In Praise of Lectures

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The Ibis was a sacred bird to the Egyptians and worshippers acquired merit by burying them with due ceremony. Unfortunately the number of worshippers greatly exceeded the number of birds dying of natural causes so the temples bred Ibises in order that they might be killed and and then properly buried.

So far as many mathematics students are concerned university mathematics lectures follow the same pattern. For these students attendance at lectures has a magical rather than a real significance. They attend lectures regularly (religiously, as one might say) taking care to sit as far from the lecturer as possible (it is not good to attract the attention of little understood but powerful forces) and take complete notes. Some lecturers give out the notes at such speed (often aided by the technological equivalent of the Tibetan prayer wheel — an overhead projector) that the congregation is fully occupied but most fail in this task. The gaps left empty are filled by the more antisocial elements with surreptitious (or not so surreptitious) conversation[1], reading of newspapers and so on whilst the remainder doodle or daydream. The notes of the lecture are then kept untouched until the holidays or, more usually, the week before the exams when they are carefully highlighted with day-glow yellow pens (a process known as revision). When more than 50% of the notes have been highlighted, revision is said to be complete, the magical power of the notes is exhausted and they are carefully placed in a file never to be consulted again. (Sometimes the notes are ceremonially burnt at the end of the student's university career thereby giving a vivid demonstration of the value placed on the academic side of fifteen years of education.)

[1] A lecture is a public performance like a concert or a theatrical event. Television allows channel hopping and conversation. At public performances, private conversation, however interesting to the participants, distracts the rest of the audience from the matter in hand. It must be added that just as good eaters make good cooks so good audiences make for good lectures. A lecturer will give a better lecture to a quiet and attentive audience than to a noisy and inattentive one.

Many students would say that there is an element of caricature in my description. They would agree that the lectures they attend are incomprehensible and boring but claim that they have to come to find out what is going to be examined. However, even if this was the case, they would still be behaving irrationally. The invention of the Xerox machine means that only one student need attend each lecture the remainder being freed for organised games, social events and so on[2]. Nor would this student need to take very extensive notes since everything done in the lecture is better done in the textbooks.

Even the least experienced observer can see that the average lecturer makes lots of little mistakes. Usually these are just 'mis-speakings' or misprints sometimes spotted by the lecturer, sometimes vocally corrected by a wide awake member of the audience, sometimes silently corrected by the note taker but often passing unnoticed into students notes to puzzle or confuse them later. The experienced observer will note that, though the general outlines of proofs are reasonably well done, the fine detail is often tackled inefficiently or vaguely with, for example, a four line proof where one line will do. A lecture takes place in real time, so to speak, with 50 minutes of mathematics occupying 50 minutes of exposition whereas a chapter of a book that takes ten minutes to read may have taken as many days to compose. When the author of a book encounters a problem she can stop and think about it; the lecturer must press on regardless. If the notation becomes too complex or it becomes clear that some variation in an early definition would be helpful the author can go back and change it; the lecturer is committed to her earlier choices. When her book is finished the lecturer can reread it and revise at leisure. She will get her friends to read the manuscript and they, viewing it with fresh eyes, will be able to suggest corrections and improvements. Finally, if she is wise, she will offer a graduate student a suitable monetary reward for each error found. Even with all these precautions, errors will still slip through, but it is almost certain that the book will provide a clearer, simpler and more accurate exposition than any lecture notes[3].

Students may feel under some obligation to go to lectures; their teachers

[2]In the past some universities made lectures compulsory. In Cambridge during the early 19th century attendance at lectures was not compulsory but attendance at Chapel was. 'The choice' thundered supporters of compulsory chapel 'is between compulsory religion and no religion at all'. 'The difference' replied one opponent '. . . is too subtle for my grasp'.

[3]At one time it was the custom for beginning lecturers to spend their first couple of years producing a perfect set of lecture notes, in effect a book. For the rest of their professional lives their lectures consisted of reading these notes out at dictation speed. Their exposition was then clear, simple and accurate but, in view the invention of printing some centuries earlier, the same result could have been obtained more efficiently.

are under no such compulsion. Yet mathematicians go to seminars, colloquium talks, graduate courses all of which are lectures under another name. Why, if lectures have all the disadvantages that I have shown, do they persist in going to them? The surprising answer is that many mathematicians find it easier to learn from lectures than from books. In my opinion there are several interlinked reasons for this.

(1) A lecture presents the mathematics as a growing thing and not as a timeless snapshot. We learn more by watching a house being built than by inspecting it afterwards.

(2) As I said above, the mathematics of lecture is composed in real time. If the mathematics is hard the lecturer and, therefore, her audience are compelled to go slowly but they can speed past the easy parts. In a book the mathematics, whether hard or easy, slips by at the the same steady pace.

