MODELING BEHAVIOR IN NOVEL STRATEGIC SITUATIONS VIA LEVEL-K THINKING

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Introduction

Recent experiments suggest that in strategic settings without clear precedents people often deviate systematically from equilibrium.

The experimental evidence suggests that in such settings a structural non-equilibrium model based on level-$k$ thinking (or as Camerer, Ho, and Chong (2004 QJE; “CHC”) call it, a “cognitive hierarchy” model) can often out-predict equilibrium.

The evidence also suggests that level-$k$ models can out-predict “equilibrium with noise” models with payoff-sensitive error distributions, such as quantal response equilibrium (“QRE”).
The talk begins with a brief introduction to level-

$k$ models and the experimental evidence in support of them.

It then illustrates their application by using level-

$k$ models to resolve several puzzles regarding people’s (usually experimental subjects’) initial responses to novel strategic situations.

The applications illustrate the generality of the level-

$k$ approach and the kinds of adaptations needed to use it in different settings.
Although the level-$k$ approach, like equilibrium, is a general model of strategic behavior, the two are complements, not competitors.

We all believe that equilibrium (or something like it) will emerge in the limit when people have had enough experience from repeated play in stable settings to learn to predict each other’s responses.

But in novel strategic situations, or in stable settings with multiple equilibria, we also need a reliable model of initial responses.

I will argue that level-$k$ models often explain more of the variation in initial responses than equilibrium or QRE, and that they are a tractable and useful modeling tool.
**Level-k models**

Level-k models were introduced to describe experimental data by Stahl and Wilson (1994 *JEBO*, 1995 *GEB*) and Nagel (1995 *AER*), and were further studied experimentally by Ho, Camerer, and Weigelt (1998 *AER*), Costa-Gomes et al. (2001 *ECMA*), Costa-Gomes and Weizsäcker (2005), Costa-Gomes and Crawford (2006 *AER*; “CGC”)

Level-k models allow behavior to be heterogeneous, but assume that each player follows a rule drawn from a common distribution over a particular hierarchy of decision rules or *types* (as they are called)

Type $L_k$ anchors its beliefs in a nonstrategic $L_0$ type and adjusts them via thought-experiments with iterated best responses: $L_1$ best responds to $L_0$, $L_2$ to $L_1$, and so on
$L1$ and higher types have accurate models of the game and are rational in that they choose best responses to beliefs (in many games $Lk$ makes $k$-rationalizable decisions)

$Lk$’s only departure from equilibrium is replacing its assumed perfect model of others with simplified models that avoid the complexity of equilibrium analysis; compare Selten (EER ’98):

“Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties…. Boundedly rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found.”
Alternative specifications of level-$k$ models have been considered:

- Stahl and Wilson have some higher types ("Worldly") best respond to noisy versions of lower types

- CHC have $Lk$ best responding to an estimated mixture of lower types, via a one-parameter Poisson type distribution

My co-authors and I prefer the simpler specification above, which is at least as consistent with the evidence and more tractable, and for which the estimated type distribution is a useful diagnostic)
In applications the type frequencies are treated as behavioral parameters, to be estimated or translated from previous analyses.

The estimated type distribution is fairly stable across games, with most weight on $L_1$, $L_2$, and perhaps $L_3$.

The estimated frequency of the anchoring $L_0$ type is usually small, so $L_0$ exists mainly as $L_1$’s model of others, $L_2$’s model of $L_1$’s, and so on; even so, the specification of $L_0$ is the main issue in defining a level-$k$ model and the key to its explanatory power.

$L_0$ needs to be adapted to the setting, as illustrated below; but the definition of higher types via iterated best responses allows a simple explanation of behavior across different settings.
Experimental evidence for level-k models

Camerer (Behavioral Game Theory, 2003, Chapter 5), CHC (Section IV) and CGC (Introduction, Section II.D) summarize the experimental evidence for level-k models in games with a variety of structures; here I give the flavor of the evidence by summarizing CGC’s results.

CGC’s experiments randomly and anonymously paired subjects to play series of 2-person guessing games, with no feedback; the designs suppress learning and repeated-game effects in order to elicit subjects’ initial responses, game by game.

