# Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications

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**Abstract:** Most applications of game theory assume equilibrium, justified by presuming either that learning will have converged to one, or that equilibrium approximates people's strategic thinking even when a learning justification is implausible. Yet several recent experimental and empirical studies suggest that people's initial responses to games often deviate systematically from equilibrium, and that structural nonequilibrium "level-*k*" or "cognitive hierarchy" models often out-predict equilibrium. Even when learning is possible and converges to equilibrium, such models allow better predictions of history-dependent limiting outcomes. This paper surveys recent theory and evidence on strategic thinking and illustrates the applications of level-*k* models in economics.

Keywords: behavioral game theory, experimental game theory, strategic thinking, level-*k* models, cognitive hierarchy models, Nash equilibrium, quantal response equilibrium, coordination, salience, strategic communication
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## 1. *Introduction*

Strategic thinking pervades human interaction. As soon as children develop enough "theory of mind" to model other people as independent decision makers, they must be taught to look both ways before crossing one-way streets—suggesting that they instinctively assume rationality when predicting others' decisions.[2] Our adult attempts to predict other people's responses to incentives are shaped by similar, though usually more subtle, rationality-based inferences.

The canonical model of strategic thinking is the game-theoretic notion of Nash equilibrium. Equilibrium is defined as a combination of strategies, one for each player, such that each player's strategy maximizes his expected payoff, given the others' strategies. Although this definition can be applied without reference to its rationale, for the purpose of modeling thinking equilibrium is best viewed as an "equilibrium in beliefs," in which players who are rational in the decision-theoretic sense have beliefs about each other's strategies that are correct, given the rational choices they imply. Rationality plus this "rational-expectations" assumption yields much more precise predictions than rationality alone, which often give a plausible and empirically reliable account of strategic behavior.[3] The precision, generality, and tractability of equilibrium analysis have made it the method of choice in strategic applications (Roger B. Myerson 1999).

However, equilibrium is better justified in some applications than others. If players have enough experience with analogous games, both theory and experimental results suggest that learning has a strong tendency to converge to equilibrium.[4] But in many applications players' interactions have only imperfect precedents, or none at all. If equilibrium is justified in such applications, it must be via strategic thinking rather than learning.[5]

Epistemic game theory gives conditions under which thinking can focus players' beliefs on an equilibrium even in their initial responses to a game. But in many games the required reasoning is too complex for a thinking justification to be behaviorally plausible.[6] The signs on trucks that say "If you can't see my mirrors, I can't see you" are a symptom of the fact that—far

[2] In this case their reliance on rationality is excessive, which is why adults have something to teach them. This example originally appeared in Camerer (2003, Chapter 1), courtesy of one of the authors.
[3] With rational expectations, even common knowledge of rationality implies only that players' strategies are rationalizable (Bernheim 1984 and David Pearce 1984), which in many games leaves behavior completely unrestricted (Section 2.2).
[4] Our statement omits some qualifications that are important only for extensive-form games.
[5] Although equilibrium is normally viewed as a model of strategy choices without reference to thinking, strategic behavior that is not shaped by learning from experience must, if it has any structure at all, reflect some kind of strategic thinking. Behavior that is not shaped by equilibrium thinking, for example, will in general track equilibrium outcome predictions only by chance.
[6] E.g. Adam Brandenburger (1992). Even in high-stakes settings where participants hire consultants, the epistemic justification of equilibrium requires at least mutual knowledge that all will follow equilibrium logic, which remains empirically questionable.

from always following equilibrium logic—we sometimes need to be reminded of the importance of considering others' cognition at all (though not, we presume, of the laws of optics).

In this paper we argue that it is often possible to improve upon equilibrium models of initial responses to games, and that better models of strategic thinking allow more useful applications. The potential value of better models is clear in applications to games without clear precedents. But such models can help even when it is plausible that learning has long since converged to an equilibrium. In applications with multiple equilibria, an equilibrium is often selected via learning dynamics for which the influence of initial responses persists indefinitely (Crawford 1995; Camerer 2003, Chapters 1 and 6).[7] And in other applications initial responses are important for their own sake, as in the FCC spectrum auction (R. Preston McAfee and John McMillan 1996).

Even researchers who grant the potential value of improving on equilibrium models of initial responses may doubt its feasibility. How can any model systematically out-predict a rational-expectations notion such as equilibrium? And how can one identify better models among the huge number of logical possibilities? We suspect that analysts sometimes assume equilibrium despite weak justification, or overestimate the scope of learning, because they hope equilibrium will still be correct on average, or fear that without equilibrium there can be no basis for analysis.

There is now a large body of experimental research to suggest that neither the hope nor the fear is justified.[8] That research shows that subjects' thinking in initial responses to games tends to avoid the fixed-point or indefinitely iterated dominance reasoning that equilibrium often requires.[9] In many games this makes their decisions deviate systematically from equilibrium.

The deviations have a large structural component, which favors rules of thumb that anchor beliefs in a strategically naïve initial assessment of others' likely responses called "level-0" or "*L0*" and then adjust them via iterated best responses, so that *L1* best responds to *L0*, *L2* to *L1*, and so on. People's rules are heterogeneous, with levels of adjustment—or "types" as they are called (no relation to private-information variables)—drawn from a distribution concentrated on

[7] Better models can also elucidate the structure of learning rules, where cognition determines which analogies between current and previous games players recognize and distinguishes reinforcement from beliefs-based and more sophisticated rules.
[8] Most empirical work in economics relies on observational data from field settings, which we discuss whenever possible. But theories of strategic behavior are notoriously sensitive to the details of the environment, and the control modern experimental methods allow often gives laboratory experiments a decisive advantage in testing such theories.
[9] As Reinhard Selten (1998) put it, "Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties…. Boundedly rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found." This does not mean learning cannot converge to something that an analyst would need fixed-point reasoning to characterize; just that such reasoning does not directly describe people's thinking.

one to three iterations.[10] The resulting "level-*k*" (Nagel 1995; Dale O. Stahl and Paul Wilson 1994, 1995, "SW" (whose term is "level-*n*"); Costa-Gomes, Crawford, and Bruno Broseta 2001, "CGCB"; Costa-Gomes and Crawford 2006, "CGC") models or the closely related "cognitive hierarchy" ("CH") model (Camerer, Ho, and Juin Kuan Chong 2004, "CHC") share the generality and much of the tractability of equilibrium analysis, but often out-predict equilibrium.

In a level-*k* model (from now on we will use "level-*k*" to include CH models, except when the distinction is important), players' types are rational in the sense of best-responding to some beliefs; they depart from equilibrium only in that the beliefs are based on simple nonequilibrium models of others. Type $k$ for $k > 1$ makes decisions that are not strictly dominated; and a level-*k* type $k$ (but not always a CH type $k$ for $k > 1$) respects *k*-rationalizability (Bernheim 1984), making decisions that in two-person games survive $k$ rounds of iterated deletion of strictly dominated strategies, though without explicitly performing iterated dominance.

As a result, in simple games the low-level types that describe most people's behavior often mimic equilibrium decisions, even though their thinking differs from equilibrium thinking. In such games a level-*k* analysis can establish the robustness of equilibrium predictions. But in more complex games level-*k* types may deviate from equilibrium, and a level-*k* analysis can then resolve empirical puzzles by explaining the systematic part of observed deviations. Importantly, level-*k* models not only predict that deviations sometimes occur; they also predict which settings evoke them; the forms they take; and given estimated type frequencies, their likely frequencies.

This paper reviews theoretical, experimental, and empirical research on strategic thinking, focusing mainly on level-*k* models. Our goals are to summarize and evaluate the evidence for such models, describe their theoretical properties, and illustrate their uses in applications to settings involving novel or complex games in which assuming equilibrium is not well justified.

Our premise is not that level-*k* models describe all or most of people's deviations from equilibrium: They stop short of that goal, although there is some evidence that the deviations they do *not* describe lack readily identifiable structure. Instead we are motivated by the scope and importance of the phenomena in applications that resist equilibrium explanation, and the fact that simple, tractable models seem capable of explaining a substantial part of them. To the extent that such applications are better served by level-*k* models, they deserve a place in the toolkit.

[10] In applications, the behavioral parameters that describe this distribution are usually estimated from the data or calibrated using previous estimates. Although estimates vary somewhat across settings and populations, in most applications a stable distribution that puts significant probability only on the lowest levels captures most deviations from equilibrium (Section 3).

We also discuss informal evidence from “folk game theory”. Our term (see also Michael Suk-Young Chwe 2011) is meant to suggest an analogy with folk physics, untrained people’s intuitive beliefs about the laws of physics. Why study folk instead of “real” game theory? Folk physics imperfectly reflects real physics, but yields insight into human cognition. Folk game theory imperfectly reflects traditional game theory, but yields insight into behavioral game theory, its empirical counterpart. As will be seen, folk game theory vividly illustrates the need for nonequilibrium models of strategic thinking and provides further support for level-*k* models.

For simplicity, we assume throughout that players have accurate models of the game and that their strategies are rational responses to some beliefs, except for errors. We also focus on normal-form games, except when we study communication. The paper is organized as follows.

Section 2 reviews the leading models of strategic thinking: equilibrium; *k*-rationalizability and finitely iterated dominance (Bernheim 1984 and Pearce 1984); quantal response equilibrium (“QRE”; Richard S. McKelvey and Thomas R. Palfrey 1995, “MP”); and level-*k* or CH models.

Section 3 reviews experimental evidence on strategic thinking in symmetric-information games. We begin with guessing games in the style of John Maynard Keynes’ (1936) beauty contest example (Nagel 1995; Ho et al. 1998, “HCW”; Antoni Bosch-Domènech et al. 2002, “BGNS”) and continue with evidence from other normal-form and guessing games (SW 1994, 1995; CGCB; CGC; Costa-Gomes and Georg Weizsäcker 2008, “CGW”). In this literature, *L0* is usually assumed to be uniform random over others’ possible decisions, as a way of capturing the strategically naïve assessments of others’ likely responses that anchor *L1*’s and, indirectly, higher types’ beliefs. The evidence from the above and other papers generally supports level-*k* models in which players anchor beliefs in a uniform random *L0*.[11] But more work is needed to evaluate the models’ domains of applicability, portability, and stability of parameter estimates across types of games; specification testing; and testing for overfitting. Section 3 concludes by reviewing existing evidence on those questions and highlighting directions for future work.

Section 4 illustrates the mechanics of level-*k* models in a simple symmetric-information “outguessing” game from folk game theory. The game has a unique mixed-strategy equilibrium, and the main strategic issue is how to respond to payoff asymmetry. The heterogeneity of

[11] The general principle that a level-*k* player anchors his beliefs in a strategically naïve assessment of others’ likely responses has been adapted to other classes of games in two alternative ways: In settings where salient decision labels have an important influence, *L0* is allowed to favor salient decisions; and in games with communication via “cheap talk”, *L0* is allowed to favor literal interpretations of messages. These adaptations are motivated and discussed in Sections 8 and 9.

players' level-*k* thinking gives a coherent account of strategic uncertainty, while avoiding equilibrium's unrealistic comparative statics implications in such games. This application also concretely motivates the uniform random specification of *L0* used in most applications.

Sections 5 through 9 illustrate the use of level-*k* models in a variety of applications for which equilibrium analysis does not always give an adequate account of behavior.

Section 5 considers games with symmetric information, using a level-*k* model that extends the uniform random specification of *L0* to require it to be independent of the realizations of others' private information. The model is used to analyze the results of experiments on zero-sum betting (CHC; Isabelle Brocas et al. 2010) and auctions with private information (Crawford and Iriberri 2007a, "CI"). It gives a unified account of the informational naïveté often observed with asymmetric information—people's failure to consider how other's responses depend on their private information, as in the winner's curse—and other aspects of nonequilibrium strategic thinking that parallel those in games with symmetric information. Section 5 next discusses CH analyses of field data with asymmetric information (Alexander Brown et al. 2010, Robert Östling et al. 2011). It concludes by discussing theoretical applications of level-*k* models to the design of optimal auctions (Crawford et al. 2009) and efficient bargaining mechanisms (Crawford 2012).

Section 6 considers symmetric-information market-entry games, where the main strategic issue is coordination via symmetry-breaking, which is important in many other applications. In Daniel Kahneman's (1998) and Amnon Rapoport and Darryl A. Seale's (2002) experimental results, subjects' aggregate choice frequencies came surprisingly close to the symmetric mixed-strategy equilibrium—a result Kahneman (quoted in CHC; see also Section 6) called "magic". Perhaps more surprisingly, subjects' ex post coordination was systematically better than in the symmetric equilibrium. Following CHC's (Section III.C) CH analysis, we use a level-*k* model to analyze the simplest possible entry game, Battle of the Sexes. A level-*k* (or CH) model resolves both puzzles, and suggests an alternative to the traditional game-theoretic view of coordination that is behaviorally more plausible and has important implications in other applications. Section 6 concludes with discussions of Avi Goldfarb and Botao Yang's (2009) and Goldfarb and Mo Xiao's (2011) CH empirical analyses of market entry with asymmetric information in the field.

Section 7 continues the level-*k* analysis of coordination in Stag Hunt-style games like those in Douglas W. Diamond and Philip H. Dybvig's (1983) classic model of bank runs. Those games have multiple symmetric, Pareto-ranked equilibria. In the Pareto-superior equilibrium players' payoffs are more vulnerable to deviations by others; and accordingly, the main strategic issue is the "assurance" needed to support that equilibrium. The workhorse model of equilibrium selection in such games has been global games analysis (Stephen Morris and Hyun Song Shin 1998; David M. Frankel et al. 2003), which replaces the original game with a payoff-perturbed version in which iterated dominance selects a unique equilibrium. In the simplest such games the equilibrium selected is the risk-dominant one (John C. Harsanyi and Selten 1987); and a global games analysis is widely believed to strengthen the argument for that conclusion. Section 7 argues that a level-*k* analysis gives stronger behavioral foundations for that conclusion in the simplest Stag Hunt-style games, but may yield different conclusions in more complex games.

Section 8 discusses work on coordination and outguessing games with salient labels. Because the labeling of players and strategies does not affect payoffs, it is traditionally excluded from consideration in equilibrium analysis. But it would be surprising if behavior did not respond to salient labels, and they influenced behavior in Thomas C. Schelling's (1960) classic experiments with coordination games and in Crawford et al.'s (2008) and Nicholas Bardsley et al.'s (2010) experiments revisiting Schelling's. In their main treatments Crawford et al. replicated Schelling's finding that subjects can use salient labels to coordinate with high frequency for games in which players' payoffs are identical. But they found that even slight payoff differences create a player-role-asymmetric tension between label salience and the inherent salience of higher own payoffs, which interferes with the use of labels to coordinate. Bardsley et al. replicated many of Schelling's findings in different settings, while also finding some puzzling results. And in Ariel Rubinstein and Amos Tversky's ("RT", Rubinstein 1999) experiments on zero-sum two-person hide-and-seek games played on non-neutral "landscapes" of salient location labels, subjects deviated systematically from the unique mixed-strategy equilibrium in patterns that respond to labeling, even though the essential uniqueness of equilibrium seems to preclude such influence.