(3) Some lecturers are too shy, some too panic stricken and a few (but very few) too vain or too lazy to respond to the mood of the audience. Most lecturers can sense when an audience is puzzled and respond by giving a new explanation or illustration. When a lecture is going well they can seize the moment to push the audience just a little further than they could normally expect to go. A book can not respond to our moods.

(4) The author of a book can seldom resist the temptation to add just one extra point. (Why should she, when purchasers and publishers prefer to deal in 'proper' books rather than slim pamphlets?) The lecturer is forced by the lecture format to concentrate on the essentials.

(5) In a book the author is on her best behaviour; remarks which go down well in lectures look flat on the printed page. A lecturer can say 'This is boring but necessary' or 'It took me three days to work this out' in a way an author cannot.

There is another advantage of lectures which is of particular importance to beginners. There is a slogan 'We learn mathematics by doing mathematics' which like many slogans conceals one truth behind another. We do not learn to play the violin by playing the violin or rock climbing by climbing rocks. We learn by watching experts doing these things and then imitating them. Practice is an essential part of learning but unguided practice is generally useless and often worse than useless. People who teach themselves to program acquire a mass of bad programming habits which (unless they wish to remain hackers all their lives) they then have to painfully unlearn. Mathematics textbooks show us how mathematicians write mathematics (admittedly an important skill to acquire) but lectures show us how mathematicians do mathematics.

In his book *Science Awakening* Van Der Waerden makes the following suggestive remarks about the decline of the ancient Greek mathematical tradition.

> Reading a proof in Apollonius requires extended and concentrated study. Instead of a concise algebraic formula, one finds a long sentence, in which each line segment is indicated by two letters which have to be located in the figure. To understand the line of thought one is compelled to transcribe these sentences in modern concise formulas. The ancients did not have this tool; instead they had the oral tradition.
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> An oral tradition makes it possible to indicate the line segments with the fingers; one can emphasise essentials and point out how the proof was found. All of this disappears in the written formulation of the strictly classical style. The proofs are logically sound, but they are not suggestive. One feels caught in a logical mousetrap, but one fails to see the guiding line of thought.
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> As long as there was no interruption, as long as each generation could hand over its method to the next, everything went well and the science flourished. But as soon as some external cause brought about an interruption in the oral tradition, and only books remained it became extremely difficult to assimilate the work of the great precursors and next to impossible to pass beyond it.

Many students simultaneously expect too little and too much from their lectures[4]. If asked they might say 'The purpose of lectures is to enable me to understand the material' or 'The purpose of lectures is to enable me to do the exercises'. Since the lectures do not achieve this end the students assume either that the lecturer is incompetent or that they are. Often both assumptions are false.

Suppose that that you visit a large town and you wish to learn how to get around. One way of learning is to go by foot on a guided tour which includes the main landmarks. At the end of the walk, even if you remember everything your guide has shown you (that is 'you have learnt the proofs by heart') you will not know the town in the way that your guide knows it. In order to know the town 'like a native' you will need to explore for yourself. Instead of using the main road to get from the market to the station you will need to try other routes and see whether they work. (Naturally you will get lost from time to time but because you have been shown routes between the main landmarks you will be able to recover your bearings.) Your guide may

[4] I went to a lecture on the violin but when I tried playing one it sounded horrid. The lecturer can't have been any good.

have explained that the road system runs the way it does because there are only three bridges across the river but only by walking the roads themselves will you be able to internalise this knowledge. However hard your guide may have tried there are clear limits to how much you can learn on the first walk. But, without that first tour given by a native, you would find it very hard to learn your way about town. Lectures by themselves can not give you a full understanding of a piece of mathematics but, without lectures to get you started, it is very hard to gain that full understanding.

In my view students should treat lectures not as a note taking exercise but as a dialogue between themselves and the lecturer. They should try to follow the argument as it emerges and not just take it down blindly. 'But' the reader will exclaim 'this is an impossible and futile council of perfection' and, after having thrown these notes into the nearest available wastepaper basket, she may well resolve her indignation into a series of questions.

What about note taking? If you look at experienced mathematicians in a lecture you will see that their note taking is an automatic process which leaves them free to concentrate on the lecture. Most mathematics lecturers follow two conventions which make automatic, or at least semi-automatic, note taking possible

(a) Everything that is written on the blackboard is to be copied down and nothing that is spoken need be taken down.

(b) It is the responsibility of the lecturer to ensure that what appears on the board forms a decent set of notes without further editing.

Semi-automatic note taking is a skill that has to be learnt, but it seems to be an easy one to acquire.