The goal is to focus on how players model others’ decisions by studying strategic thinking “uncontaminated” by learning (“Eureka!” learning is possible, but can be tested for and is rare).
In CGC’s guessing games each player has his own lower and upper limit, both strictly positive (finite dominance-solvability)

Each player also has his own target, and his payoff increases with the closeness of his guess to his target times the other’s guess

Targets and limits vary independently across players and games, with targets both less than one, both greater than one, or mixed

(In previous guessing experiments, the targets and limits were always the same for both players, and they varied at most across treatments)
CGC’s large strategy spaces and the independent variation of targets and limits across games enhance the separation of types’ implications to the point where many subjects’ types can be precisely identified.

<table>
<thead>
<tr>
<th>Types' guesses in the 16 games, in (randomized) order played</th>
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11
Of the 88 subjects in CGC’s main treatments, 43 made guesses that complied \textit{exactly} (within 0.5) with one type’s guesses in from 7 to 16 of the games (20 L1, 12 L2, 3 L3, and 8 Equilibrium)

(The other 45 subjects made guesses that conformed less closely to a type, but econometric estimates of their types are also concentrated on L1, L2, L3, and Equilibrium, in roughly the same proportions (Table 1))

For example, CGC’s Figure 2 shows the “fingerprints” of the 12 subjects whose guesses conformed most closely to L2’s

72\% of these guesses were exact; only deviations are shown:
CGC’s Figure 2. "Fingerprints" of 12 Apparent L2 Subjects
The size of CGC’s strategy spaces, with from 200 to 800 possible exact guesses per game, and the fact that each subject played 16 different games, makes exact compliance very powerful evidence for the type whose guesses are tracked.

If, say, a subject chooses 525, 650, 900, 546 in games 1-4, we “know” that he’s $L_2$.

Further, because the definition of $L_2$ builds in risk-neutral, self-interested rationality, we also know that the subject’s deviations from equilibrium are “caused” not by irrationality, risk aversion, altruism, spite, or confusion, but by his simplified model of others.
Because $Lk$ makes $k$-rationalizable decisions, it is tempting to take the high frequencies of $Lk$ guesses as evidence that subjects are explicitly performing finitely iterated dominance (this is a very common interpretation of the spikes in Nagel’s (1995) data).

But CGC’s design separates $Lk$ types from the analogous iterated dominance types ($Dk-1$, not separated from $Lk$ in Nagel’s design).

More detailed analysis shows that CGC’s subjects are following $Lk$ types that mimic iterated dominance, not doing iterated dominance.

More detailed analysis also shows that CGC’s subjects whose guesses are closest to equilibrium are actually following types that mimic equilibrium in some of games, not following equilibrium logic.
Applications

Level-k models have now been used to resolve a variety of puzzles:

- CHC: coordination via structure, market-entry games, speculation and zero-sum betting, money illusion in coordination
- Ellingsen and Östling (2006): Aumann’s (1990) critique and why one-sided communication works better in games like Battle of the Sexes but two-sided communication works better in Stag Hunt
- Crawford, Gneezy, and Rottenstreich (2007, not yet available): coordination via focal points based on structure and framing
- Crawford (2007): coordination via preplay communication
This talk gives four illustrations, selected for their economic interest and to illustrate the modeling issues that arise in level-\(k\) analyses:

- CHC’s analysis of “magical” ex post coordination in market-entry games (simple normal-form games with binary choices)
- Crawford and Iriberri’s (2007 *AER*) explanation of systematic deviations from the unique mixed-strategy equilibrium in zero-sum two-person hide-and-seek games (non-neutral framing)
- Crawford and Iriberri’s (2007 *ECMA*) analysis of systematic overbidding in independent-private-value and common-value auctions (incomplete information)
- Crawford’s (2003 *AER*) analysis of preplay communication of intentions in zero-sum games (extensive-form games)
“Magical” coordination in market-entry games

**Puzzle:** Subjects in market-entry experiments (e.g. Rapoport and Seale (2002)) regularly achieve better ex post coordination (number of entrants closer to market capacity) than in the symmetric mixed-strategy equilibrium, the natural benchmark; this led Kahneman (1988, quoted in CHC) to remark, “…to a psychologist, it looks like magic”

**Resolution:** CHC show that the magic can be explained by a level-$k$ model: The heterogeneity of strategic thinking allows more sophisticated players to mentally simulate less sophisticated players’ entry decisions and (approximately) accommodate them. The more sophisticated players behave like Stackelberg followers, breaking the symmetry with coordination benefits for all
The basic idea can be illustrated in a Battle of the Sexes game:

The unique symmetric equilibrium is in mixed strategies, with $p = \Pr\{H\} = \frac{a}{1+a}$ for both players.