To explain RT's puzzling hide-and-seek results, CI (2007b) proposed a level-*k* model in which *L0*'s strategically naïve initial assessment of others' likely responses deviates from the uniform random specification by favoring salient decision labels, following the same principles in either player role. Crawford et al. proposed a similar level-*k* model to describe some of their coordination results, in which *L0* now responds to payoff salience as well as label salience.

By contrast, some of Crawford et al.'s and Bardsley et al.'s other results appear to reflect a notion Bardsley et al. (p. 40) call "team reasoning", whereby "each player chooses the decision rule which, if used by all players, would be optimal for each of them". Section 8 concludes with a discussion of directions for future work to identify the ranges of applicability of level-*k* and team reasoning models, and to further explore the foundations of level-*k* models with salience.

Section 9 considers level-*k* models of strategic communication that has no direct payoff consequences, or cheap talk. Equilibrium analysis as in Crawford and Sobel (1982) misses important features of real communication via natural language. The fact that the receiver has rational expectations about the sender's motives implies that in two-person games of pure conflict with known preferences, cheap talk messages must be uninformative, and must be ignored. And the fact that messages do not directly affect payoffs precludes any role for their literal meanings. Yet deceptive messages are common in real conflicts, and sometimes successful; and messages' literal meanings play a prominent role in how they are interpreted.

Crawford (2003) introduced a level-*k* model of one-sided preplay communication in a two-person game of pure conflict. Here it would be behaviorally odd if a player's assessment of the meaning of a message did not start with its literal interpretation, and Crawford accordingly assumed that *L0* is truthful for senders and credulous for receivers. Crawford also introduced the possibility that with given probabilities, some players in each role are *Sophisticated* and play (Bayesian) equilibrium strategies in a game in which both they and their possibly *Sophisticated* partners take into account the likelihoods that senders and receivers are level-*k* or *Sophisticated*. The model gives a richer and more realistic account of real communication, in which, depending on the population frequencies, *Sophisticated* players may gain from exploiting level-*k* players, and *Sophisticated* senders can sometimes deceive even *Sophisticated* receivers.

Section 9 next discusses Ellingsen and Östling's (2010) and Crawford's (2007) level-*k* analyses of communication of intentions in coordination and other games. They use similar level-*k* models to elucidate long-standing puzzles about how the effectiveness of communication varies with its structure and with the payoff structure in experiments and presumably in the field.

Section 9 next considers Wang et al.'s (2010) experimental analysis of communication of private information in sender-receiver games with partially conflicting preferences. In such games Crawford and Sobel (1982) showed that cheap talk can convey information in equilibrium but that information transmission must be noisy; and that more information transmitted in the most informative equilibrium, the closer are players' preferences. Wang et al. found, as in previous experiments, that most senders exaggerate the truth in the direction that would, if believed, move receivers toward senders' ideal action. Despite senders' exaggeration, their messages contain some information, measured by the correlation between *S* and *A*; and most receivers are credulous, responding to the sender's message even more than they should. But in spite of those deviations from equilibrium, Wang et al.'s results support the equilibrium-based comparative-statics prediction that more information will be transmitted, the closer the sender's and receiver's preferences. Wang et al.'s analysis of subjects' decisions and information searches gives strong support to Crawford's (2003) level-*k* model, and reconciles subjects' non-equilibrium behavior with the validation of the equilibrium-based comparative-statics prediction.

Section 9 concludes with discussions of Ulrike Malmendier and Devin Shanthikumar's (2007, 2009) CH empirical analyses of the interaction between stock analysts and traders.

Section 10 is the conclusion.

## 2. *Theoretical Models of Strategic Thinking*

This section reviews the leading models of strategic thinking: equilibrium plus noise, *k*-rationalizability and finitely iterated strict dominance, and QRE; followed by level-*k* and CH models, which are the primary focus. Like equilibrium, the alternatives are general models, applicable to any game; and can be viewed as models of thinking as well as decisions.

### 2.1. *Equilibrium plus Noise*

Any notion that is to be taken to data must allow for errors. The obvious way to do so in an equilibrium analysis, equilibrium plus noise, simply adds errors to equilibrium predictions. The errors are usually assumed to have a given distribution with zero mean and estimated precision, with likelihoods sensitive to the payoff costs of deviations as in the logit distribution. The resulting model resembles QRE (Section 2.3) in allowing cost-sensitive errors; but it is unlike QRE in that the costs are evaluated assuming that others play equilibrium strategies exactly.

Except in the simplest games, a player can only find his equilibrium decision via fixed-point or indefinitely iterated dominance reasoning. The experimental evidence (Section 3) suggests that the more complex that reasoning, the less likely it is to directly describe people's thinking.

In games with multiple equilibria, equilibrium plus noise is *incomplete* in that it does not specify a unique prediction conditional on the values of its behavioral parameters. This has been dealt with by estimating an unrestricted probability distribution over equilibria, but in our view it is usually preferable to complete the model by adding a refinement such as risk- or payoff-dominance (Harsanyi and Selten 1987).[12] That makes equilibrium yield predictions specific enough to be useful, and puts it on an equal footing with other models of strategic thinking.

In many applications equilibrium plus noise fits subjects' initial responses well. But in others, even if equilibrium is unique, initial responses deviate systematically from equilibrium, in ways that are sensitive to a subject's out-of-equilibrium payoffs not only when others play their equilibrium strategies but also when they do not. QRE, level-*k*, and CH models all attempt to account for the sensitivity of such deviations, in different ways.

2.2. *Finitely Iterated Strict Dominance and* k-*Rationalizability*

A common reaction to implausibility of equilibrium's thinking justification is to maintain some or all of its reliance on rationality and iterated knowledge of rationality, while relaxing its strong rational-expectations assumption. This yields the notions of rationalizability and *k*-rationalizability (Bernheim 1984 and Pearce 1984). *k*-rationalizability reflects the implications of finite levels of iterated knowledge of rationality: A 1-rationalizable strategy is one for which there is a profile of others' strategies that makes it a best response; a 2-rationalizable strategy is one for which there is a profile of others' 1-rationalizable strategies that makes it a best response; and so on. In two-person games a strategy is *k*-rationalizable if and only if it survives *k* rounds of iterated deletion of strictly dominated strategies. (There are subtle differences in *n*–person games, unimportant for our purposes.) Rationalizability is equivalent to *k*-rationalizability for all *k*, reflecting common knowledge of rationality with no further restrictions on beliefs.[13]

Equilibrium, by contrast, reflects the implications of common knowledge of rationality plus at least mutual knowledge of beliefs. Any equilibrium strategy is *k*-rationalizable for all *k*, but not all combinations of rationalizable strategies are in equilibrium. However, in games that are strictly dominance-solvable in *k* rounds or less, *k*-rationalizability implies that players have the same beliefs, so that any combination of *k*-rationalizable strategies is in equilibrium. In two-person games, a player can find his set of *k*-rationalizable strategies via *k* rounds of iterated strict dominance, without the need for fixed-point reasoning. Thus, *k*-rationalizability is cognitively less taxing than equilibrium, especially for small *k*.

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[12] In Crawford et al. (2010, Section 8) we argue that estimating a probability distribution over equilibria risks overfitting. With some qualifications, Harsanyi and Selten defined payoff-dominance as Pareto-efficiency within the set of equilibria. In pure coordination games this coincides with team reasoning (Bardsley et al. 2010; Section 8.3). Level-*k* and CH models ensure completeness despite multiple equilibria (Sections 2.4-2.5; see also Sections 5, 6, 8, and 9).

[13] Unlike equilibrium, rationalizability and *k*-rationalizability restrict individual players' strategies, not their relationship.

With or without multiple equilibria, rationalizability and *k*-rationalizability are incomplete, in general implying only set-valued restrictions on individual players' strategies. In games that are *not* strictly dominance-solvable in *k* rounds, *k*-rationalizability allows a range of deviations from equilibrium, which for low *k* are often consistent with the patterns in subjects' observed deviations. However, this consistency is obtained via predictions that may be so vague that they are useless. In the outguessing and coordination games discussed below, *any* strategy is a best response to some beliefs. One can then construct a "helix" of beliefs, consistent at all levels with iterated knowledge of rationality, and even rationalizability does not restrict behavior at all. But for most *k*-rationalizable or rationalizable outcomes the beliefs in the helix rest on rationality-based inferences at implausibly high levels and/or cycle unrealistically across levels.

2.3. *Quantal Response Equilibrium*

To capture the sensitivity of subjects' deviations from equilibrium to their out-of-equilibrium payoffs when others may deviate from their equilibrium strategies, MP (see also Robert W. Rosenthal 1993) proposed the notion of QRE.[14] In a QRE players' decisions are noisy with a specified distribution, logit in most applications, tuned by a precision parameter. The density of a decision is increasing in its expected payoff, evaluated taking the noisiness of others' decisions into account—QRE's key difference from equilibrium plus noise. A QRE is thus a fixed point in the space of decision distributions, with each player's distribution a noisy best response to the others'. As precision increases QRE converges to equilibrium without noise; and as precision approaches zero QRE converges to uniform randomization over all feasible strategies.

The fact that QRE responds to the noise in others' decisions is essential to its ability to improve upon equilibrium plus noise. But this response makes QRE's predictions highly sensitive to distributional assumptions—more so than in quantal response models of individual decisions or in other models of strategic thinking. Philip Haile et al. (2008) show that by varying the distribution QRE can "explain" any given dataset that has only one observation per game-player pair. Goeree et al. (2005) have shown, however, that QRE with a plausible monotonicity

[14] MP suggest using QRE for both initial responses and limiting outcomes, with increasing precision as a reduced-form model of learning. But although QRE has until recently been the most popular model of initial responses, not all researchers consider it suitable for that purpose. Jacob K. Goeree and Charles A. Holt (2004) suggest reserving QRE for limiting outcomes, and instead propose a "noisy introspection" ("NI") model to describe initial responses. NI relaxes QRE's equilibrium assumption by assuming that players form beliefs by iterating noisy best responses as in a level-*k* model, except that players respond to the noise in others' responses. Higher-order beliefs are assumed to reflect increasing amounts of noise, converging to uniform randomness. In special cases NI coincides with QRE or *L1*. NI is cognitively less taxing than QRE because it requires no fixed-point reasoning, but more taxing than level-*k* or CH because it requires more iterated best responses, and to noisy beliefs. We don't discuss NI further because it is seldom applied beyond Goeree and Holt (2004).

restriction on responses to payoffs but no further distributional assumptions does imply some restrictions even with one observation per game-player pair; and that QRE implies some cross-game restrictions even without monotonicity. QRE is still more than usually sensitive to the error distribution; there is little theory to guide its specification (but see Lars-Göran Mattsson and Jörgen Weibull 2002); and the frequent use of the logit is guided mostly by fit and custom.

A QRE player must respond to a distribution of others' responses and find his part of a fixed point in a large space of response distributions. If equilibrium reasoning is cognitively taxing, QRE reasoning is doubly taxing; and QRE is less behaviorally plausible as a model of thinking.

In applications QRE's precision is either calibrated from previous analyses or determined by fitting the model to data. The logit QRE or "LQRE" that results from assuming a logit distribution often fits initial responses better than equilibrium plus noise (MP; Goeree and Holt 2001; Goeree et al. 2005). But in some settings LQRE fits worse than equilibrium plus noise, sometimes with errors that deviate from equilibrium in the wrong direction to fit the data (Chong et al. 2005; CI 2007b, Online Appendix; Östling et al. 2011, Section II.C).[15]

2.4. *Level-k Models*

Aside from the John Maynard Keynes (1936) quotation that heads Section 3, level-*k* models seem to have been proposed first by Nagel (1995) and SW (1994, 1995). In a level-*k* model players anchor their beliefs in a strategically naïve initial assessment of others' likely responses to the game called "*L0*", and then adjust them via thought-experiments with iterated best responses: *L1* best responds to *L0*, *L2* to *L1*, and so on. Players' levels are heterogeneous, but each player's level is usually assumed to be drawn from a common distribution.

Even though *L0* normally has low or zero frequency, its specification has an important influence. In most applications, including those in Sections 3 to 7, *L0* is assumed to be uniform random over others' feasible decisions; or sometimes when there are more than two players, over the relevant summary of others' decisions.[16] This reflects the model's compartmentalization of a player's thinking into a strategically naïve initial assessment of others' likely responses to the game followed by strategic thinking via a series of iterated best responses. An *L1* player, for instance, is aware that he is playing a game in which his payoff is influenced by others' decisions as well as his own; but the *L0* by which he evaluates his decisions' expected payoffs reflects a

[15] QRE, or LQRE, is seldom easily adapted to theoretical analysis, and usually must be solved for numerically.
[16] Sections 8 and 9 discuss alternative specifications for games with salient labels or communication, as explained in footnote 11.

strategically naïve assessment of others' responses to the incentives the game creates. An *L2* player assesses others' responses less naively, but his *L1* model of others is still simpler than his model of himself. Note that on this interpretation there is no presumption that *L0* players exist: *L0* is simply *L1*'s model of others, *L2*'s model of *L1*'s model of others, and so on.

*Lk* rules rest on the cognitively simple operation of iteration of best responses to a naïve prior, and so avoid the criticisms that epistemic reasoning based on iterated knowledge of rationality or finding a QRE's fixed point in distribution space are too taxing for a realistic model of thinking.[17] Except for *L1*'s response to a uniform random *L0*, *Lk* rules need not respond to the noisiness of others' responses. A level-*k* model avoids QRE's sensitivity to distributional assumptions by treating deviations from equilibrium as part of the deterministic structure, rather than as errors or responses to errors. Because *Lk* rules respect simple dominance, level-*k* models limit the probability of violations of simple dominance more than equilibrium plus noise or QRE, where those probabilities can approach 50%. The fact that *Lk* rules may deviate systematically from equilibrium in ways that are sensitive to out-of-equilibrium payoffs, often allows a level-*k* model to out-predict equilibrium plus noise or LQRE.

Because *Lk* respects *k*-rationalizability, and *k* can vary across players, a level-*k* model can be viewed as a heterogeneity-tolerant refinement of *k*-rationalizability. It avoids rationalizability or *k*-rationalizability's unrealistic rationality-based inferences and cycles in beliefs by smoothing beliefs in an evidence-based way. Unlike unrefined equilibrium or QRE, level-*k* models are generically complete even in games with multiple equilibria.

