? Show your calculation. (2 points)

\[
PV = 60,000 - 63,000 + \frac{(64,000 - 63,000)}{1 + 0.25}
\]

\[
= -3,000 + 800
\]

\[
= -$2,200
\]
b) If the firm decided it was unprofitable to keep this worker, is there a risk to the firm if it lays the worker off? Be specific. (2 points)

Yes, it could be sued for age discrimination.  

The Age Discrimination in Employment Act prohibits firms from laying off workers based on age.  

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c) Suppose that the firm instead decides to offer a buyout to the worker, consisting of an immediate payment to the worker if he or she voluntarily quits. What is the range of buyout payments that the firm would be willing to make? (Buyouts have to be for a positive amount of money.) (2 points)

The buyout could range from $1 to $2200.  

(The firm is willing to pay up to the $2200 loss it would incur by keeping the worker.  

Note: Also accept $0 < B < $2200  

or $0 < B < $2200.
d) Suppose that the worker's best alternative to deciding to stay for another two years at this firm is to work elsewhere earning $62,000 in each year before retiring in two years. Writing the value of the buyout the firm offers as B, write down the present value of staying at the firm (PV_{stay}), the present value of taking the buyout and going to the other employer (PV_{go}), and an inequality that determines whether the worker will accept the buyout of $B$. (3 points)

\[ PV_{\text{stay}} = 63\,000 + \frac{63\,000}{1 + 0.25} = 113,400 \]
\[ PV_{\text{go}} = 62\,000 + \frac{62\,000}{1 + 0.25} + B = 111,600 + B \]
Accept the buyout if \( 111,600 + B > 113,400 \) or \( B > 1,800 \)

e) Can the firm make both itself and the worker better off by offering a buyout? What is the range of values for B that would leave both the worker and firm better off if the worker accepts the buyout and leaves? (3 points)

\[ \text{Yes} \]
\[ 1,800 < B < 2,200 \text{ would make both worker and firm better off if the worker accepts the buyout} \]

Note: Take off 1 point if uses an equality i.e.
\[ 1,800 \leq B \leq 2,200 \text{ because } B = 1,800 \text{ or } B = 2,200 \text{ leaves only 1 party better off} \]
3. (14 points) Suppose that a firm gives firm specific training to workers during the early part of their career with the firm. The value marginal product of a typical worker is shown below.

a) In the top graph plot a possible profile for wages \( W(t) \) at the firm and explain in a sentence or two. Also specifically explain what would happen if the firm set wages equal to value marginal product in each period. How does your choice of wages to pay the worker fix this problem? (6 points)

During training firm pays \( W > V \) and after training it pays a wage \( W \leq V \) to make profit to pay back the subsidy. It does this to share the costs of training.

If \( W(t) = V(t) \) for all \( t \), it would be tempted to lower \( W \) in later periods so the wage worker would command in other jobs. Workers would be ripped off and so they should not accept this offer.

- The solution above solves this because firm wants to keep the worker after training to earn some profits to pay for training subsidy.

T is retirement date, \( V(t) \) is Value Marginal Product at time \( t \).
b) Draw a likely profile for Rent on the bottom graph above. Explain in a few sentences why Rent against tenure $t$ has the shape that it does. (3 points)

$$R(0) = 0 \text{ due to competition in the labor market}$$
(Firm makes 0 profit over the worker's tenure at the firm)

$$R(t) = 0 \text{ because by retirement date the firm has earned back all of the subsidies it provided during training and there are no future profits for the firm to earn back.}$$

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c) Suppose that a short recession hits, and $V(t)$ drops somewhat for a few periods. Draw a line for "Rent*" in the bottom figure above showing what could happen to rent in this situation. Please label it Rent*. (3 points)

\[ \text{For Rent* see graph on last page.} \]

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d) Whom should the firm most likely lay off in case of a recession? Be specific. (2 points)

The youngest and oldest workers
4. (18 points) A firm is trying to establish a wage \(-a+bE\) where \(E\) is worker effort and \(a\) and \(b\) are to be chosen by the firm. The worker maximizes utility which is given by wage \(-\frac{1}{3}E^3\) or \(U = (a+bE) - 0.33333E^3\), where the latter term reflects the cost of effort to the worker. The firm’s output is given by \(Q = E\). The product sells for $9 per unit.

a) Assume that the worker’s utility must be at least zero for him or her to accept the job.

Calculate the profit maximizing values of \(a\) and \(b\), and the optimal effort \(E\) and profit \(\pi\) that results. (14 points)

**Worker:**

\[
\max_{E} \quad a + bE - \frac{1}{3}E^3
\]

**E.O.C.:** \(b - \frac{1}{3}E^2 = 0\)

\(E^* = \sqrt[3]{\frac{3b}{1}}\) \(\quad (1)\)

**S.O.C.:** \(-2E^* < 0\) so utility maximum.

**Firm:**

\[
\max_{a,b} \quad qE^* - a - bE^* \quad s.t.\ (1)
\]

and \(U = 0\) or \(a + bE^* - \frac{1}{3}E^3 = 0\) \(\quad (2)\)

Substitute \(E^* = \sqrt[3]{\frac{3b}{1}}\) \(\Rightarrow\)

**Firm:**

\[
\max_{b} \quad qE^* - \frac{1}{3}E^3 \quad s.t.\ (1)
\]

**E.O.C.:** \((q - E^*) \frac{dE^*}{db} = 0\)

From \((1)\) \(\frac{dE^*}{db} = \frac{1}{2}b^{-\frac{1}{2}} > 0\) so

\[q - E^*^2 = 0\]

\[
E^* = \sqrt[3]{\frac{3b}{1}}
\]

Substitute into \((1)\) \(\Rightarrow\)

\(b^* = q\)

\((2) \Rightarrow q^* = -q(3) + \frac{1}{3}(3)^3\)

\[= -27 + 9 = -18\]

\(\pi = qE^* - \frac{1}{3}E^3 = 27 - \frac{1}{3}(3)^3\)

\[= 18\]
b) (4 points) Draw a graph (with $ on the vertical axis and effort $E$ on the horizontal axis) that shows the optimal wage $a+bE$, the worker’s disutility of effort, and net revenues, and illustrate where the optimal effort is.