The equilibrium expected coordination rate is $2p(1-p) = \frac{2a}{(1+a)^2}$; and players’ payoffs are $a/(1+a) < 1$. 
In the level-\(k\) model, each player follows one of, say, four types, \(L1, L2, L3,\) or \(L4,\) with each role filled from the same distribution.

Assume (as in most previous analyses) that \(L0\) chooses its action uniformly randomly, with \(\Pr\{H\} = \Pr\{D\} = \frac{1}{2}\)

\(L1s\) mentally simulate \(L0s’\) random decisions and best respond, thus choosing \(H;\) \(L2s\) choose \(D,\) \(L3s\) choose \(H,\) and \(L4s\) choose \(D\)

The model’s predicted outcome distribution is determined by the outcomes of the possible type pairings and the type frequencies.
Assume that the frequency of $L0$ is 0, and the type frequencies are independent of player roles and payoffs (as they “should” be).

Players’ level-$k$ ex ante (before knowing type) expected payoffs are equal, proportional to the expected coordination rate.

Combining $L1$ and $L3$ and denoting their total probability $\nu$, the level-$k$ coordination rate is $2\nu(1-\nu)$, maximized when $\nu = \frac{1}{2}$ at $\frac{1}{2}$.
The mixed-strategy equilibrium coordination rate, $2a/(1+a)^2$, is maximized when $a = 1$ at $\frac{1}{2}$, but converges to 0 like $1/a$ as $a \to \infty$

For $v$ near $\frac{1}{2}$, empirically plausible, the level-$k$ coordination rate is higher than the equilibrium rate even for moderate values of $a$, dramatically higher for higher values of $a$

Even though decisions are simultaneous and there is no actual communication, the predictable heterogeneity of strategic thinking allows some players (say $L2s$) to mentally simulate others’ ($L1s$) entry decisions and accommodate them, as Stackelberg followers would (but less accurately, because others’ types are unobserved)
The level-$k$ model yields a view of coordination radically different from the traditional view:

Although players are rational (in the decision-theoretic sense), equilibrium (let alone equilibrium selection principles such as risk- or payoff-dominance) plays no direct role in their strategic thinking.

Coordination, when it occurs, is an almost accidental (though statistically predictable) by-product of non-equilibrium thinking.
Systematic deviations from equilibrium in hide-and-seek games with non-neutrally framed locations


Typical seeker’s instructions (hider’s instructions analogous):

*Your opponent has hidden a prize in one of four boxes arranged in a row. The boxes are marked as shown below: A, B, A, A. Your goal is, of course, to find the prize. His goal is that you will not find it. You are allowed to open only one box. Which box are you going to open?*
RTH’s framing of the hide and seek game is non-neutral in two ways:

- The “B” location is distinguished by its label
- The two “end A” locations may be inherently focal

(This gives the “central A” location its own brand of uniqueness as the “least salient” location—mathematically analogous to the uniqueness of “B” but as we will see, psychologically different)
RTH’s design is important as a tractable abstract model of a non-neutral cultural or geographic frame, or “landscape”

Similar landscapes are common in “folk game theory”:

● “Any government wanting to kill an opponent…would not try it at a meeting with government officials.”
  (comment on the poisoning of Ukrainian presidential candidate—now president—Viktor Yushchenko)
  (The meeting with government officials is analogous to RTH’s B, but there’s nothing in this example analogous to the end locations)

● “…in Lake Wobegon, the correct answer is usually ‘c’.”
  (Garrison Keillor (1997) on multiple-choice tests)
  (With four possible choices arrayed left to right, this example is very close to RTH’s design)
RTH’s design made it into an episode of the CBS series *Numb3rs*, “Assassin” (clip at [http://www.youtube.com/watch?v=HCinK2PUsyk](http://www.youtube.com/watch?v=HCinK2PUsyk)):

Charlie: Hide and seek.

Don: What are you talking about, like the kids’ version?