In empirical applications it is assumed that *L1* and higher types make errors, often taken to be logit as in equilibrium plus noise or LQRE. Applications sometimes also allow for the possibility that some people play their equilibrium strategies, and/or that some are *Sophisticated* in the sense of playing (Bayesian) equilibrium strategies in a game that takes into account that other players are either level-*k* or *Sophisticated*. The population type frequencies are estimated or calibrated from previous analyses. As expected, the estimated frequency of *L0* is usually zero or small. The type distribution is fairly stable across settings, with most weight on *L1*, *L2*, and perhaps *L3*.

[17] In our view people stop at low levels mainly because they believe others will not go higher, not due to cognitive limitations; but the evidence on this is not yet conclusive. Allowing level-*k* types to consider the possibility that others are the same level leads to fixed-point problems like those with equilibrium, which we have argued are the main reason people deviate from equilibrium. Closing the loop by requiring rational expectations leads back to equilibrium, and so is empirically a dead end.

2.5. *Cognitive Hierarchy ("CH") Models*

In CHC's closely related CH model, *Lk* best responds not to *Lk-1* alone but to a mixture of lower types. The distribution of rules is usually approximated as Poisson, with its parameter estimated from the data or calibrated from previous estimates. *Lk* rules' beliefs are assumed to be derived from that distribution by Bayesian updating, assuming other players' levels are lower than the player's own. Thus, a CH *L1* is the same as a level-*k* *L1*, but a CH *L2* or higher type may differ from its level-*k* counterpart. A CH *L1* or higher type makes undominated decisions like its level-*k* counterpart. But unlike a level-*k* *Lk*, a CH *Lk* may not respect *k*–rationalizability.

Section 2.4's observations about the cognitive ease of level-*k* types mostly carry over to CH types. In particular, CH *L2* or higher types need not find fixed points or respond to others' noise.

In a CH model, unlike in a level-*k* model, *L1* and higher types are usually assumed not to make errors. Instead the uniform random *L0*, which the Poisson distribution constrains to have positive frequency, doubles as an error structure for higher types, though this is not in any way essential. Like a level-*k* model, given the assumed distribution a CH model makes point or mean predictions that do not depend on its estimated precision. But unlike a level-*k* model, and to some extent like QRE, the form of the distribution influences the model's point predictions.

In some applications the Poisson distribution is not very restrictive (CHC 2004, Section II.B; and Chong et al. 2005). But in others it seems excessively restrictive (Chong et al. 2005; CGC; CI 2007ab). Like a level-*k* model, a CH model limits the probability of violations of simple dominance more than equilibrium plus noise or QRE. If the Poisson specification is correct, CH *Lk* beliefs, unlike level-*k* *Lk* beliefs, become more accurate as *k* increases (CHC, Section II.A; Chong et al. 2005, Section 2.1). Even if the Poisson specification is incorrect, if the CH model is defined flexibly enough a higher *k* implies a more accurate model of others' fitted decisions. But because levels higher than *L3* are rare, this seems relatively unimportant.

The fact that CH *Lk* rules may deviate systematically from equilibrium often allows a CH model to out-predict equilibrium plus noise or LQRE.

## 3. *Keynes' Beauty Contest: Experimental Evidence from Guessing and Other Normal-Form Games*

> "...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average

preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

—Keynes (1936, Chapter 12)

Keynes' famous analogy suggests an *n*-person game whose players must outguess each other's responses to a payoff-irrelevant landscape of judgments about prettiness. Equilibrium analysis is not very helpful because it rules out the influence of such landscapes, and there are too many equilibria. Instead the analogy suggests a level-*k* model. His "fourth, fifth and higher degrees" is more than evidence suggests is realistic but may be only a coy reference to himself.[18]

This section reviews evidence on strategic thinking from experiments that elicit initial responses to normal-form symmetric-information games. The evidence we present here is representative of other evidence from symmetric-information games, with exceptions noted and discussed below. We begin with Nagel's (1995), HCW's, and BGNS's analyses of symmetric *n*-person guessing games directly inspired by Keynes' analogy. We then discuss SW's (1994, 1995) analyses of symmetric two-person matrix games and CGCB's analysis of asymmetric two-person matrix games. We next discuss CGC's analysis of (mostly asymmetric) two-person guessing games. We close with a summary and directions for future work.

Readers uninterested in the detailed evidence can skip ahead to Sections 3.4 and 3.5.

3.1. *Guessing or Beauty Contest Games*

In Nagel's and HCW's games, *n* subjects (*n* = 15-18 in Nagel, *n* = 3 or 7 in HCW) made simultaneous guesses between lower and upper limits (0 and 100 in Nagel, 0 and 100 or 100 and 200 in HCW). In BGNS some of the same games were played in the field, by 7500+ volunteers recruited through the newspapers *Financial Times*, *Spektrum der Wissenchaft*, or *Expansión*. In each case the subject who guessed closest to a target (*p* = 1/2, 2/3, or 4/3 in Nagel; *p* = 0.7, 0.9, 1.1, or 1.3 in HCW; *p* = 2/3 in BGNS) times the group average guess won a prize. Each

[18] Tellingly, in one of the first reviews of John von Neumann and Oskar Morgenstern (1944), Jacob Marschak (1946) quoted this Keynes passage and said (with reference to their theory of zero-sum two-person games) "…it seems to us that properly stated differences in degrees of knowledge or intelligence of individual players can also be regarded as rules of the game."

treatment had identical targets and limits for all players. The structures were publicly announced, to justify comparing the results with symmetric-information predictions.

Although Nagel's and HCW's subjects played a game repeatedly, their first-round guesses can be viewed as initial responses if they treated own influences on future guesses as negligible, as is plausible for all but HCW's three-subject groups. BGNS's subjects played only once.

With symmetric information, in all but one treatment the game is dominance-solvable in a finite (limits 100 and 200) or infinite (limits 0 and 100) number of rounds, with a unique equilibrium in which all players guess their lower (upper) limit when $p < 1$ ($p > 1$). The epistemic argument for this "all–0" equilibrium is stronger than usual, in that it depends "only" on (sometimes infinitely) iterated knowledge of rationality, not on mutual knowledge of beliefs.

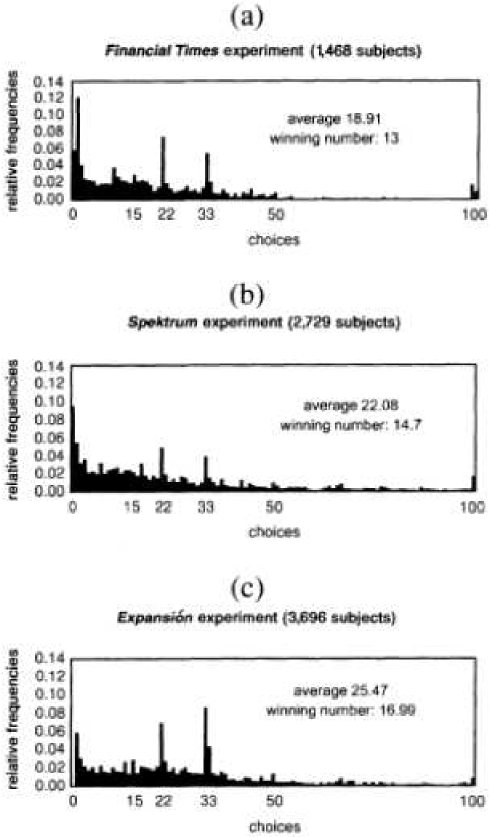


FIGURE 1. RELATIVE FREQUENCIES OF CHOICES IN THREE NEWSPAPER EXPERIMENTS

**Figure 1. Bosch-Domènech et al.'s (2002) Figure 1**

BGNS's Figure 1 illustrates the results (see also Nagel's Figure 1 and HCW's Figures 2A-H and 3A-B). Subjects seldom made equilibrium guesses initially. Most guesses respected at most three rounds of iterated dominance, although more were needed to reach equilibrium. The guess distributions have spikes that track $50p^k$ for $k = 1, 2, 3$ across the different targets $p$ in the treatments. Like the spectrograph peaks that foreshadow the existence of chemical elements, the spikes suggest that subjects' deviations from equilibrium have a coherent structure, one that is discrete and individually heterogeneous. The deviations are inconsistent with "equilibrium plus noise" or "equilibrium taking noise into account" as in QRE, for any reasonable distribution.

Nagel's, HCW's, and BGNS's designs are distinguished by their large strategy spaces, which greatly increase the informativeness of results. But for the purpose of studying strategic thinking, it is a weakness that their subjects' initial responses were limited to one game. One observation yields very limited information about the rule a subject was following.

There are two plausible interpretations of how the spikes' locations vary across treatments. In one, subjects follow "level-*k*" rules based on a uniform random *L0* as in Section 2.4, interpreted in this *n*-person game as representing a strategically naïve estimate of the group average guess.[19] *Lk* then iterates best responses *k* times, so that in these games *Lk*+1 guesses $[(0+100)/2]p^{k+1} = 50p^{k+1}$. In the other interpretation, a subject does *k* rounds of iterated dominance and then best responds to a uniform prior over the average of others' remaining strategies, a rule we call *Dk*, which guesses $([0+100p^k]/2)p = 50p^{k+1}$. Theorists often interpret Nagel's, HCW's, and BGNS's results as showing that subjects explicitly performed iterated dominance, but the results equally well support the interpretation that subjects followed level-*k* rules that only implicitly respect it.

In other games *Dk* and *Lk*+1 respond similarly to dominance, both yielding *k*-rationalizable strategies (the different indices are a quirk of notation). Thus each completes *k*-rationalizability via a specific selection. But the distinction matters in some applications. *Dk* and *Lk*+1 are weakly separated in SW's, HCW's, and CGCB's experiments. They are strongly separated in CGC's experiments, and the results favor level-*k*'s iterated best responses over iterated dominance.

Nagel's, HCW's, and BGNS's designs have another weakness for our purpose, in that their subjects' influences on others' payoffs were negligible ex ante. When the authors think about the stock market, we know that "it" isn't thinking about us, and that greatly simplifies our thinking. Most games in applications are more like Warren Buffet thinking about the stock market. Results

[19] In this paper we focus mainly on two-person games, but in *n*-person games it may matter whether *L0* is independent across players, and the limited evidence (HCW; Costa-Gomes et al. 2009) suggests that people have highly correlated models of others. Here we take *L0* to model others' average guess, implicitly assuming perfect correlation as the evidence suggests.

for *n*-person guessing games give limited insight into the reciprocal strategic thinking such games require. We now turn to experiments in which individual influences are more important.

3.2. *Other Normal-Form Games*

SW (1994, 1995) reported experiments in which each subject played a series of 10 or 12 different symmetric 3×3 matrix games. As in the remaining experiments we discuss here, subjects were randomly and anonymously paired to play the games without feedback, to suppress learning and repeated-game effects and elicit initial responses game by game. "Eureka!" learning was possible, but probably rare. The fact that a series of responses was elicited for each subject greatly increases the power of the design, but the small strategy spaces sacrifice some power.

SW's (1994) data analysis used a mixture model combining a type they called *Naïve Nash*, our equilibrium plus noise (Section 2.1) with a uniform random *L0* type; an *L1* as in Section 2.4; and an *L2* that differs from Section 2.4's in best responding to a noisy *L1* (which SW motivate as a weighted average of their *L1* and their uniform random *L0*). SW (1994) found 35 of 40 subjects for which one type had posterior probability at least 0.90: 18 *L2*, 9 *Naïve Nash*, and 8 *L1*.

SW (1995) generated a new dataset from a design close to SW's (1994), but analyzed it by adding to the mixture a noiseless *Equilibrium* type; a *Rational Expectations* type that best responds to the model's predicted partners' choice frequencies; and a *Worldly* type, which best responds to an estimated mixture of a noisy *L1* and their noiseless *Equilibrium*. SW (1995) found 38 of 40 subjects for which one type had posterior probability at least 0.90: 17 *Worldly*, 9 *L1*, 6 *L0*, 5 *Naïve Nash*, and 1 *L2*. Thus they found no *Rational Expectations* subjects and almost completely rejected *L2* in favor of *Worldly*. SW's (1994) estimates are more consistent than SW's (1995) estimates with other analyses, earlier and later. We suspect that SW's (1995) richly parameterized *Worldly* type, which implicitly assumes subjects share the analysts' understanding of others' responses via its dependence on parameters estimated from the data, overfits the data.

CGCB reported experiments in which each subject played a series of 18 different asymmetric 2×2, 2×3, and 2×4 matrix games, retaining SW's small strategy spaces but further increasing the number of observations per subject and avoiding symmetric games that might blur the cognitive distinction between own and other's decisions. CGCB's data analysis allows types *L1*, *L2*, and *L3* as defined in Section 2.4 (best responding to noiseless lower-level types); *D1* and *D2* as in Section 3.1; an *Equilibrium* type like SW's *Naïve Nash*; and a *Sophisticated* type that best responds to potential partners' observed choice frequencies (a nonparametric analog of SW's

1995 *Rational Expectations* type), as a proxy for subjects whose understanding of strategic behavior transcends mechanical rules such as the other types. CGCB's estimates of the type distribution are quite similar to SW's (1994). They also resemble SW's (1995) estimates, except that, excluding *Worldly*, they identify more subjects as *L1*, *L2*, or *D1*.[20]

3.3. *Two-Person Guessing Games*

CGC's design combines Nagel's, HCW's, and BGNS's large strategy spaces with SW's and CGCB's series of different games, with subjects again randomly and anonymously paired without feedback. CGC's subjects played a series of 16 different but related two-person guessing games. Each player has his own lower and upper limit, both strictly positive, which implies that the games are finitely dominance-solvable. Each player also has his own target, and his payoff increases with the closeness of his guess to his target times the other player's guess. Importantly, unlike in Nagel's and HCW's guessing games, the targets and limits vary independently across players and games. The games are mostly asymmetric; and the targets and limits are sometimes both less than one, sometimes both greater than one, and sometimes "mixed".

CGC's games have essentially unique equilibria, whose locations are determined by players' lower (upper) limits when the product of targets is less (greater) than one.[21] Consider for instance a game in which the first player's target and limits are 0.7 and [300, 500] and the second player's are 1.5 and [100, 900]. The product of targets is 1.05 > 1, and it is not hard to show that the equilibrium is therefore determined by players' upper limits. In equilibrium the first player guesses his upper limit of 500; but the second guesses only 1.5×500 = 750 < his upper limit 900. No guess is dominated for the first player, but any guess outside [450, 750] is dominated for the second. Given this, any guess outside [315, 500] is iteratively dominated for the first player; and so on until the equilibrium at (500, 750) is reached after 22 rounds of iterated dominance.

The main difficulty in analyzing the data from such experiments is identifying subjects' decision rules within the enormous set of logically possible rules. As in previous studies, CGC assumed that each subject's decisions follow one of a small set of a priori plausible rules up to logit errors, and econometrically estimated which rule best fits his decisions. Their specification includes *L1*, *L2*, and *L3* (Section 2.4); *D1* and *D2* (Section 3.1); *Equilibrium*; and *Sophisticated*.