Would it better not to take notes? Some mathematicians never take notes but most find that note taking helps them concentrate on the job in hand. (When the audience at a seminar stop taking notes the experienced seminar speaker knows that they have lost interest and are now using her as a gently babbling source of white noise whilst they think their own thoughts.) Further even the largest blackboard will eventually be erased and notes allow you to glance back to earlier parts of the lecture.

What should you do if you get lost? The first and most important thing is to remember that most mathematicians are lost most of the time during lectures. (If you do not believe me, ask around.) Attending a mathematics lecture is like walking through a thunderstorm at night. Most of the time you are lost, wet and miserable but at rare intervals there is a flash of lightening and the whole countryside is lit up. Once you realise that your plight is neither an infallible sign of your incurable stupidity nor a clear indication of the lecturer's total incompetence but simply a normal occurrence, it is clear how you should act. You should continue taking notes watching all the time for a point where the lecturer changes the subject (or finishes a proof or whatever) and you can rejoin her exposition as an active partner.

It is obvious that if you study your lecture notes after the lecture *with the object of understanding the point where the lecturer has got to* you will have a better chance of understanding the next lecture. If you are one of the majority of the students who find this a counsel of perfection then you could at least use the five minutes before the next lecture rereading the last part of your notes. (If you do not do even do this, at least ask yourself why you do not do this.)

What should you do if you understand nothing at all of what is going on? At an advanced level it is possible for an entire course of 24 lectures to be devoted to the proof of a single theorem. If you get really lost in such a course (and probably by the end everybody except, perhaps, the lecturer will be really lost) you stay lost. However first and second year undergraduate lectures consist of a set of short topics chained together in some reasonable order. Even if you completely fail to understand one topic there is no reason why you should not understand the next (even if you do not understand the proof of Cauchy's theorem you can still use it). On the other hand if incomprehensible topic succeeds incomprehensible topic then taking notes in the hope that all will become clear when you revise is not an adequate response. You should swallow your pride and consult your director of studies.

What about questions? There are three types of questions that an audience can ask.

(a) *Questions of Correction* If you think the lecturer has missed out a minus sign or written α when she meant β then you should always ask. No lecturer likes to spend a blackboard of calculations sinking further into the mire because her audience has failed to point out an error on line one. Sometimes very polite students wait until after a lecture to point out errors with the result that the lecturer knows that she has made an error but that she cannot correct it. So the rule is ask and ask at once.

(b) *Questions of Incomprehension* It takes considerable courage to admit that you do not understand something in front of other people. However if you do not understand something it is likely that many others in the audience will be in the same boat and you will have their silent thanks. You will usually also have the audible and honest thanks of the lecturer since, as I have indicated above, most lecturers prefer to keep in touch with the audience[5]. (There is a small and unfortunate minority who would prefer to lecture to an empty room, but give your lecturer the benefit of the doubt

[5]I have often thought that the technology of the TV game-show should be adapted to the lecture theatre. Each seat would have a concealed button which the auditors could press when they wanted the lecturer to slow down. The 'votes' could be added and the

and ask.)

(c) *Questions of Extension* If you are in the happy position of understanding everything the lecturer says then you may wish her to go further into a topic. Your modest request to hear more about the general case is unlikely to go down well with the rest of the audience who are still struggling with the particular case. Such questions should be left until after the lecture when the lecturer will be happy to oblige (few mathematicians can resist an invitation to talk more about their subject). If you find yourself asking more than one question per lecture, examine your motives.

It is noticeable that at seminars it is often the most distinguished mathematicians who ask the simplest (if they were not so distinguished, one might say naive) questions. It is, I suppose, possible that they only began to ask such questions after they became distinguished, but I believe that a willingness to ask when they do not know is a characteristic of many great minds[6].

Mathematical sayings tend to have multiple attributions (perhaps because mathematicians remember processes rather than isolated facts like names). The ancient Greeks attributed the following saying to Euclid among others. Ptolomey, King of Egypt, asked Euclid to teach him geometry. 'O King' replied Euclid 'in Egypt there are royal roads and roads for the common people, but there are no royal roads in geometry.' Mathematics is hard, there are no easy ways to understanding but the lecture, properly used, is the easiest way that I know.

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`http://www.dpmms.cam.ac.uk/~twk/`.

My home page includes other guides to things like writing essays and applying for Cambridge fellowships.]

result shown on a dial visible only to the lecturer who would then be in the position of a motorist trying to keep to the speed limit.

[6]Though there is no unique recipe for greatness. When the very great physicist Bohr was visiting the great physicist Landau in Moscow he was invited to give a talk to the graduate students with Landau translating. Bohr concluded his talk with the assertion 'I attribute my success to the fact that I have never been afraid to let my students tell me what a fool I am'. The Russian translation ended 'I attribute my success to the fact that I have never been afraid to tell my students what fools they are'.