Charlie: A mathematical approach to it, yes. See, the assassin must hide in order to accomplish his goal, we must seek and find the assassin before he achieves that goal.

Megan: Ah, behavioral game theory, yeah, we studied this at Quantico.

Charlie: I doubt you studied it the way that Rubinstein, Tversky and Heller studied two person constant sum hide and seek with unique mixed strategy equilibria.

Megan: No, not quite that way.

Don: Just bear with him.
Hide-and-seek has a clear equilibrium prediction, which leaves no room for framing to systematically influence the outcome.

Yet in a large sample from around the world, framing has a strong and systematic effect, with Central A most prevalent for hiders (37%) and even more prevalent for seekers (46%).

(The other boxes are chosen roughly equally often in both roles)

Folk game theory also deviates from equilibrium logic: Any game theorist would respond to the Yushchenko quote: “If investigators thought that way, a meeting with government officials is precisely where a government would try to kill an opponent.”
Puzzles:

● Hiders’ and seekers’ responses are unlikely to be completely non-strategic in such simple games. So if they aren’t following equilibrium logic, what are they doing?

● Hiders are as smart as seekers, on average, so hiders tempted to hide in central A should realize that seekers will be just as tempted to look there. Why do hiders allow seekers to find them 32% of the time when they could hold it down to 25% via the equilibrium mixed strategy?

● Further, why do seekers choose central A even more often than hiders? (Although the payoff structure is asymmetric, this asymmetry of choice distributions is not explained by QRE, which coincides with equilibrium in RTH's games.)
Resolution:
A level-k model with a role-independent $L0$ that probabilistically favors salient locations yields a simple explanation:

- Given $L0$'s attraction to salient locations, $L1$ hiders choose *central A* to avoid $L0$ seekers and $L1$ seekers avoid *central A* in searching for $L0$ hiders
- For similar reasons, $L2$ hiders choose *central A* with probability between 0 and 1 and $L2$ seekers choose it with probability 1
- $L3$ hiders avoid *central A* and $L3$ seekers choose it with probability between zero and one
- $L4$ hiders and seekers both avoid *central A*

For plausible type distributions (estimated 19% $L1$, 32% $L2$, 24% $L3$, 25% $L4$—almost hump-shaped), the model explains the prevalence of *central A* for hiders and its greater prevalence for seekers
The role asymmetry in behavior, which (despite the games’ payoff asymmetry) is a mystery from the viewpoint of equilibrium, QRE, or any other theory I am aware of, follows naturally from hiders’ and seekers’ asymmetric responses to L0’s role-symmetric choices.

The analysis suggests that our first epigraph (“Any government wanting to kill an opponent…would not try it at a meeting with government officials”) reflects the reasoning of an L1 poisoner, or equivalently of an L2 investigator reasoning about an L1 poisoner.
Although our empirically based prior about the hump shape and location of the type distribution imposes some discipline, the freedom to specify $L0$ leaves room for doubt about overfitting and portability.

To see if our proposed level-$k$ explanation is more than a “just-so” story, we compare it on the overfitting and portability dimensions with the leading alternatives:

- Equilibrium with intuitive payoff perturbations (salience lowers hiders’ payoffs, other things equal; while salience raises seekers’ payoffs)
- QRE with similar payoff perturbations
- Alternative level-$k$ specifications
We test for overfitting by re-estimating each model separately for each of RTH’s six treatments and using the re-estimated models to “predict” the choice frequencies of the other treatments.

Our favored level-$k$ model has a modest prediction advantage over the alternative models, with mean squared prediction error 18% lower and better predictions in 20 of 30 comparisons.
A more challenging test regards portability, the extent to which a model estimated from subjects’ responses to one game can be extended to predict or explain other subjects’ responses to different games.

We consider the two closest relatives of RTH’s games in the literature:

- O’Neill’s (1987 *PNAS*) famous card-matching game
- Rapoport and Boebel’s (1992 *GEB*) closely related game

These games both raise the same kinds of strategic issues as RTH’s games, but with more complex patterns of wins and losses, different framing, and in the latter case five locations.