[20] Matthias Sutter et al. (2010) replicate CGCB's results for individuals and also for three-person teams.

[21] The discontinuity of the equilibrium correspondence when the product of targets equals one stress-tests equilibrium, which responds much more strongly to the product of the targets than behaviorally plausible alternative decision rules do. In this class of games, the discontinuity also enhances the separation of *Equilibrium* from alternative rules.

CGC's large strategy spaces and independent variation of targets and limits across games enhance the separation of types' implications to the point where many subjects' types can be directly and precisely identified from their guesses, without econometrics. Of the 88 subjects in CGC's main treatments, 43 complied exactly (within 0.5) with one type's guesses in from 7 to 16 games (20 *L1*, 12 *L2*, 3 *L3*, and 8 *Equilibrium*). Because the types specify precise, well-separated guess sequences in a very large space, those subjects' guesses allow one intuitively to "accept" the hypothesis that they followed their apparent types. Because the types build in risk-neutral, self-interested rationality and perfect models of the game, the deviations from equilibrium of the 35 whose apparent types are *Lk* can confidently be attributed to nonequilibrium beliefs rather than irrationality, risk aversion, altruism, spite, or confusion.[22] Finally, because the types build in a uniform random specification of *L0*, that specification is directly confirmed by the data.

CGC's other 45 subjects made guesses that follow a type less closely. But for 31 of them, violations of simple dominance had frequencies less than 20% (versus 38% for random guesses), suggesting that their behavior was coherent. Econometric type estimates for these 45 subjects are concentrated on *L1*, *L2*, *L3*, and *Equilibrium* in roughly the same proportions as for the 43 with very high compliance with a type's guesses. But for these subjects there is room for doubt about whether CGC's econometric specification omits relevant types and/or overfits via accidental correlations with included types. CGC addressed such doubts via a semiparametric specification test, described in Section 3.5, which confirms that *L1*, *L2*, *L3*, and perhaps *Equilibrium* are truly present in the population; but that omitted types describe only 1-2% of the population. Thus in this setting, deviations from equilibrium other than *L1*, *L2*, or *L3* have little discernible structure.

### 3.4. *Lessons for Modeling Strategic Thinking*

Nagel's, HCW's, and BGNS's results show that initial responses can deviate systematically from equilibrium, even when equilibrium reasoning requires "only" iterated dominance (though possibly to many rounds). The deviations resemble neither equilibrium plus noise nor QRE for any reasonable distribution. Subjects' thinking is heterogeneous, so no model that imposes

[22] By contrast, in designs with small strategy spaces, or in which each subject plays a single game, even a perfect fit may not distinguish a type from nearby omitted types. The subjects in CGC's main treatments (Baseline and OB) were trained in and quizzed on how their payoffs were determined and how to identify their and their partner's best-responses, but not taught any decision rule. Their high rates of exact compliance with level-*k* types reflect their own thinking. Although an earlier version of Sotiris Georganas et al. (2010) claimed that CGC's subjects were trained to use iterated best responses, that is not true.

homogeneity will do justice to their behavior.[23] Subjects' thinking falls into discrete classes and may violate rationalizability, but it respects *k*–rationalizability for low values of *k* (1, 2, or 3).

SW's, CGCB's, and CGC's analyses confirm and sharpen these lessons. Their results suggest that half or more of subjects' decisions in these symmetric-information normal-form games are explained by a level-*k* model anchored on a uniform random *L0*, with only the types *L1*, *L2*, *L3*, and *Equilibrium*. Estimates that make adequate allowance for errors also suggest that *L0* subjects exist mainly as *L1*'s model of others, *L2*'s model of *L1*'s model, and so on.[24] And CGCB's and CGC's analyses suggest that there are few if any *Sophisticated* or *Dk* subjects.[25] Finally, CGC's specification test (Section 3.5) suggests that at least in their games, behavior that doesn't follow *L1*, *L2*, *L3*, or *Equilibrium* lacks readily discernible structure, so that it may be reasonable to treat such behavior as errors. SW's, CGCB's, and CGC's conclusions are consistent with most other studies of initial responses, just more precise. (We discuss exceptions in Section 3.5.)

3.5. *Directions for Future Work*

The experiments just reviewed yield insights into strategic thinking in a variety of symmetric-information normal-form games, and give substantial support to level-*k* models. But the evidence is from classes of games studied in isolation, with most data analyses based on particular econometric models of decisions alone. More work is needed to evaluate the credibility of the models' explanations and to assess their domains of applicability, their portability, and the stability of their parameter estimates across types of games. We now review some of what has been done along those lines, outline what still needs to be done, and describe the research strategies we think are most likely to be useful.[26]

We begin with CGC's data analysis, which provides concrete illustrations of several points. Some conclusions can be read directly from their estimated model. Although CGC's model nests equilibrium plus noise, only 11 of 88 subjects in their main treatments are estimated as *Equilibrium* (and even they may be following rules that only mimic *Equilibrium*; CGC, pp.

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[23] Allowing for heterogeneity of people's thinking turns out to be essential in the applications discussed in Sections 5, 6, 8, and 9.

[24] CGC's econometric model implicitly allows *L0*s, because any type with zero estimated precision mimics uniform randomness. For all but 10 of the 88 subjects in their main treatments, the subject's estimated type was significantly better than a random model of guesses at the 5% level. Thus at the most, 11% of the subjects were better described by *L0* than their estimated types. Unconstrained estimates that separate *L0* from the error structure almost always have very low or zero frequencies of *L0*.

[25] Thus to the extent that subjects respect iterated dominance, it is not because they explicitly perform it, but because they follow level-*k* rules that respect it. This conclusion is reinforced by CGC's (2012) data on "robot/trained subjects," where 7 of 19 subjects trained and rewarded as *D1*s and having passed a *D1* understanding test, "morphed" into *L2*s, *D1*'s closest *Lk* relative.

[26] We focus here on settings where salience is not important, but we revisit some of these issues with salience in Section 8.

1753-1754). Thus the analysis indicates clearly that strategic thinking is far from homogeneous, and that a model that allows heterogeneity has a low population frequency of *Equilibrium*.

CGC's econometric model allows logit errors for each type, including *Equilibrium*. CGC's payoff function has a convenient certainty-equivalence property (p. 1748) that limits differences between *Equilibrium* plus logit noise and LQRE, but they are not quite the same because the limits create asymmetries in the error distributions that they respond to differently. Even so, the multiple spikes in the subjects population's observed guess distributions make it clear that the usual population-homogeneous specifications of LQRE would be similarly rejected.

There is no conflict with *k*–rationalizability, at least for $k = 1$, because the rules with positive estimated frequencies all respect it. But the estimated model shows that CGC's subjects' thinking has a structure that completes *k*–rationalizability by making more specific predictions.

It would be of interest to discriminate between level-*k* and CH models, but so far no study has clearly separated their predicted decisions.[27] But most level-*k* estimates that make adequate allowance for errors assign *L0* a much lower frequency than CH estimates do, often zero. Thus a CH model's Poisson constraint will often be binding, and a level-*k* model, which imposes no such restriction, will fit better (see however CHC 2004, Section II.B; and Chong et al. 2005).

The above observations all work within CGC's econometric specification, and therefore yield little information on whether any subjects' decisions could be better explained by types omitted from the specification; or whether any subjects' estimated types are artifacts of overfitting, via accidental correlations with included types that are in fact irrelevant. To address these issues, CGC conducted a semiparametric specification test, comparing the likelihoods of subjects' estimated types with those of estimates based on 88 "pseudotypes", each constructed from a single subject's guesses in the 16 games. The logic of the test is that if an important type had been omitted from the specification, then there would be a corresponding cluster of subjects whose decisions are better "explained" by each other's pseudotypes than by any type included in the specification. Conversely, if an irrelevant type had been included, subjects' decisions would be no better "explained" by it than by chance. These tests confirm that there were no important types omitted from CGC's specification, and reaffirm most of CGC's *L1*, *L2*, or *Equilibrium* type estimates for individual subjects. However, the tests call into question most of the estimates

[27] In CGC's analysis, by a quirk of design, CHC's CH *L2* and *L3* are both confounded with CGC's level-*k* *L2* (CGC, fn. 36, p. 1763). A CH *Lk*'s best response to a mixture of lower types seems more realistic than a level-*k* *Lk*'s best response to one lower type, but the issue is not which would be the better econometric specification, but which better describes the rules people use.

that subjects followed other types, and they led CGC to leave 33 of their 88 subjects unclassified, mostly because those subjects' decisions had no discernible structure.

We now discuss leading examples of additional ways to test a model's specification and evaluate the credibility of its explanation of behavior.

One way is to study cognition via measures that complement decisions, such as monitoring subjects' searches for hidden information or monitoring their neural activity.

CGCB and CGC (see also Crawford's 2008 survey), Brocas et al. (2010), and Joseph Tao-Yi Wang et al. (2010) monitored subjects' searches for hidden payoff information in various types of games. Noting that level-*k* and other decision rules can be viewed as describing how people reason, they used algorithmic models of how a subject's rule (equilibrium or not) drives his thinking to derive restrictions on how the subject searches. They then used subjects' searches, along with their decisions, to better estimate what rules they were following. The results generally confirm and enrich these authors' level-*k* or CH interpretations of subjects' decisions.[28]

Meghana A. Bhatt and Camerer (2005), Giorgio Coricelli and Nagel (2009), and Bhatt et al. (2010) studied strategic thinking via fMRI, again finding further support for level-*k* models.

Another way is to directly elicit subjects' beliefs along with their decisions. Costa-Gomes and Weizsäcker (2008, "CGW") reported experiments in which subjects made decisions and stated beliefs about others' decisions in 14 asymmetric two-person 3×3 games. They elicited beliefs via a quadratic scoring rule, which is incentive compatible when the decision-maker is a risk-neutral expected-utility maximizer. Their econometric analysis viewed stated beliefs as another kind of decision and treated the two as symmetrically as possible, with beliefs subject to error as well as chosen decisions. Elicited beliefs provide a complementary lens through which to study subjects' thinking. Most subjects' stated beliefs were close to *L2*'s, as if they thought others would behave as *L1*s; but most of their decisions were close to *L1*'s. This result challenges the usual view that the setting causes beliefs, which then cause decisions. Instead it suggests that the decision rule is the fundamental, which in conjunction with the setting drives both beliefs and decisions. Imposing the untested restriction that decisions best respond to stated beliefs is risky.[29]

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[28] CGC's design separates the search implications of level-*k* and CH *L2*s and *L3*, and the search data clearly favor the level-*k* versions.CH *Lk* rules are information hogs, but subjects focus on the more limited searches that suffice for level-*k* *Lk* rules.

[29] Pedro Rey-Biel (2009) partially replicates CGW's results; but his results also suggest that in constant-sum games, where equilibrium reasoning need not depend on strategic thinking, equilibrium may predict better than level-*k* or other theories.

A third way to study cognition is to monitor subjects' chats within teams with common decisions and goals. Konrad B. Burchardi and Stefan P. Penczynski (2011, "BP"; see also Penczynski 2011) adapt David J. Cooper and John H. Kagel's (2005) chat method, with player roles filled by two-subject teams with common payoffs whose chat deliberations were monitored along with their decisions. If team members could agree on a decision it was implemented; if not each submitted a proposal, which was implemented with probability one-half. Thus a member had an incentive both to convey his own thinking to his partner, and thus to the experimenters, and to propose the decision he thought was optimal. BP and Penczynski elicited responses within subjects to one of Nagel's (1995) games and to RT's hide-and-seek game with non-neutral framing of locations (Section 8; Rubinstein 1999; CI 2007b), also adding some structural variety.

Recall that *L0* has usually been taken to be uniform random, as a way of capturing players' strategically naïve assessments of others' likely decisions. Although most of the evidence generally supports uniform randomness, it does not allow a test of this independent of the model's other assumptions.[30] BP's chats do allow such a test. In the chats for Nagel's guessing game, more than half of BP's subjects based their reasoning on some kind of *L0*, a majority with a uniform random *L0*. Most of BP's subjects also followed rules that iterated best responses to whatever their *L0* was (in level-*k*, not CH fashion). But BP's analysis calls into question the usual assumption that *L0* is homogeneous in the population.

Although the estimated population type distributions are fairly stable across the different types of games that have been studied, evidence from classes of games studied in isolation allows few firm conclusions about the models' generalizability and portability across types of games.[31] A fourth way to study cognition and to evaluate the credibility of a model's explanations is to create designs that add structural variety, preferably within subjects.

BP found that individual subjects' estimated types are only weakly correlated across Nagel's guessing game and RT's hide-and-seek game with non-neutral framing of locations (Section 8).[32]

[30] We note two exceptions: HCW estimated the mean of a parameterized *L0* distribution, finding in Nagel's guessing games that it differs significantly from a uniform distribution. And those of CGC's subjects whose guesses conformed almost exactly to *Lk* types defined for a uniform *L0* provide some support for that assumption.

[31] SW's, Nagel's, CGCB's, CGC's, and CGW's designs have some structural variety, limited to particular classes of games.

[32] We have some reservations about BP's econometric specification, which allows a heterogeneous *L0* with bounded normal errors, but as in some CH estimations, no errors in higher types' decisions, which are constrained to be exact best responses to the hypothesized beliefs. Making *L0* serve both as anchor and error structure greatly increases the risk of misspecification. Even without assuming a Poisson type distribution, BP estimate an *L0* frequency of 22-37%, far higher than estimates with unconstrained distributions. We are also concerned that BP's *n*-person guessing game raises issues about strategic thinking like the extent to which subjects' models of others are correlated, which are far from the issues raised by salience in RT's games. Games with different structures but that raise similar strategic issues might have been more suitable for a first study.

CI (2007b) checked portability across RT's and two other hide-and-seek games with non-neutral framing, finding some consistency across games as explained in Section 8.2.

Adding structural variety in a different way, Georganas et al. (2010) elicited subjects' decisions in some of CGC's guessing games and some new "undercutting" games. They also considered alternative definitions of *L0*. They found further support for level-*k* and CH models within each class of game, but moderate correlation of subjects' estimated types across games.[33]

In judging these and other results on the portability of models across different kinds of games, the notion of a "general" theory may require some adjustment: Allowing behavioral parameters an influence opens the possibility that they may vary with the setting. This, we believe, should not disqualify theories with parameters, but it highlights the need for exploratory work with specifications flexible enough to allow credible identification of the kinds of model that may be useful. We may eventually be able to predict how the parameters will vary with the environment; but even if not, the estimated models capture useful empirical regularities.

Another final route to progress uses models of strategic thinking to interpret evidence from observational data, whenever the field setting has a structure clear enough to make it possible to use such models to do that. We discuss field applications below whenever possible.

