Problem Set #5: Ramsey Model with Endogenous Labor and Government

1. Consider the following model of an economy:

\[ Y_t = (A_th_t)^\alpha K_t^{1-\alpha} \]  
Production Function

\[ K_{t+1} = (1 - \delta)K_t + I_t \]  
Capital Accumulation

\[ C_t + I_t = Y_t \]  
Resource Constraint

\[ \sum_{\nu=0}^{\infty} \beta^\nu \left\{ \ln C_t - \frac{\phi \nu}{1 + \nu} h_t^{(1+\nu)/\nu} \right\}, \nu > 0 \]  
Utility of Representative Household

\[ A_t = (1 + x)^t \]  
Growth of Technology

K is capital, h is hours worked per capita, I is investment, C is consumption, and Y is output. For simplicity, we have assumed no population growth, and we have normalized population to unity.

A. Find the first-order conditions and rewrite them (if necessary) to represent an Euler equation for labor supply similar to the one we have used for consumption.

B. Find the conditions that characterize the balanced growth path. Does this utility function generate constant hours worked in steady-state despite growth in technology? Demonstrate.
2. Now consider the following model of an economy:

\[ Y_t = (A h_t)^{\alpha} K_t^{1-\alpha} \]  
\[ K_{t+1} = (1 - \delta)K_t + I_t \]  
\[ C_t + I_t + G_t = Y_t \]

\[ \sum_{i=0}^{\infty} \beta^i \{ \ln(C_t + \chi G_t) + \phi \ln(1 - h_t) \} \]

\( K \) is capital, \( h \) is hours worked per capita, \( I \) is investment, \( C \) is consumption, \( G \) is government spending, and \( Y \) is output. For simplicity, we have assumed constant technology and no population growth. (We normalized the levels of both to be unity.) Assume the government finances its spending with lump-sum taxes.

Suppose that government spending starts at 0 and then increases to \( G' \). Compare the effect on steady-state interest rates, capital-labor ratios, the consumption-output ratio, the investment-output ratio and hours worked for the case where \( \chi = 0 \) versus \( \chi = 1 \). Give economic intuition for your results.