We test for portability by using the leading alternative models, estimated from RTH’s data, to “predict” subjects’ initial responses in O’Neill’s and Rapoport and Boebel’s games.
In O’Neill’s game, players simultaneously and independently choose one from four cards: A, 2, 3, J

One player, say the row player (the game was presented to subjects as a story, not a matrix) wins if there is a match on J or a mismatch on A, 2, or 3; the other player wins in the other cases

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<tr>
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<th>A (s)</th>
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<td>J (s)</td>
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O’Neill’s Card-Matching Game

O’Neill’s game is like a hide-and-seek game, except that a player is a hider (h) for some locations and a seeker (s) for others

Even so, it is clear how to adapt L0 or payoff perturbations to the game
A, 2, and 3 are strategically symmetric, and equilibrium (without perturbations) has \( \Pr\{A\} = \Pr\{2\} = \Pr\{3\} = 0.2, \Pr\{J\} = 0.4 \)

Discussions of O’Neill’s data have been dominated by an “Ace effect,” whereby when the data are aggregated over all 105 rounds, row and column players respectively played A 22.0% and 22.6% of the time

(O’Neill speculated that “players were attracted by the powerful connotations of an Ace”)

But it’s difficult (impossible?) to find a behaviorally plausible level-\( k \) model in which row players play A more than the equilibrium 20%
Fortunately, for initial responses it turns out that there is no Ace effect. Instead there is a strong Joker effect, a full order of magnitude larger:

- 8% A, 24% 2, 12% 3, 56% J for rows
- 16% A, 12% 2, 8% 3, 64% J for columns

These frequencies can be gracefully explained by a level-$k$ model in which $L0$ probabilistically favors the salient A and J cards (J’s unique payoff role may make it even more salient than A)

Our analysis suggests that the Ace effect in the aggregated data is due to learning, not salience; if anything is salient, it’s the Joker
Systematic overbidding in experimental independent-private-value and common-value auctions

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<th>Equilibrium predictions</th>
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<td>Independent-Private-Value Auctions</td>
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<td>Common-Value Auctions</td>
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**Puzzle:** Systematic overbidding (relative to equilibrium) has been observed in subjects’ initial responses to all kinds of auctions (Goeree, Holt, and Palfrey (2002 JET), Kagel and Levin (1986 AER, 2000), Avery and Kagel (1997 JEMS), Garvin and Kagel (1994 JEBO))

But the literature has proposed completely different explanations of overbidding for private- and common-value auctions:

- Risk-aversion and/or joy of winning for private-value auctions
- Winner’s curse for common-value auctions
Resolution:

Our level-\(k\) analysis extends Kagel and Levin’s (1986 *AER*) and Holt and Sherman’s (1994 *AER*) analyses of “naïve bidding”

It also builds on Eyster and Rabin’s (2005 *ECMA*; “ER”) analysis of “cursed equilibrium”

The key issue is how to specify \(L_0\); there are two leading possibilities:

- **Random \(L_0\)** bids uniformly on the interval between the lowest and highest possible values (even if over own realized value)
- **Truthful \(L_0\)** bids its expected value conditional on its own signal (meaningful here, but not in all incomplete-information games)

In judging these, bear in mind that they describe only the starting point of a subject’s strategic thinking; we have found it best to make \(L_0\) as dumb as possible, letting higher \(L_k\)s model strategic thinking

The model constructs separate type hierarchies on these \(L_0\)s, and allows each subject to be one of the types, from either hierarchy

(“Random (Truthful) \(L_k\)” is \(L_k\) defined by iterating best responses from Random (Truthful) \(L_0\); not itself random or truthful)
Given a specification of $L0$, the optimal bid must take into account:

- Value adjustment for the information revealed by winning (in common-value auctions only)

- The bidding trade-off between the higher price paid if the bidder wins and the probability of winning (in first-price auctions only)

With regard to value adjustment, Random $L1$ does not condition on winning because Random $L0$ bidders bid randomly, hence independently of their values; Random $L1$ is “fully cursed” (ER’s term).

All other types do condition on winning, in various ways, but this conditioning tends to make bidders’ bids strategic substitutes, in that the higher others’ bids are, the greater the (negative) adjustment.

(Thus, to the extent that Random $L1$ overbids, Random $L2$ tends to underbid (relative to equilibrium): if it’s bad news that you beat equilibrium bidders, it’s even worse news that you beat overbidders)

The bidding tradeoff, by contrast, can go either way.
The question, empirically, is whether the distribution of heterogeneous types’ bids (e.g. a mixture of Random $L1$ overbids and Random $L2$ underbids) fits the data better than the leading alternatives.