The experimental papers discussed here reflect an encouraging trend toward using the power and flexibility of experimental design to assess the generalizability, portability, and scope of models of strategic thinking. Turning to what still needs to be done and the research strategies we think are most likely to be useful, we believe further progress will best be served by taking full advantage of the methods now available to supplement the analysis of decisions with other measures of cognition; and expanding the variety of structures for which we have reliable evidence on thinking, with a particular focus on integrative work evaluating model performance across classes of games rather than in isolation. Such work should use detailed, individual-level data analyses that exploit the power of experimental design to reveal subjects' thinking as directly as possible, not shying away from econometrics to supplement design but substituting the power of design for sophisticated econometrics as much as possible, and avoiding untestable structural assumptions that risk bias.[34] The main goals should be learning about the ranges of

[33] They also found some positive correlation of subjects' types with measures of their cognitive ability. We suspect that the relatively low correlation of types across games is due in part to Georganas et al.'s simplification of CGC's instructions and omission of CGC's understanding test, which we believe is crucial for results that are representative of cognition in the field.

[34] The common practice of using progressively more sophisticated econometrics to reanalyze existing datasets, some with only one observation per subject, seems particularly unlikely to yield useful information.

applicability of existing models and the variation of behavioral parameters across settings and populations, and identifying the need for adaptations or new models—all with the view of bringing us closer to a truly general model of strategic thinking.

## 4. *M.M. Kaye's* Far Pavilions*:*
## *Payoff Asymmetries in Outguessing Games*

> "…ride hard for the north, since they will be sure you will go southward where the climate is kinder…."
>
> —Koda Dad, in M.M. Kaye (1978, p. 97)

This section illustrates the application of level-*k* models by analyzing the symmetric-information "outguessing" game suggested by the above quotation. As will be seen, the heterogeneity of level-*k* players' thinking gives a coherent account of strategic uncertainty in outguessing games, while avoiding equilibrium's unrealistic comparative statics implications in such games. The thinking reflected by the quotation also motivates the uniform random specification of *L0* used here and in the applications in Sections 5 to 7.

Early in M.M. Kaye's novel *The Far Pavilions*, the main male character, Ash/Ashok, is trying to escape from his pursuers along a north-south road. Both Ash and his pursuers must choose, in effect simultaneously, between north and south. South is warm, but north are the Himalayas, with winter coming. Ash's mentor, Koda Dad, nonetheless advises Ash to ride north. Ash follows Koda Dad's advice, the pursuers go south, and Ash escapes.

Imagine that if the pursuers catch Ash, they gain two units of payoff and Ash loses two; and that they both gain one extra unit for choosing south, whether or not Ash is caught. This yields the payoff matrix in Figure 2. The main strategic issue the game poses is how best to respond to the payoff asymmetry. Its unique mixed-strategy equilibrium gives one answer, which we will now contrast with the answer suggested by a level-*k* model.

| | | **Pursuers** | |
|---|---|---|---|
| | | **South (*q*)** | **North** |
| **Ash** | **South (*p*)** | **-1** (Ash), **3** (Pursuers) | **1** (Ash), **0** (Pursuers) |
| | **North** | **0** (Ash), **1** (Pursuers) | **-2** (Ash), **2** (Pursuers) |

**Figure 2.** ***Far Pavilions*** **Escape**

| Type | Ash | Pursuers |
|---|---|---|
| ***L0*** | Uniform random | Uniform random |
| ***L1*** | South | South |
| ***L2*** | North | South |
| ***L3*** | North | North |
| ***L4*** | South | North |
| ***L5*** | South | South |

**Table 1. *Lk* types' decisions in *Far Pavilions* Escape**

Examples like this are as common in experimental game theory as in fiction, but fiction sometimes more clearly reveals the thinking behind a decision. In the quotation Koda Dad is advising Ash to choose the *L3* response to a uniform random *L0*.[35] To see this, note that if the pursuers expect Ash to go south because it's "kinder", they must be modeling Ash as an *L1* responding to a uniform random *L0*; for south's payoff advantage is decisive only if there is no difference in the probability of being caught.[36] Moreover, because Koda Dad says the pursuers will be sure Ash will go south, he must be modeling them as *L2* and advising Ash to choose the *L3* response to a uniform random *L0*. The level-*k* model implies decisions as in Table 1. It predicts the outcome in the novel exactly, provided that Ash is *L3* and the Pursuers are *L2*.

How does this level-*k* prediction compare with an equilibrium model? *Far Pavilions* Escape has a unique equilibrium in mixed strategies, in which Ash's Pr{South} $p^* = 1/4$, and the Pursuers' Pr{South} $q^* = 3/4$. Thus in equilibrium the novel's observed outcome {Ash North, Pursuers South} has probability $(1 – p^*)q^* = 9/16$: less than 1, but much better than a random ¼.

This comparison is unfair because the level-*k* model has been allowed an omniscient narrator telling us how players think while equilibrium does not use such information. But in applications where such information is unavailable, as in most applications, we can derive the level-*k* model's implications as in Table 1, and estimate or calibrate the population type frequencies. Suppose for example that each player role is filled from a 50-30-20 mixture of *L1*s, *L2*s, and *L3*s and there are no errors. Then Ash goes north with probability ½ and pursuers go south with probability 4/5. Assuming independence, the observed outcome {Ash North, Pursuers South} then has probability 2/5: less than the equilibrium frequency 9/16, but still better than a random ¼.

[35] Until recently we were aware of no level higher than *L3* anywhere in literature or folk game theory; but Johanna Thoma has now identified a plausible *L4* in William Boyd (2006, pp. 250-251): a spy novel, perhaps not coincidentally.We remain unaware of examples of fixed-point reasoning.

[36] There is a plausible alternative interpretation in which the pursuers' model of Ash ignores strategic considerations, and given this, uses the principle of insufficient reason. A uniform random *L0* can also be viewed as approximating random sampling of a player's payoffs for alternative strategy combinations, unstratified by other players' strategy choices.

More importantly, the heterogeneity of level-*k* players' thinking gives a coherent account of the uncertainty in outguessing games, while avoiding an unrealistic comparative statics implication of the mixed-strategy equilibrium. In games like *Far Pavilions* Escape, the equilibrium responds to the payoff asymmetry of south and north in a decision-theoretically intuitive way for pursuers ($q^* = 3/4 > ½$, their equilibrium probability with no north-south asymmetry); but in a counterintuitive way for Ash ($p^* = 1/4 < ½$). Yet in initial responses to games like this, subjects' choices tend to follow decision-theoretic intuition in both roles.[37] MP and Goeree et al. (2005) discuss experiments with 2×2 Matching Pennies games with payoff perturbations in only one player role. These yield initial aggregate choices that reflect decision-theoretic intuition in the role with perturbed payoffs; but in the other role, for which the intuition is neutral, choices deviate from equilibrium in the direction that increases expected payoff, given the response in the first role. MP (Figures 6 and 7) and Goeree et al. (2005) show that LQRE with fitted precisions tracks these qualitative patterns, although it sometimes underpredicts the magnitudes of deviations from equilibrium, especially for the player whose payoff is perturbed. A level-*k* or CH model, either calibrated or estimated from the data, also tracks those patterns.

## 5. *Groucho's Curse: Zero-Sum Betting and Auctions with Asymmetric Information*

> "I sent the club a wire stating, 'Please accept my resignation. I don't want to belong to any club that will accept people like me as a member'."
>
> —Groucho Marx (1959, p. 321), Telegram to the Beverly Hills Friar's Club

> "Son," the old guy says, "No matter how far you travel, or how smart you get, always remember this: Someday, somewhere," he says, a guy is going to come to you and show you a nice brand-new deck of cards on which the seal is never broken, and this guy is going to offer to bet you that the jack of spades will jump out of this deck and squirt cider in your ear. But, son," the old guy says, "do not bet him, for as sure as you do you are going to get an ear full of cider."
>
> —Obadiah ("The Sky") Masterson, quoting his father in Damon Runyon (1932)

This section shows how to extend level-*k* models to allow the informational asymmetries commonly found in field applications, and uses them to discuss laboratory and field evidence. It shows that a model with the uniform random *L0* used in most applications with symmetric

[37] Ash's counterintuitive choice would not contradict this pattern if he were a subject because his revealed type is in the minority. Crawford and Dennis E. Smallwood (1984) discuss the comparative statics of mixed-strategy equilibria in perturbed Matching Pennies games, showing that this role-asymmetric intuitiveness is general when both players' payoffs are perturbed.

information, but now assumed to be independent of the realizations of others' private information, gives a simple account of the results of experiments on zero-sum betting (CHC; Isabelle Brocas et al. 2010) and auctions with private information (Crawford and Iriberri 2007a, "CI"). This account allows a unified treatment of informational naïveté—people's failure to take into account that others' responses will depend on their own private information, as in the winner's curse—and other observed aspects of nonequilibrium strategic thinking in games with asymmetric information. The section next discusses CH analyses of field data from settings with asymmetric information (Alexander Brown et al. 2010, Robert Östling et al. 2011). It concludes by discussing theoretical applications of level-*k* models to the design of optimal auctions (Crawford et al. 2009) and efficient mechanisms for bilateral trading (Crawford 2012).

5.1. *Zero-Sum Betting*

Experiments on zero-sum betting build on Paul R. Milgrom and Nancy Stokey's (1982) no-trade theorem: Suppose traders are weakly risk-averse and have concordant beliefs, and the initial allocation is Pareto-efficient relative to the information available at the time. Then even if traders receive new private information, no weakly mutually beneficial trade is possible; and if traders are strictly risk-averse, no trade at all is possible. Any such trade would make it common knowledge that all had benefited, contradicting the Pareto-efficiency of the original allocation. With reference to our first quotation, this result has been called the Groucho Marx theorem.

The fact that speculative zero-sum trades are common in real markets has a number of possible explanations, one of which is nonequilibrium thinking. Brocas et al.'s (2010) experiments on zero-sum betting have the control required to distinguish between alternative models of strategic thinking and models based on other factors such as hedging or the joy of gambling (see also CHC 2004, Section VI; Rogers et al. 2009; and the earlier papers they cite).

| **Player/state** | **A** | **B** | **C** |
|---|---|---|---|
| **1** | 25 | 5 | 20 |
| **2** | 0 | 30 | 5 |

**Figure 3. Zero-Sum Betting**

Brocas et al.'s design used simple three-state betting games in the spirit of Milgrom and Stokey's market model, including the one in Figure 3. The rules and the information structure were publicly announced, so that the design could be used to test theories that assume common knowledge. Each of the two players, 1 and 2, is given information about which of three ex ante equally likely states has occurred, A, B, or C. As indicated by the heavy borders in Figure 3, player 1 learns either that the state is {A or B} or that it is C; while player 2 learns either that the state is A, or that it is {B or C}. Once informed, the players choose simultaneously between two decisions: Bet or Pass. A player who chooses Pass earns 10 no matter what the state. If one player chooses Bet while the other chooses Pass, they both earn 10 no matter what the state. If both choose Bet, they get the payoffs in Figure 3, depending on which state has occurred.

This game has a unique trembling-hand perfect Bayesian equilibrium.[38] In this equilibrium, player 1 told C will Bet because 20 > 10, and player 2 told A will Pass because 0 < 10. Given that, player 1 told {A or B} will Pass, because player 2 will Pass if told A, so betting given {A or B} yields player 1 at most 5 < 10. Given that, player 2 will Pass if told {B or C}, because player 1 will Pass if told {A or B}, so betting given {B or C} yields player 2 at most 5 < 10. This covers all contingencies and completes the characterization of equilibrium, showing that the game is weakly dominance-solvable in three rounds. In equilibrium no betting takes place in any state.

Despite this clear conclusion, in Brocas et al.'s and previous experiments, subjects bet approximately half the time, with strong regularities in betting patterns across roles and states. To explain these results, Brocas et al. proposed a level-*k* model in which, following CHC (Section VI), *L0* bids uniformly randomly, independent of its private information. Recall that *L0* is a player's strategically naïve model of others' responses—others whose private information he does not observe. One could still imagine *L0*s that reflect an influence of others' information on their responses via contingent reasoning, e.g. by assuming others never bid above their true values. But such reasoning is seldom consistent with results from other settings, and CHC's specification greatly enhances the model's explanatory power, as will be seen. As in previous analyses, Brocas et al. took *L1* to best respond to *L0,* and *L2* to best respond to *L1*. Following CI (2007a), we call an *L1* that best responds to a random *L0* a "random *L1*" even though it need not itself be random; and we call an *L2* that best responds to a random *L1* a "random *L2*".[39]

Given this specification, random *L1* player 1s will Bet if told {C} because it yields 20 > 10 if player 2 Bets. Unlike in equilibrium, random *L1* player 1s will also Bet if told {A or B} because it yields 25 in state {A} and 5 in state {B}; random *L0* player 2s will Bet with probability one-half in {A} or {B}; and the two states are equally likely, so Betting yields expected payoff (25 +

[38] Trivial equilibria also exist, in which players do not bet because their partners do not bet, though this is weakly dominated.
[39] Compare Milgrom and Stokey's (1982, p. 23) Case A "Naïve Behavior," in which a player simply best responds to his prior. This refusal to draw contingent inferences from others' willingness to bet is implied by random *L1*'s random model of others. Milgrom and Stokey's Case B "First-Order Sophistication" is then equivalent to CI'our random *L2*.

5)/2 = 15 > 10. Random *L1* player 2s will Pass if told {A}, because it yields 0 < 10. Unlike in equilibrium, random *L1* player 2s will Bet if told {B or C}, because it yields 30 in state {B} and 5 in state {C}; random *L0* player 1s will Bet with probability one-half in {B} or {C}; and the two states are equally likely, so Betting yields expected payoff (30 + 5)/2 = 17.5 > 10. Similarly, Random *L2* or *L3* player 1s will Pass if told {A or B} but Bet if told {C}; Random *L2* player 2s will Pass if told {A} but Bet if told {B or C}; and Random *L3* player 2s will Pass in any state.

Brocas et al.'s data analysis finds clusters of subjects corresponding to *L1*s, *L2*s, and *L3*s, and a fourth cluster of apparently irrational subjects. Their mixture of level-*k* types tracks the patterns of subjects' decisions much better than any alternative model, including equilibrium plus noise.[40]

In related work, Camerer (2003, Chapter 6) and Tomasz Strzalecki (2010) conduct level-*k* analyses of Rubinstein's (1989) electronic mail game, showing that the models' bounded depths of reasoning make plausible predictions that are independent of the tail assumptions on higher-order beliefs that lead to Rubinstein's behaviorally unrealistic equilibrium-based predictions for that game. Rogers et al. (2009) conduct a horse race between LQRE and CH for similar betting games, in which a flexible "truncated heterogeneous LQRE" model fits better than CH or LQRE.

