A Level-*k* Model for Games with Asymmetric Information

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Level-*k* models

In a level-*k* model people follow rules of thumb that:

- Anchor their beliefs in a naïve model of others' response to the game, called *L0*, often uniform random over feasible decisions;

and

- Adjust their beliefs via a small number (*k*) of iterated best responses, so *L1* best responds to *L0*, *L2* to *L1*, and so on.

People's levels are usually heterogeneous, and the population level frequencies are treated as behavioral parameters and either estimated from the data or calibrated from previous estimates.

Estimates vary with the setting and population, but normally the estimated frequency of *L0* is small or zero and the distribution of levels is concentrated on *L1*, *L2*, and *L3*.

- *Lk* (for $k > 0$) is decision-theoretically rational, with an accurate model of the game; it departs from equilibrium only in deriving its beliefs from an oversimplified model of others' responses.

- *Lk* (for $k > 0$) respects *k*-rationalizability (Bernheim 1984 *ECMA*), hence in two-person games its decisions survive *k* rounds of iterated elimination of strictly dominated strategies.

- Thus *Lk* mimics equilibrium decisions in *k*-dominance-solvable games, but may deviate systematically in more complex games.

- A level-*k* model (with zero weight on *L0*) can be viewed as a heterogeneity-tolerant refinement of *k*-rationalizability.

- But unlike *k*-rationalizability, a level-*k* model makes precise predictions, given the population level frequencies: not only that deviations from equilibrium will sometimes occur, but also which settings evoke them and which forms they are likely to take.

A level-*k* model for direct games with asymmetric information

- Following Camerer, Ho, and Chong (2004 *QJE*) and Crawford and Iriberri (2007 *ECMA*), I take *L0*'s decisions to be uniform over the feasible decisions, and *independent of its own value*.

- One can imagine more refined specifications, e.g. with an *L0* buyer's bid (seller's ask) uniform below (above) its value instead of over the entire range, thus eliminating dominated strategies.

- But *L0* is not an actual player: It is a player's naïve model of other players—others whose values he does not observe.

- It is logically possible that *Lk* players initially reason contingent on others' possible values, but behaviorally far-fetched.

- A level-*k* model with *L0* uniform over the feasible decisions and *independent of own value* captures people's aversion to fixed-point and complex contingent reasoning in a tractable way.

This extended level-*k* model has a long history:

> "Son…One of these days in your travels, a guy is going to show you a brand-new deck of cards on which the seal is not yet broken. Then this guy is going to offer to bet you that he can make the jack of spades jump out of this brand-new deck of cards and squirt cider in your ear. But, son, do not accept this bet, because as sure as you stand there, you're going to wind up with an ear full of cider."
>
> —Obadiah ("The Sky") Masterson, quoting his father in Damon Runyon (*Guys and Dolls: The Stories of Damon Runyon*, 1932)

Here, Dad is worried that Son, while decision-theoretically rational, will stick with his prior in the face of an offer that is "too good to be true": just as an *L1* does in this extended level-*k* model.

Milgrom and Stokey's (1982 *JET*) "No-Trade Theorem" shows that if traders in an asset market start out in market equilibrium—Pareto-efficient, given their information—then giving them new information, fundamentals unchanged, cannot lead to new trades.

For, any such trades would make it common knowledge that both had benefited, contradicting the efficiency of the initial equilibrium.

This result has been called the Groucho Marx Theorem:

> "I sent the club a wire stating, 'Please accept my resignation. I don't want to belong to any club that will accept people like me as a member'."
>
> —Groucho Marx, Telegram to the Beverly Hills Friars' Club

In speculating on why zero-sum trades occur despite the theorem, Milgrom and Stokey contrast Groucho's equilibrium inference with their rules Naïve Behavior, which sticks with its prior but otherwise behaves rationally, just as this model's *L1* does; and First-Order Sophistication, which best responds to Naïve Behavior, just as this model's *L2* does.

There is a growing body of evidence that this extended level-*k* model gives a realistic account of the main patterns of people's strategic thinking and "informational naiveté", their failure to attend to how others' incentives depend on their private information.

Crawford and Iriberri (2007 *ECMA*) showed that the model gives a coherent account of subjects' overbidding and vulnerability to the winner's curse in initial responses in classic auction experiments.

Brown, Camerer, and Lovallo (2012 *AEJ Micro*) use the model to explain film-goers' failure to draw negative inferences from studios' withholding weak movies from critics before release.

Brocas, Carillo, Camerer, and Wang (2014 *REStud*; see also Camerer et al. 2004 *QJE*) report powerful experimental evidence for this level-*k* model from three-state betting games (~zero-sum):

<table>
<tr><th>player/state</th><th>A</th><th>B</th><th>C</th></tr>
<tr><td>1</td><td colspan="2">25 5</td><td>20</td></tr>
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player/state	**A**	**B**	**C**
1	25	5	20
2	0	30	5

There are three ex ante equally likely states, A, B, C.

Player 1 privately learns either that the state is {A or B} or that it is C; simultaneously, player 2 privately learns either that the state is A or that it is {B or C}.

Players then simultaneously choose to Bet or Pass.

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Players then simultaneously choose to Bet or Pass.

A player who chooses Pass, or who chooses Bet while the other chooses Pass, earns 10 in any state.

If both players choose Bet, they get their respective payoffs in the table for whichever state occurs.

All this is publicly announced (to induce common knowledge).

The betting game has a unique trembling-hand perfect Bayesian equilibrium, identifiable via iterated weak dominance. (There's also an imperfect equilibrium in which both players always Pass.)

Round 1 (Bet, Pass):

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Round 2:

player/state	A	B	C
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1	25	5	20
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Round 2:

player/state	A	B	C
1	25	5	20
2	0	30	5

Round 3:

player/state	A	B	C
1	25	5	20
2	0	30	5

In equilibrium, no betting takes place in any state (although player 1 is willing to bet in state C).

Despite this clear equilibrium prediction, half of Brocas et al.'s subjects Bet, in patterns that varied systematically with the player role and state (as in several similar previous experiments).

L1 respects simple dominance:

player/state	**A**	**B**	**C**
1	25	5	20
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But if all subjects were *L1s*, 100% of player 1s and 67% of player 2s would be willing to bet, many more than in Brocas et al.'s data.

Further, 100% of subjects would be willing to bet in states B and C, which is also not true in Brocas et al.'s data.

However, *L2* respects two rounds of iterated weak dominance:

player/state	**A**	**B**	**C**
1	25	5	20
2	0	30	5

And *L3* respects three rounds of iterated weak dominance (= trembling-hand-perfect equilibrium in this 3-dominance-solvable game):

player/state	**A**	**B**	**C**
1	25	5	20
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player/state	**A**	**B**	**C**
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And *L3* respects three rounds of iterated weak dominance (= trembling-hand-perfect equilibrium in this 3-dominance-solvable game):

player/state	**A**	**B**	**C**
1	25	5	20
2	0	30	5

Brocas et al. find clusters of subjects whose behavior corresponds to each of *L1*, *L2*, and *L3*; and also a cluster of “irrational” players.

The level-*k* model fits subjects’ decisions (and information searches) better than equilibrium or any homogeneous model.