Name: ___________________

Student ID ________________

There are 9 written problems in this exam, worth a total of 95 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the marker.

If you use pencil, the exam cannot be regraded. The graders reserve the right to regrade the entire test if you submit the test for regrading.

SHOW ALL YOUR WORK, AND WRITE YOUR ANSWERS IN A WAY THAT THE GRADERS SHOULD BE ABLE TO UNDERSTAND! IN QUESTIONS WITH MATH YOU SHOULD ADD SOME WRITTEN EXPLANATION FOR WHAT YOU ARE DOING!

You have 180 minutes to complete this test. Good luck.

For graders’ use:

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1. (15 points) a) You want to produce 10 units of output a week, and can do this by hiring either 10 high school graduates or 4 workers with a community college degree. The current wage rates for the two types of workers are $6 per hour and $12 per hour respectively. Draw a graph showing the isoquant for output of 10 and a series of isocost lines. Indicate the slopes of the lines either directly on the graph or in text below the graph. Based on your graph, indicate which type of worker you should hire. (6 points)

b) Now calculate the cost to output ratios for the two types of workers to check your answer to a) is correct. Explain in a sentence how you can use this ratio to figure out what type of worker to hire. (3 points)
c) In more realistic cases your firm may decide that it needs to hire both high school and community college graduates to produce the same 10 units of output per hour. Draw a typical isoquant and isocost lines for such a situation, and indicate clearly the cost-minimizing combination of the two types of workers. (You don’t have to give specific numbers such as slopes in this graph. Instead, just show the general shapes of the isoquant and isocost lines.) (6 points)

2. (10 points) a) Explain in a few sentences the problem of adverse selection in hiring. (3 points)

b) Suppose that in the overall labor market unskilled workers earn $W_u$ and skilled workers earn $W_U + G$, where $G > 0$. Explain how you could set up a probation system that would ensure that only skilled workers apply. (Assume that workers maximize the sum of wages over two periods.) Show your reasoning carefully. (7 points)
Show your work in all of your answers!

3. (10 points) a) List 2 advantages of piece rates over fixed salaries, and 4 disadvantages. (6 points)

b) List two advantages of using promotion tournaments instead of piece rates, and two disadvantages of tournaments. (4 points)
4. (16 points) You have been hired by a company to create a piece rate system for workers that will maximize profits. Workers’ utility is given by 
\[ U = \text{earnings} - cE^2 \] where \( E \) is effort expended by the worker and \( c > 0 \).
Because there is a recession, the best alternative for workers is to stay at home, earning 0 and exerting zero effort. Therefore, workers will work for the company if utility is at least 0.
Revenue per worker is simply \( E \). You are asked to design a piece rate system where earnings = \( a + bE \).
a) Derive the profit-maximizing values of \( a \) and \( b \). Also calculate workers’ optimal effort \( E^* \) and profit per worker \( \pi^* \). (8 points)
b) Suppose that there are two types of workers in the economy, those with \( c = 1 \) and those with \( c = 2 \). Suppose that the firm could identify each worker before hiring him or her. Which would lead to the more profit per worker, hiring \( c = 1 \) workers only or \( c = 2 \) workers only? Show your work. (4 points)

c) In part b) we assumed, perhaps unrealistically, that the firm can identify workers of each type before hiring them. Now consider what to do if the firm cannot tell the workers apart. Prove that the firm can ensure that only workers with \( c = 1 \) will apply for the job if it sets the two components of the wage, \( a \) and \( b \), using \( c = 1 \) in the formulas you derived in a. (Hint: Recall that we are in a recession so the best alternative for each worker is to stay at home, with zero earnings and effort.) (4)
5. (11 points) a) Your firm provides general training in period 1 of a 2-period employment relation with workers. The alternative wage that workers can make each period is $W_{\text{alt}}$. The VMP of workers during training and after training is $W_{\text{alt}} - c$ and $W_{\text{alt}} + m$ respectively. Workers maximize the undiscounted sum of wages they earn in the two periods. Derive the profit-maximizing wages that you should pay in periods 1 and 2 that will ensure that workers will be willing to accept a job with your firm. (6 points)
b) Suppose that \( m > c \) at your company right now. What will be the two-period (undiscounted) profit from hiring one worker and paying the optimal wages derived in a? What forces might cause this to change over time, and what do you predict will happen in the long run to \( m \), \( c \) and \( W_{\text{ult}} \)? (5 points)

6. (6 points) a) What are the two reasons why it is difficult for American firms to lay off older workers during an economic downturn? (Be specific) (4 points)
b) Explain what a window plan is for buying out older workers. Explain why a firm might want to use this type of buyout plan. (2 points)

7. (16 points) Senior management asks you to devise a promotion tournament for two workers, where the worker who produces more earns $W_1$ and the worker who produces less earns a smaller amount $W_2$. The two workers have identical productivity (on average) and the same utility function:

$$U = E(wage) - m_i^3$$

for workers $i=k,j$ where $m_i$ is effort.

Each worker’s output is given by

$$q_i = m_i + e_i$$

where $m$ is effort and $e$ is a random luck factor. Each unit of output brings in $3$ dollars of revenues to your company. There are no costs apart from labor.

$$x = e_k - e_j$$
takes on values between $-1/4$ and $+1/4$ with a uniform probability distribution.

To ensure that both workers accept the job, you must pay an expected wage for a given level of effort to ensure that expected utility equals 0.

Solve the workers’ problems and the firm’s problem. DEFINE ALL NEW VARIABLES THAT YOU USE THAT ARE NOT MENTIONED IN THIS QUESTION. (16 points)
Show your work in all of your answers!
8. (8 points) List two reasons to promote workers from within the firm and two reasons why it might be better to hire senior workers from outside.

9. (3 points) Towards retirement workers may develop an increased incentive to “shirk”, that is, goof off. Suppose that the employer pays a fixed wage W for all years the worker is at a firm. What is the cost to the worker of being fired, and why might this cost go down towards retirement?