Carrillo and Palfrey (2009) analyze two-person games where players with privately known strengths can decide whether to fight or compromise. In equilibrium players always fight because as in zero-sum betting, they have opposing interests about when to compromise. But subjects compromise 50-70% of the time; and more often, the higher the compromise payoff. Finally, in a horse race among LQRE, CH, and "cursed equilibrium" (Erik Eyster and Matthew Rabin 2005, "ER"; Section 5.2), the results favor a blend of LQRE and cursed equilibrium.

5.2. *Auction Experiments*

There is a rich literature on sealed-bid asymmetric-information auction experiments, which developed independently of the literature on game experiments, despite similar goals. In auction experiments subjects' initial responses tend to exhibit overbidding relative to the risk-neutral Bayesian Nash equilibrium, whether the auction is first- or second-price, independent-private-value or common-value (e.g. Kagel and Levin 1986, "KL", and Goeree et al. 2002). The literature proposes to explain overbidding by "joy of winning" or risk-aversion in independent-private-value auctions, or by the winner's curse in common-value auctions. Those explanations

[40] *L3*s in this 3-dominance-solvable game correspond to *Equilibrium* players. Brocas et al.'s analysis of their lookup data reinforces their level-*k* interpretation of their decision data, and is evidence against LQRE or even CH.

are only loosely related to those proposed for deviations from equilibrium in other games, and it would plainly be useful to unify the explanations of nonequilibrium behavior in common-value auctions and other asymmetric-information games where informational inferences are relevant; and also to unify the explanations for common-value and independent-private-value auctions.

KL and ER took a first step, formalizing the intuition behind the winner's curse in models in which "naïve" bidders do not adjust their value estimates for the information revealed by winning, but otherwise follow Bayesian equilibrium. ER's notion of cursed equilibrium, in which people do not fully take into account the correlation between others' decisions and private information but otherwise follow equilibrium logic, generalizes KL's model to allow levels of value adjustment from equilibrium with full adjustment to "fully-cursed" equilibrium with no adjustment; and from auctions to other kinds of asymmetric-information games.[41] KL's and ER's models allow players to deviate from equilibrium only in their informational inferences, and reduce to equilibrium in independent-private-value auctions or symmetric-information games.

CI (2007a) propose a level-*k* analysis to unify the explanations of deviations from equilibrium in initial responses to independent-private-value or common-value auctions or to other kinds of games, without invoking joy of winning, risk-aversion, or cognitive biases.

The main issue is how to specify *L0*. In auctions there are two leading possibilities: *Random L0*, as in the above analysis of zero-sum betting, bids uniformly between the lowest and highest possible values, independent of its own value. *Truthful L0* bids its expected value conditional on its own signal, which is meaningful in auctions. CI build separate type hierarchies on these *L0*s, stopping for simplicity at *L2*: *Random* (*Truthful*) *Lk* is defined by iterating best responses from *Random* (*Truthful*) *L0*. CI allow each subject to be one of the types, from either hierarchy.

CI show that most conclusions of equilibrium auction theory are robust to deviations from equilibrium structured by a level-*k* model. An *Lk* type's optimal bid must take into account value adjustment for the information revealed by winning in common-value auctions, and the bidding trade-off between the higher price paid if the bidder wins and the probability of winning in first-price auctions. The level-*k* model allows a tractable characterization of those issues, which closely parallels Milgrom and Robert J. Weber's (1982) equilibrium-based characterization.

With regard to value adjustment, Random *L1* does not update on winning because its Random *L0* model of others bids independently of their values, and so Random *L1* is fully cursed

[41] ER also show that cursed equilibrium can explain zero-sum betting with a probability that is positive but less than one.

in ER's sense. Thus the random type hierarchy unifies an explanation of informational naïveté with explanations of nonequilibrium strategic thinking that have had success in other settings. Other types condition on winning in various ways. This conditioning tends to make bidders' bids strategic substitutes, in that if it's bad news that you beat equilibrium bidders, it's even worse to have beaten overbidders; thus the higher are others' bids, the greater the value adjustment. The bidding tradeoff, by contrast, can go either way, just as in an equilibrium analysis.

Most equilibrium results survive, qualitatively, in the level-*k* analysis. The main exceptions are those that rely on bidders' ex ante symmetry. They are altered, even though players are objectively symmetric, because level-*k* players have simpler models of others than of themselves.

CI ask whether an estimated mixture of level-*k* types fits the initial response data from classic auction experiments better, taking the numbers of parameters into account, than equilibrium plus noise; cursed equilibrium; or for private-value auctions, LQRE. In three of four leading cases, a level-*k* model does better than the alternatives. In the fourth case, KL's first-price auction, the most flexible cursed-equilibrium specification has a small advantage; but that disappears when the cursed-equilibrium specification's number of parameters is made comparable to that of the level-*k* model. Except in KL's second-price auctions, where many subjects seem to have been confused, the estimated type frequencies are similar to those estimated for non-auction experiments, with results generally favoring the random over the truthful type hierarchy.

5.3. *Acquiring a Company and the Winner's Curse*

Charness and Levin (2009, "CL") report experiments that seek to test whether the winner's curse is due to nonequilibrium strategic thinking or cognitive failures of individual optimization when it requires complex inferences. CL's experiments are based on William F. Samuelson and Max H. Bazerman's (1985) "Acquiring a Company" game, a game-theoretic analog of a "lemons" market. There are two risk-neutral players, a bidder and a responder. The responder owns an indivisible object and the bidder makes a single bid for it. If the bidder accepts the bid the object is transferred at the bid price; if not, there is no deal. The value of the company to the responder is an integer between 0 and 100 inclusive, with each of these values equally likely. Only the responder observes his value, before he must decide whether to accept; but the bidder knows that, whatever the value, it is 50% larger for him than for the responder; and this fact and the value distribution are common knowledge.

This game has an essentially unique perfect Bayesian equilibrium, in which the bidder bids zero and the responder rejects that bid, but would accept any offer greater than his value. In equilibrium the bidder draws a contingent inference from the responder's willingness to accept, like those required to overcome the winner's curse or avoid losing a zero-sum bet. If the bidder offers $x > 0$, the responder will accept only if his value is less than $x$, so conditional on acceptance, given the uniform distribution, the responder's expected value is $x/2$ and the bidder's is $3x/4$. Thus accepting loses the bidder $x/4$ on average, his optimal bid is 0, and no transfer will occur, even though it is common knowledge that a transfer is mutually beneficial at some prices.

Despite this clear equilibrium prediction, experimental subjects often trade, at prices that seem to split the expected gains from trade from an ex ante point of view, even though the responder observes his value before the game is played. CL's design uses "robot" treatments in which a bidder's decisions determine his payoffs the same way a rational responder's decisions determine his payoffs in Acquiring a Company, but the robot responder is framed not as another player but as part of the game. (John S. Carroll et al. 1988 ran the same treatment, their "beastie run", with similar results.) But because the robot problem involves no decisions by others, cursed equilibrium or level-*k* players, taken literally, should get its informational inferences right. But CL's subjects in the robot treatment do not get it right, overbidding as much as in a normal (non-robot) treatment. On that basis CL argue that cursed equilibrium or level-*k* models do not explain the overbidding commonly observed in normal Acquiring a Company treatments.

In our view Carroll et al.'s and CL's results do not undermine the support for level-*k* or cursed-equilibrium models from more conventional experiments. Those models were originally formulated for settings in which the main difficulty is predicting and responding to others' decisions, simplifying other aspects of the problem to explore their implications as transparently as possible. There is no reason to expect such formulations to translate unmodified to settings in which the complexity has been shifted from the "other people" part of the problem to the "own decisions" part. CL's failure to find significant differences across normal and robot treatments would not be evidence against the most obvious generalizations of level-*k* or cursed-equilibrium models.[42] CL's results simply highlight the need for more general models that addresses both the cognitive difficulty of predicting others' responses to a game and drawing inferences from them,

[42] However, we doubt that such mechanical generalizations would be empirically very useful, because we suspect people use different cognitive routines in games against real other players than in decision problems based on theoretical other players.

and the difficulty of drawing analogous inferences in decision problems. Future work should further map the domains of applicability of existing models and identify specific models that explain why people do not always make rational or equilibrium responses, integrating what cursed-equilibrium or level-*k* models get right with better models of the aspects of cognition they do not adequately address, such as reasoning contingent on future events.

Asen Ivanov et al. (2010; "ILN") continue CL's investigation. Their design is based on Jeremy Bulow and Paul Klemperer's (2002) Maximum Game, a second-price common-value auction in which both bidders' values equal the maximum of their independent and identically distributed value signals. The Maximum Game is weakly dominance-solvable in two steps, with truthful bidding its unique equilibrium. ILN run three treatments, each with two parts. In part I subjects are randomly paired to play the game with value signals sampled without replacement, enough times that a subject's bids generate a function mapping each possible value into a bid. In part II subjects play the game with the same possible value signals in a different, randomized order, but now against a computer "robot" that uses the subject's own bidding function from part I. ILN argue that if overbidding in part I is due to non-equilibrium beliefs, subjects who overbid in part I should overbid less in part II because the best-responses to their part I bids are then lower than their part I bids. Most of ILN's subjects overbid for at least one value signal in part I. ILN used criteria based on the data to select the subset of their subjects whose data they analyzed. For those subjects much of the part I overbidding persists in part II, and there was no significant difference in the median response for any signal. ILN conclude from this failure to find significant differences that beliefs-based theories cannot explain their results.

Costa-Gomes and Makoto Shimoji (2011) critique ILN's criteria for selecting subjects and reanalyze ILN's data without excluding any subjects. They show that ILN's prediction of less overbidding in part II can only be applied to the overbidding of one third of their subjects, and that half of such data actually conforms to the prediction, undermining ILN's rejection of beliefs-based theories. Costa-Gomes and Shimoji also show that ILN's theoretical comparative-statics argument is incomplete for at least one beliefs-based model, in that their predictions are based solely on random *L1*, but as in CI's (2007a) analysis of auctions, random *L2* can deviate from equilibrium in the opposite direction, reversing the comparative statics.

Salvatore Nunnari et al. (2011) also reanalyze ILN's data without excluding subjects. Noting that ILN's design very weakly separates alternative theories, they show that if one allows decision noise, ILN's results are fully consistent with a wide range of beliefs-based models, including cursed equilibrium, CH, QRE, or hybrids of CH and QRE.

We also note that ILN's inferences are based entirely on their failure to find significant differences in the direction they argue is predicted by beliefs-based models. Yet the Maximum Game requires very subtle inferences. As Bulow and Klemperer said of its equilibrium predictions, "…the Maximum Game, provides a good illustration of how a different choice of value function…can make it easy to obtain extreme 'perverse' results." ILN's data are extremely noisy, with 25% of subjects' bids 10 or more (in many cases, thousands) times higher than the largest possible value.[43] With such a low signal-to-noise ratio, negative inferences based on a failure to find significant differences across treatments could be used to reject any theory. ILN's paper makes clear the importance of experimental analyses that, like most of those discussed in Section 3, compare specific positive predictions based on complete models of behavior, evaluated via individual-level analyses of datasets from designs that generate coherent results.

5.4. *Field Studies: Movie Opening and Lowest Unique Positive Integer Games*

Alexander Brown, Camerer, and Dan Lovallo (2012) use field data to study an asymmetric-information signaling game with verifiable signals. Film distributors face a choice between "cold opening" a movie and pre-releasing them to critics in the hope that favorable reviews will increase profits. In perfect Bayesian equilibrium, cold-opening should not be profitable, because moviegoers will infer low quality for cold-opened movies; as a result there should be no cold opening, except possibly by the worst movie type. Yet distributors sometimes cold-open movies, and in a set of 1303 widely released movies, cold opening increased domestic box office revenue (though not foreign or for DVD sales) by 10-30% over movies of similar quality that were reviewed before release. Further, ex post fan ratings on the Internet Movie Database were lower for cold-opened movies. Brown et al. use a CH model to explain these results. Both features suggest that moviegoers had unrealistically high expectations for cold-opened movies, which is hard to explain using theories others than CH or level-*k*. However, movie distributors do not appear to take advantage of moviegoers' lack of sophistication, since only 7% of movies were opened cold despite the expected-profit advantage.

Östling et al. (2011) study a novel set of field data from a Swedish gambling company, which ran a competition for a short period of time involving a "lowest unique positive integer" or LUPI

[43] This exposed subjects to large negative payoffs for which they could not be held liable, thus losing control of preferences.

game. (They also studied experimental data from parallel treatments.) In the LUPI game, players strategically simultaneously pick positive integers and the player who chose the lowest unique (that is, not chosen by anyone else) number wins a prize. Except for the uniqueness requirement, the game is like a first-price auction with the lowest bid winning.

The game would have symmetric information except that participants had no way to know how many others would enter on a given day. The authors deal with this by adapting Myerson's (2000) Poisson games model, in which fully rational players face Poisson-distributed uncertainty about the number of players. They characterize the LUPI game's unique symmetric Poisson-Nash equilibrium, and compare it to the predictions of versions of QRE and CH models, using both the field data and data from experiments using a scaled-down version of the LUPI game.

In both the field and the laboratory, participants choose very low and very high numbers too often relative to the Poisson-Nash equilibrium, and avoid round and salient numbers.[44] However, initial responses are surprisingly close to the equilibrium, even though the setting surely prevents participants from computing it. Learning brings them even closer to equilibrium in later periods.

In comparing the data to the predictions of versions of QRE and CH, Östling et al. assume that both have power rather than the usual logit error distributions, and they allow the CH types to best respond to the noise in others' decisions.[45] They find that relative to the Poisson-Nash equilibrium, power QRE predicts too few low-number choices, deviating from equilibrium in the wrong direction, while CH predicts the pattern observed in the field data.

### 5.5. *Level-k Mechanism Design*

A number of recent papers reconsider mechanism design taking a "behavioral" view of individual decisions or probabilistic judgment, but there are few analyses of design outside the equilibrium paradigm. Replacing equilibrium with a model that better describes responses to novel games should allow us to design more effective mechanisms. It also suggests an evidence-based way to assess the robustness of mechanisms, something previously left to intuition.[46]

Crawford et al. (2009) relax the equilibrium assumption in mechanism design, maintaining standard rationality assumptions regarding decisions and probabilistic judgment. They conduct a

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[44] Salience plays a similar role in CI's (2007b) analysis of hide-and-seek games (Section 8.1).

[45] A standard CH model would not fit the LUPI data well: *L1* would choose 1, *L2* 2, *L3* 3 or less, and *Lk k* or less. But best responding to power errors sometimes makes *L2*'s modal choice higher than 5 (Östling et al., Figure A6)). This is not a criticism of Östling et al.'s CH analysis, but a general limitation of the structural features of models like level-*k* or CH.