In three of the four leading cases, a level-$k$ model has an advantage over equilibrium, cursed equilibrium, and/or QRE.

For the remaining case (Kagel and Levin’s first-price auction), the most flexible specification of cursed equilibrium has a small advantage.

Except in Kagel and Levin’s second-price auctions, the estimated type frequencies are similar to those found in other experiments:

Random and Truthful $L0$ have low or zero estimated frequencies, and the most common types are Random $L1$, Truthful $L1$, Random $L2$, and sometimes Equilibrium or Truthful $L2$.

(With independent private values, most of the examples that have been studied experimentally do not separate level-$k$ from equilibrium bidding strategies, hence our choice to study GHP’s results.)
The level-$k$ analysis accomplishes several things:

- Provides a more unified explanation for systematic patterns of non-equilibrium bidding in private and common-value auctions.
- Explores how to extend level-$k$ models to an important class of incomplete-information games.
- Explores the robustness of equilibrium auction theory to failures of the equilibrium assumption.
- Links experiments on auctions and on strategic thinking.
Preplay communication of intentions in zero-sum games

Consider a simple perturbed matching pennies game, viewed as a model of the Allies’ choice of where to invade Europe on D-Day

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<tr>
<td>Normandy</td>
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- Attacking an undefended Calais (closer to England) is better for the Allies than attacking an undefended Normandy, and so better for the Allies on average.

- Defending an unattacked Normandy is worse for the Germans than defending an unattacked Calais, and so worse for the Germans on average.
Now imagine that D-Day is preceded by a message from the Allies to the Germans regarding their intentions about where to attack.

Imagine that the message is (approximately!) cheap talk.

An Inflatable “Tank” from Operation Fortitude
Puzzle: In an equilibrium analysis of a zero-sum game preceded by a cheap-talk message regarding intentions, the sender must make his message uninformative, and the receiver must ignore it. Thus the underlying game must be played according to its mixed-strategy equilibrium, and communication can have no effect.

Yet intuition suggests that in many such situations:

- The sender’s message and action are part of a single, integrated strategy
- The sender tries to anticipate which message will fool the receiver and chooses it nonrandomly
- The sender’s action differs from what he would have chosen with no opportunity to send a message

Moreover, in my highly stylized version of D-Day:

- The deception succeeded (the Allies faked preparations for invasion at Calais, the Germans defended Calais and left Normandy lightly defended, and the Allies then invaded Normandy)
- But the sender won in the less beneficial of the two possible ways
Admittedly, D-Day is only one datapoint (if that)....

But there’s an ancient Chinese antecedent of D-Day, Huarongdao, in which General Cao Cao chooses between two roads, trying to avoid capture by General Kongming (thanks to Duozhe Li of CUHK for the reference to Luo Guanzhong’s historical novel, *Three Kingdoms*).

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<td><strong>Huarong</strong></td>
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- Cao Cao loses 2 and Kongming gains 2 if Cao Cao is captured.
- Both Cao Cao and Kongming gain 1 by taking the Main Road, whether or not Cao Cao is captured—it’s important to be comfortable, even if (especially if?) if you think you’re about to die.
In Huarongdao, essentially the same thing happened as in D-Day: Kongming lit campfires on the Huarong road; Cao Cao was fooled by this into thinking Kongming would wait for him on the *Main Road*; and Kongming captured Cao Cao, but only by taking the bad Huarong road (The ending was happy: Kongming later let Cao Cao go)

In what sense did the “essentially the same thing” happen?

In D-Day the message was literally deceptive but the Germans were fooled because they “believed” it (either because they were credulous or because they inverted it one too many times)

Kongming's message was literally truthful—he lit fires on the Huarong Road and ambushed Cao Cao there—but Cao Cao was fooled because he inverted it

Although the sender’s and receiver’s message strategies and beliefs were different, the end result—what happened in the underlying game—was the same: The sender won, but in the less beneficial way
Why was Cao Cao fooled by Kongming’s message?

One advantage of using fiction as data (aside from not needing human subjects approval) is that it can reveal cognition without eye-tracking:

● *Three Kingdoms* gives Kongming’s rationale for sending a deceptively truthful message: “Have you forgotten the tactic of ‘letting weak points look weak and strong points look strong’?”