[46] A mechanism that implements the desired outcome in dominant strategies or after a small number of rounds of iterated dominance will evoke the desired response from most level-*k* types that are likely to be observed. It may therefore perform better in practice than a mechanism that can theoretically implement better outcomes, but only in equilibrium.

level-*k* analysis of optimal sealed-bid auctions with symmetric bidders who have independent private values, for which Myerson (1981) gives a complete equilibrium-based analysis. Their model follows CI's (2007a) level-*k* analysis of behavior in auction experiments, with either a random *L0* that bids uniformly over the range of possible bids or a truthful *L0* that bids its private value. Bidders are drawn from a given population of level-*k* types, known to the designer.

With independent private values, revenue-equivalence fails because level-*k* types respect simple dominance, hence a second-price auction reproduces the expected revenue of equilibrium bidders, while a first-price auction may do better, because types like Random *L1* tend to overbid. The optimal reserve price may be large with equilibrium bidders but small with level-*k* bidders, or vice versa. In theory, a designer can use exotic auction forms to exploit level-*k* bidders' nonequilibrium beliefs to obtain very large expected revenues. But as Crawford et al. note, a general formulation of the design problem must take a position on how the design influences the rules that describe bidders' behavior, and develop methods to deal with that influence.

Crawford (2012) relaxes the equilibrium assumption in favor of a similar level-*k* model in Myerson and Mark Satterthwaite's (1983) analysis of efficient bilateral trading mechanisms with independent private values. In the leading case of uniform value distributions, when attention is restricted to incentive-compatible mechanisms (defined for level-*k* rather than equilibrium traders), the double auction Myerson and Satterthwaite showed to be equilibrium-incentive-efficient in that case remains efficient in the set of level-*k*-incentive-compatible mechanisms for a wide class of level-*k* models. But the revelation principle fails with level-*k* traders. Just as incentive-compatible mechanisms may counteract the excessive aggressiveness of *L1* traders in the double auction, non-incentive-compatible direct mechanisms may increase the efficiency of trading above the Myerson-Satterthwaite bound for incentive-compatible mechanisms by tacitly exploiting traders' non-equilibrium beliefs to make them bargain less aggressively.

## 6. *Kahneman's Entry Magic: Coordination via Symmetry-Breaking*

"…to a psychologist, it looks like magic."
—Kahneman (1988), quoted in CHC

This section discusses level-*k* models of symmetric-information market entry games, in which *n* subjects choose simultaneously between entering ("In") or staying out ("Out") of a market with given capacity. In yields a given positive profit if the number of entrants exceeds a given market capacity; but a negative profit if too many enter. Out yields profit of 0, no matter how many enter. In these games efficient coordination requires breaking the symmetry of players' roles, so that the capacity will be exactly filled. Because subjects cannot distinguish one another, it is not sensible to predict systematic differences in their behavior, and the natural equilibrium benchmark is therefore the unique, symmetric mixed-strategy equilibrium, in which each player enters with a given probability that makes all players indifferent between In and Out. That equilibrium is inefficient, yielding an expected number of entrants approximately equal to market capacity, but with a positive probability that either too many or too few enter. The "magic" to which Kahneman refers is that even in their initial responses to the game, subjects' independent decisions came surprisingly close to the aggregate choice frequencies of the symmetric equilibrium. Perhaps more surprisingly, their ex post coordination was systematically better (number of entrants stochastically closer to market capacity) than in the equilibrium.

The section begins, following CHC's (Section III.C) CH analysis, by using a level-*k* model to analyze Battle of the Sexes, which is like a two-person entry game with capacity one. A level-*k* model resolves both puzzles by showing that the heterogeneity of strategic thinking mimics the mixed-strategy equilibrium's decision frequencies and allows some players to mentally simulate others' decisions and accommodate them, which in these games yields better coordination than in the mixed-strategy equilibrium. The model suggests a behaviorally plausible alternative to the traditional view of coordination, which has important implications in other applications. The section concludes by discussing of Avi Goldfarb and Botao Yang's (2009) and Goldfarb and Mo Xiao's (2011) CH empirical analyses of market entry with asymmetric information in the field.

6.1. *A Level-k Analysis of Two-Person Entry/Battle of the Sexes Games*

Consider a two-person Battle of the Sexes game with $a > 1$, as in Figure 4. The unique symmetric equilibrium is in mixed strategies, with $p \equiv \Pr\{\text{In}\} = a/(1+a)$ for both players. The mixed-strategy equilibrium expected coordination rate is $2p(1 - p) = 2a/(1+a)^2$, and players' equilibrium expected payoffs are $a/(1+a)$. This expected coordination rate is maximized when $a = 1$, where it takes the value ½. With $a > 1$ the expected payoffs are $a/(1+a) < 1$: worse for each player than his worst pure-strategy equilibrium. As $a \to \infty$, $2a/(1 + a)^2 \to 0$ like $1/a$.

Now consider a level-*k* model in which each player follows one of four types, *L1*, *L2*, *L3*, or *L4*, with each role filled by a draw from the same distribution. For simplicity assume the frequency of *L0* is 0, and that *L0* chooses uniformly randomly, with Pr{In} = Pr{Out} = 1/2.

| | **In** | **Out** |
|---|---|---|
| **In** | **0**, **0** | ***a***, **1** |
| **Out** | **1**, ***a*** | **0**, **0** |

**Figure 4. Battle of the Sexes**

| **Type pairings** | ***L1*** | ***L2*** | ***L3*** | ***L4*** |
|---|---|---|---|---|
| ***L1*** | **In, In** | **In, Out** | **In, In** | **In, Out** |
| ***L2*** | **Out, In** | **Out, Out** | **Out, In** | **Out, Out** |
| ***L3*** | **In, In** | **In, Out** | **In, In** | **In, Out** |
| ***L4*** | **Out, In** | **Out, Out** | **Out,In** | **Out, Out** |

**Table 2. Outcomes in Battle of the Sexes**

*L1*s mentally simulate *L0*s' random decisions and best respond, thus, with $a > 1$, choosing In; *L2*s choose Out; *L3*s choose In; and *L4*s choose Out. The predicted outcome distribution is determined by the outcomes of the possible type pairings (Table 2) and the type frequencies. If both roles are filled from the same distribution, players have equal ex ante payoffs, proportional to the expected coordination rate. *L3* behaves like *L1*, and *L4* like *L2*. Lumping *L1* and *L3* together and letting $v$ denote their total probability, and lumping *L2* and *L4* together, the expected coordination rate is $2v(1 - v)$, maximized at $v = ½$ where it takes the value ½. Thus for $v$ near ½, which is behaviorally plausible, the coordination rate is near ½. For more extreme values the rate is worse, converging to 0 as $v \to 0$ or 1. But because the equilibrium rate of $2a/(1 + a)^2 \to 0$ like $1/a$, even for moderate values of $a$, the level-*k* coordination rate is higher.[47]

The level-*k* analysis suggests a view of tacit coordination quite different from the traditional view, and illustrates the importance of the heterogeneity of strategic thinking the model allows. With level-*k* thinking, equilibrium and refinements like risk- or payoff-dominance play no role in

[47] This analysis highlights a drawback of level-*k* models, in that without payoff-sensitive errors, their predictions are independent of $a$ as long as $a > 1$, while behavior is often sensitive to such parameter variations. Adding payoff-sensitive errors would help to remedy this, but probably not enough to make the models fully descriptive of observed behavior. CHC (Section III.C) and Chong et al. (2005, Section 2.1) argue that in this context, CH models fit better than level-*k* models because they yield smooth monotonicity of entry rates as market capacity increases, while a level-*k* model implies a step function. However in most of the datasets CHC consider, unlike in their CH model, there are congestion effects that allow payoff-sensitive logit errors like those in a typical level-*k* analysis to smooth entry as well. As CHC show, their CH model can produce entry which is monotonic in market capacity and approximates the mixed-strategy equilibrium; but it can also explain the facts that subjects tend to over-enter at low market capacities and under-enter at high capacities, which equilibrium cannot.

players' thinking. Coordination, when it occurs, is an accidental (though statistically predictable) by-product of the use of nonequilibrium decision rules. Even though players' decisions are simultaneous and independent, the heterogeneity of strategic thinking allows more sophisticated players such as *L2*s to mentally simulate the decisions of less sophisticated players such as *L1*s and accommodate them, just as Stackelberg followers would do. Mental simulation doesn't work perfectly, because an *L2* is as likely to be paired with another *L2* as an *L1*. Neither would it work if strategic thinking were homogeneous. But it's very surprising that it works at all.

6.2. *Field Studies: CH Analyses of Entry Games*

The same issues arise in Goldfarb and Yang's (2009) and Goldfarb and Xiao's (2011) field studies of asymmetric-information entry games. These studies are of particular interest because they are among the first studies of nonequilibrium models of strategic thinking using field data.

Goldfarb and Yang (2009) apply an asymmetric-information CH model to explain choices by managers at 2,233 Internet Service Providers (ISP) in 1997 whether or not to offer their customers access through 56K modems versus the standard then, 33K modems. There were two possible 56K technologies, one by Rockwell Semiconductor and one by US Robotics. Thus an ISP manager had four alternatives: (i) adopt neither technology, (ii) adopt Rockwell's, (iii) adopt US Robotics's, or (iv) adopt both. Controlling for market and ISP-specific characteristic, Goldfarb and Yang adapted the CH model to describe the heterogeneity in ability or strategic sophistication among the ISP managers in these decisions. They assumed (departing from the usual *L0* specification) that an *L0* manager maximizes profits on the assumption that he will be a monopolist; an *L1* manager on the assumption that his competitors will be *L0*s; an *L2* manager on the assumption that his competitors will be an estimated mixture of *L0*s and *L1*s, and so on. They found significant heterogeneity of sophistication among managers, with an estimated $\tau$, the average $k$ in a CH model, of 2.67—seemingly higher than most previous estimates, but their *L0* is in some respects akin to an *L1*, which would bring it more in line with previous estimates.

Goldfarb and Yang's CH model fits no better than a Bayesian equilibrium plus noise model, but their CH estimates have interesting and plausible implications. Interestingly, they suggest that relative to equilibrium, heterogeneity of strategic thinking slowed the diffusion of the new 56K technology, with more sophisticated managers *less* likely to adopt, anticipating more competition. Managers behaved more strategically, in the sense of higher estimated $k$s, if they competed in larger cities, with more firms, or in markets with more educated populations.

Finally, those managers estimated as more strategic in 1997 were more likely to survive through April 2007. We note however that in a CH model, though not a level-*k* model, a higher *k* implies a more accurate model of others' fitted decisions, and hence higher predicted expected profits against the heterogeneous population of players it faces. Thus, in a CH model a firm that does well in the market must have had a higher *k*. Only a model that allows the possibility that a firm might err by perceiving others as being of a higher level than they are allows independent inferences about a firm's level of sophistication and its beliefs about others' sophistication.

Goldfarb and Xiao (2011) applied a similar CH model to explain managers' choices whether to enter local U. S. telecommunications markets after the *Telecommunications Act* of 1996, which allowed free competition. Using Goldfarb and Yang's (2009) *L0* specification, they found that more experienced or better educated managers did better, entering markets with fewer competitors, on average; having better survival rates; and having higher revenues conditional on survival. Estimated sophistication rises from 1998 to 2002. The CH model fits much better than an equilibrium plus noise model in 1998, but only slightly better in 2002, in keeping with the view that models like CH are best suited to initial responses to novel situations.

## 7. *Bank Runs: Coordination via Assurance*

> "A crude but simple game, related to Douglas Diamond and Philip Dybvig's (1983) celebrated analysis of bank runs, illustrates some of the issues involved here. Imagine that everyone who has invested $10 with me can expect to earn $1, assuming that I stay solvent. Suppose that if I go bankrupt, investors who remain lose their whole $10 investment, but that an investor who withdraws today neither gains nor loses. What would you do? Each individual judgment would presumably depend on one's assessment of my prospects, but this in turn depends on the collective judgment of all of the investors.
>
> Suppose, first, that my foreign reserves, ability to mobilize resources, and economic strength are so limited that if any investor withdraws I will go bankrupt. It would be a Nash equilibrium (indeed, a Pareto-dominant one) for everyone to remain, but (I expect) not an attainable one. Someone would reason that someone else would decide to be cautious and withdraw, or at least that someone would reason that someone would reason that someone would withdraw, and so forth. This…would likely lead to large-scale withdrawals, and I would go bankrupt. It would not be a close-run thing. …Keynes's beauty contest captures a similar idea.
>
> Now suppose that my fundamental situation were such that everyone would be paid off as long as no more than one-third of the investors chose to withdraw. What would you do then? Again, there are multiple equilibria: everyone should stay if everyone else does, and

> everyone should pull out if everyone else does, but the more favorable equilibria [sic] seems much more robust."
>
> —Lawrence H. Summers (2000)

Summers (2000) views bank runs as an *n*-person coordination game with Pareto-ranked equilibria, a kind of generalized Stag Hunt game as in Diamond and Dybvig's (1983) model. This section uses level-*k* models to study coordination in such games.

Summers' game can be represented by a payoff table as in Figure 5. The summary statistic measures whether the required number of investors stays In. In Summers's first example, all investors must stay In to prevent collapse, so the summary statistic equals In if and only if all but the representative player stay In. In his second example two-thirds of the investors must stay In, so the summary statistic equals In if and only if this is the case, including the player himself. Each example has two pure-strategy equilibria: "all-In" and "all-Out". All-In is Pareto-superior to all-Out, but it is also more fragile, with payoffs more vulnerable to deviations by others.

| | | **Summary statistic** | |
|---|---|---|---|
| | | **In** | **Out** |
| **Representative player** | **In** | **1** | **-10** |
| | **Out** | **0** | **0** |

**Figure 5. Bank Runs**

Summers' discussion presumes that some equilibrium will govern play. To capture his intuitions in a model it is necessary to complete equilibrium by adding a refinement that selects a unique equilibrium in these games, such as Harsanyi and Selten's (1987) notions of payoff-dominance or risk-dominance. Payoff-dominance selects an equilibrium that is not Pareto-dominated by any other equilibrium, in this case "all-In", for any population size *n*.

Risk-dominance selects the equilibrium with the largest "basin of attraction"—the set of initial beliefs that yield convergence of best responses to that equilibrium, assuming players' beliefs are independent. In two-person games like Summers' examples, risk-dominance therefore selects the equilibrium that results if each player best responds to a uniform prior over others' strategies. For Summers' payoffs, whether all investors or only two-thirds must stay In to prevent collapse, risk-dominance selects the all-Out equilibrium for any *n*. Even with much less extreme payoffs, say with -1.5 replacing -10, and with only two-thirds In needed to prevent collapse, risk-dominance selects the all-Out equilibrium for any *n*, because no *n* makes the probability at least 0.6 that at least two-thirds of *n*-1 independent Bernoulli trials yield In.