● It also gives Cao Cao's rationale for inverting Kongming’s message: “Don’t you know what the military texts say? ‘A show of force is best where you are weak. Where strong, feign weakness.’ ”

Cao Cao must have bought a used, out-of-date edition….

(Cao Cao’s rationale resembles *L1* thinking, in that it assumes that the sender assumes that his message will be taken at face value

But Kongming’s rationale resembles *L2 thinking*)
We can now restate the puzzle more concretely, for both examples:

- Why did the receiver allow himself to be fooled by a costless (hence easily faked) message from an *enemy*?

- If the sender expected his message to fool the receiver, why didn't he reverse it and fool the receiver in the way that would have allowed him to win in the *more* beneficial way?

(Why didn't the Allies feint at Normandy and attack at Calais? Why didn't Kongming light fires and ambush Cao Cao on the main road?)

Was it a coincidence that the same thing happened in both cases?
A level-$k$ analysis suggests that it was more than a coincidence

Assume that Allies’ and Germans’ types are drawn from separate distributions, including both boundedly rational, or Mortal, types and a strategically rational, or Sophisticated, type (interesting but rare)

Sophisticated types know everything about the game, including the distribution of Mortal types; and play equilibrium in a “reduced game” between Sophisticated players, taking Mortals' choices as given

Mortal types, like other boundedly rational types, use step-by-step procedures that generically determine unique, pure strategies, avoid simultaneous determination of the kind used to define equilibrium
Mortal types’ behaviors regarding the message are anchored on analogs of \( L_0 \) based on truthfulness or credulity, as in the informal literature on deception.

\( L_1 \) or higher Mortal Allied types always expect to fool the Germans, either by lying (like the Allies) or by telling the truth (like Kongming); given this, all such Allied types send a message they expect to make the Germans think they will attack Normandy; and then attack Calais.

If we knew the Allies and Germans were Mortal, we could now derive the model’s implications from an estimate of type frequencies.

But the analysis can usefully be extended to allow the possibility of Sophisticated Allies and Germans.
To do this, note first that *Mortals*’ strategies are determined independently of each other’s and *Sophisticated* players’ strategies, and so can be treated as exogenous (but they affect others’ payoffs).

Then plug in the distributions of *Mortal* Allies’ and Germans’ independently determined behavior to obtain a “reduced game” between possibly *Sophisticated* Allies and Germans.

Because *Sophisticated* players’ payoffs are influenced by *Mortal* players’ decisions, the reduced game is no longer zero-sum, its messages are not cheap talk, and it has incomplete information (The sender’s message, which is ostensibly about his intentions, is in fact read by a *Sophisticated* receiver as a signal of the sender’s type).
The equilibria of the reduced game are determined by the population frequencies of Mortal and Sophisticated senders and receivers. There are two leading cases, with different implications:

- When Sophisticated Allies and Germans are common—not that plausible—the reduced game has a mixed-strategy equilibrium whose outcome is virtually equivalent to D-Day’s without communication.

- When Sophisticated Allies and Germans are rare, the game has an essentially unique pure equilibrium, in which Sophisticated Allies can predict Sophisticated Germans’ action, and vice versa. In this equilibrium, Sophisticated Allies send the message that fools the most common kind of Mortal German (depending on how many believe messages or, like Cao Cao, invert them) and attack Normandy, while Sophisticated Germans defend Calais (because they know that Mortal Allies, who predominate in this case, will attack Calais).

(For more subtle reasons, there is no pure-strategy equilibrium in which Sophisticated Allies feint at Normandy and attack Calais.)
In the pure-strategy equilibrium, the Allies’ message and action are part of a single, integrated strategy; and the probability of attacking Normandy is much higher than if no communication was possible.

The Allies choose their message nonrandomly, the deception succeeds most of the time, but it allows the Allies to win in the less beneficial of the possible ways.

Thus for plausible parameter values, without postulating an unexplained difference in the sophistication of Allies and Germans, the model explains why even *Sophisticated* Germans might allow themselves to be “fooled” by a costless message from an enemy.

In a weaker sense (resting on a preference for pure-strategy equilibria and high-probability predictions), the model also explains why *Sophisticated* Allies don’t feint at Normandy and attack Calais, even though this would be more profitable if it succeeded.