As Summers suggests, coordination on the all-In equilibrium is behaviorally implausible in his examples, even for small *n*. Neither does risk-dominance fully reflect his (or our) intuition, because all-In is "much more robust" in the second example, but all-Out remains risk-dominant.

Many people are skeptical of risk-dominance as a model of strategic thinking. Many of them, possibly including Summers (p. 7, footnote 9) are reassured by the fact that equilibrium selection in bank-runs games can be predicted via a "global games" analysis as in Morris and Shin (1998) or Frankel et al. (2003), which has become the standard model of equilibrium selection in bank-runs games. Global games replaces the original symmetric-information game with a version with privately observed payoff perturbations that satisfy certain distributional assumptions. In bank-runs games and some other games, the perturbed version is dominance-solvable with a unique equilibrium. Thus the global games analysis implies unique equilibrium selection without recourse to a refinement. But in the simplest bank-runs games, including Summers' examples, global games again selects the original game's risk-dominant equilibrium.

We now argue that a level-*k* analysis has stronger behavioral foundations than the global games approach. The two approaches yield similar conclusions in the simplest bank-runs games; but a level-*k* analysis yields conclusions that differ in important ways in other games.

Consider a level-*k* analysis of the game in Figure 5. *L1* best responds to the distribution of others' decisions from independent draws from a uniform random *L0*. Given the symmetry of the game, *L1* players all have the same response, as do *L2* players, and so on. Thus any level-*k* model in which the frequency of *L0* is 0 selects the risk-dominant equilibrium with probability 1, as in the global-games analysis (see for example CHC, Section III.B).

This result can be viewed as establishing the robustness of a global-games analysis, working with the originally specified game rather than an artificial perturbed game, and avoiding the need to assume more iterated dominance than most people's thinking respects.

But in more complex games a level-*k* model may yield conclusions that differ from global games. More importantly, *L0* makes it easy to combine realistic models of strategic thinking with more nuanced views of market psychology, such as how players model the correlation of others' decisions, an issue on which evidence raises doubts about the standard view (HCW; Costa-Gomes et al. 2009) but which global games analyses have ignored.[48]

[48] The correlation of players' models of others is irrelevant in defining payoff-dominance. Risk-dominance is traditionally defined assuming independence, but its definition is easily modified to allow such correlation. Perfect correlation makes level-

As in Section 6's market-entry games, level-*k* models suggest a view of coordination that seems behaviorally more plausible than the traditional refinement-based approach. Level-*k* players use the same rules to choose their strategies with or without multiple equilibria, and coordination when it occurs is a statistically predictable by-product of how empirically grounded rules of thumb interact with the game—though this time there is no "magic", because no symmetry-breaking is required. Further, a level-*k* model also predicts the likelihood of coordination failure and the forms it may take.

## 8. *Salient Labels in Outguessing and Coordination Games*

Because the labeling of players and strategies does not affect payoffs, it is traditionally excluded from consideration in equilibrium analysis; but it would be surprising if behavior did not respond to salient labels. Salience strongly influenced behavior in Schelling's (1960) classic experiments with coordination games, and in Crawford et al.'s (2008) and Bardsley et al.'s (2010) replications. And in Rubinstein and Tversky's ("RT", Rubinstein 1999) experiments on zero-sum two-person hide-and-seek games played on non-neutral "landscapes" of salient location labels, subjects deviated systematically from the unique mixed-strategy equilibrium in patterns that responded to the labeling, even though the essential uniqueness of equilibrium seems to preclude any such influence. Salience also plays a prominent role in folk game theory.

This section discusses level-*k* and team reasoning models of its effects in outguessing and coordination games. It concludes with a discussion of directions for future work to identify the ranges of applicability of the models and explore the foundations of level-*k* models with salience.

### 8.1. *Hide and Seek Games with Salient Labels*

> "Any government wanting to kill an opponent…would not try it at a meeting with government officials."
>
> —comment, quoted in Chivers (2004), on the poisoning of Ukrainian presidential candidate—now ex-president—Viktor Yushchenko

> "…in Lake Wobegon, the correct answer is usually 'c'."
>
> —Garrison Keillor (1997) on multiple-choice tests (quoted in Yigal Attali and Maya Bar-Hillel (2003)

---

*k* players perceive examples like Summers' as quasi-two-person games. Depending on the payoffs and the fragility of the all-In equilibrium, this can make all-In more or less likely to prevail than when players' models of others are independent.

The Yushchenko and Lake Wobegon quotations refer to simultaneous-move zero-sum two-person games with unique mixed-strategy equilibria. In the first, the players are an assassin choosing one of several dinners at which to try to poison Yuschenko, one of which is with officials of the government suspected of wanting to poison him; and an investigator who can check only one of the dinners. In the second the players are a test designer deciding where to hide the correct answer and a clueless test-taker trying to guess where it is hidden. In each case the key issue is how to react to a pattern of salience in the labeling of strategies that does not affect payoffs. The thinking in the quotations is plainly strategic, but equilibrium in zero-sum two-person games leaves no room for reactions to the labels. Further, the thinking is plainly not equilibrium: A game theorist would respond to the first quotation, "If that's what people think, a meeting with government officials is exactly where the government *would* try to poison him."

RT conducted experiments with zero-sum, two-person "hide-and-seek" games with patterns of salience that closely resemble those in the Yushchenko and Lake Wobegon quotations. A typical seeker's instructions (Rubinstein 1999) were: "Your opponent has hidden a prize in one of four boxes arranged in a row. The boxes are marked as shown below: A, B, A, A. Your goal is, of course, to find the prize. His goal is that you will not find it. You are allowed to open only one box. Which box are you going to open?" A hider's instructions were analogous. RT's design is an abstract model of a game played on a non-neutral cultural or geographic landscape. The frame has no payoff consequences, but it is non-neutral in that the "*B*" location is distinguished by its label and the two "*end A*" locations may be inherently salient as well.[49]

RT's hide-and-seek game has a unique equilibrium prediction, whose logic is transparent and which leaves no room for framing to influence the outcome. Even so, framing had a strong and systematic effect in RT's experiments, qualitatively the same in their six experiments around the world where labeling created salience without positive or negative connotations, with *Central A* (or its analogs in other treatments) most prevalent for hiders (37% in the aggregate) and *Central A* even more prevalent for seekers (46%).[50] These results pose two puzzles. On average hiders are as smart as seekers, so hiders tempted to hide in *central A* should realize that seekers will be

[49] This gives the "central A" location its own brand of uniqueness as the "least salient" location. Mathematically this uniqueness is no different from the uniqueness of "*B*", but CI's (2007b) analysis suggests that its psychological effects are quite different.

[50] This statement depends on identifying analogies among RT's treatments as explained in CI (2007b, Section 1). One might argue that because any strategy, pure or mixed, is a best response to equilibrium beliefs, deviations do not violate the theory. But systematic deviations from equilibrium choice frequencies must (with high probability) have a cause that is partly common across players. They are therefore symptomatic of systematic deviations from equilibrium probabilities.

just as tempted to look there. Why then do hiders allow seekers to find them 32% of the time when they could hold it down to 25% by using the equilibrium mixed strategy? And why are the results role-asymmetric, with seekers choosing *central A* more often (46%) than hiders (37%)?

RT took the nonequilibrium patterns in their data as evidence that their subjects did not think strategically (CI 2007b, p. 1733, footnote 3). But it would be surprising if responses to such simple games were completely non-strategic, and the fact that subjects' behavior patterns were qualitatively the same in six independent treatments suggests that they have a common structure.

What kind of model can explain the role-asymmetric patterns in RT's data? RT's game and the role-asymmetric responses it evoked are an interesting test case, because although its payoff structure is asymmetric, models like equilibrium and QRE, which in this game coincides with equilibrium for any distribution and precision, imply role-symmetric responses. Even a level-*k* model with a uniform random *L0* coincides with equilibrium.

CI (2007b) used RT's data to compare adaptations of equilibrium, LQRE, and level-*k* models that allow salience an effect. They used RT's six treatments in which decision labels had neutral connotations, which presumably influenced behavior only via salience, to avoid confounding subjects' strategic thinking with their responses to positive or negative connotations.

CI adapted equilibrium or LQRE to this setting by adding payoff perturbation parameters that reflect plausible instinctive reactions to salience: a payoff gain for choosing a salient location for seekers, or a payoff loss for hiders.

Depending on the flexibility of the specification, LQRE gets the role-asymmetry in the data qualitatively wrong, or estimates an infinite precision and so turns itself back into an equilibrium with perturbations model, which also misses the role-asymmetry, in a less extreme way.

CI adapted the level-*k* model to settings where salience is important by assuming that *L0*'s strategically naïve initial assessment of others' likely responses to the game deviates from uniform randomness by favoring salient locations, either B or one of the end As, with no other changes. CI took *L0* to be determined by the same general principles in each player role, which in this setting means that *L0* is the same in each role, although its implications for *L1* differ across roles due to hiders' and seekers' different payoffs. CI made no assumption about whether B or the end As are more salient. But the facts that *L0* is the same in each role and that all that matters about it are the best responses it yields for hiders' and seekers' *L1*s imply that estimating B's and the end As' relative salience adds one binary parameter (CI, Figure 3): the minimum possible flexibility that allows salience to have an influence. Finally, the idea that level-*k* rules are meant to generalize across games suggests that they should generalize across player roles within this game, which suggests the restriction that the population distribution of rules is the same for hiders and seekers. That and the assumption that *L0* is determined by the same general principles in each role requires the model to explain the role-asymmetric patterns in the data endogenously, rather than via unexplained parameter variation across hiders and seekers.

Unrestricted estimates of the level-*k* rule distribution are almost hump-shaped (0% *L0*, 19% *L1*, 32% *L2*, 24% *L3*, 25% *L4*), as is plausible for a homogeneous population. Counter to our intuition, the end locations are estimated to be more salient than the B location. This *L0* and rule distribution tracks the observed prevalence of *central A* for hiders, its even greater prevalence for seekers, and the other main patterns in RT's data.[51] Despite RT's intuition, the analysis suggests that their subjects were unusually sophisticated, with an average *k* a level higher than usual; they just didn't follow the fixed-point logic of equilibrium. (The Yushchenko quotation, by contrast, reflects only the reasoning of an *L2* investigator reasoning about an *L1* poisoner.)

The level-*k* model with *L0* adapted to respond to salience is the only one of which we are aware that responds to the hide and seek game's asymmetric payoff structure in a way that can explain the robust role-asymmetric patterns in RT's data. CI's model has been criticized for having too much flexibility for its explanation to be credible. But all that matters about its *L0* is the best responses it yields for *L1*s, so the adaptation adds one binary parameter: the minimum flexibility that allows salience an influence, less than the adapted equilibrium or LQRE model.

8.2. *Portability to Other Outguessing Games*

To address the criticism that adapting *L0* to respond to salience gives the level-*k* model too much flexibility, CI (2007b) compared the equilibrium with payoff perturbations and adapted level-*k* models' portability, the extent to which a model estimated for one game can describe responses to other games.[52] They ported the models to the two closest relatives of RT's games that have been studied experimentally: Barry O'Neill's (1987) card-matching game and Rapoport and Richard B. Boebel's (1992) closely related outguessing game. Both games raise

[51] *L1* hiders then choose *central A* to avoid *L0* seekers and *L1* seekers avoid *central A*. *L2* hiders choose *central A* with probability between 0 and 1, breaking payoff ties randomly; and *L2* seekers choose it with probability 1. *L3* hiders avoid *central A* and *L3* seekers choose it with probability between zero and one. *L4* hiders and seekers avoid *central A*. The heterogeneity of *L1*, *L2*, and *L3* yields a role-asymmetric aggregate pattern that reconciles the prevalence of *central A* for hiders with its greater prevalence for seekers.

[52] CI also tested the models for overfitting by using them to compute estimates separately for each of RT's six treatments and using the estimated models to "predict" the results of the other five. The results moderately favored the level-*k* model.

the same strategic issues as RT's games, but with more complex patterns of wins and losses, different framing, and in the latter case five locations.

Here we discuss only the results for O'Neill's game. In it, players simultaneously choose one of four cards: A, 2, 3, J. One player wins if there is a match on J or a mismatch on A, 2, or 3; otherwise the other wins. The game is like hide and seek, but with each player a hider for some locations and a seeker for others. Without payoff perturbations it has a unique equilibrium, in which each player plays A, 2, and 3 with probability 0.2 and J with probability 0.4.

CI defined salience-sensitive versions of equilibrium for O'Neill's game by introducing payoff perturbations as for RT's game: a player gains (loses) payoff for a salient location in which he is a seeker (hider). They then used O'Neill's data to estimate the perturbations.[53] Even with these estimates, equilibrium with perturbations cannot explain subjects' initial responses better than equilibrium without perturbations, which explains them poorly.

CI adapted the level-*k* model to O'Neill's game by defining a non-strategic *L0* that favors A and J, which (as both face cards and end locations) are intuitively more salient than 2 and 3. With no assumption about which is more salient, the adaptation again adds a single binary parameter. CI used O'Neill's data to estimate that parameter, but instead of re-estimating the rule distribution they re-used the distribution estimated for RT's data.

Discussions of O'Neill's data (e.g. MP) have been dominated by an "Ace effect", whereby subjects in both player roles, aggregated over all 105 periods, played Ace with more than the equilibrium probability 0.2. O'Neill and others have speculated that this was a reaction to the Ace's salience. If this Ace effect extended to the initial periods of O'Neill's data, no behaviorally plausible level-*k* model could explain it, because no such model can make the players who win by matching on Joker play Ace with more than the equilibrium probability 0.2.[54] However, for the initial periods the Ace was in fact played with probability far less than 0.2. Taking up the slack was a hitherto unremarked positive Joker effect, an order of magnitude larger than the Ace effect. CI's adapted level-*k* model readily explains this Joker effect and the other patterns in the initial-period data, using RT's rule frequencies and an *L0* that favors the A and J cards.[55]

[53] Because O'Neill's experiment had only 25 subjects per role, CI treated a subject's first five periods of play as initial responses.
[54] A look at the payoff matrix shows that that would require either *L0* playing 2 and 3 more than A, or more than 60% *L3*s or *L4*s.
[55] Because CI's analysis is static and the Ace effect is negative for the first five periods, the Ace effect in the data aggregated over all 105 periods was almost surely a by-product of learning rather than the salience of the Ace.

CI's analysis traces the portability of the level-*k* model from RT's to O'Neill's and Rapoport and Boebel's games to the fact that the model compartmentalizes strategic thinking into an iterated best response component that is reasonably stable across those games and an *L0* based on portable nonstrategic intuitions about salience. If *L0* were strategic it would interact with each new game's structure in a new, high-dimensional way, and one could seldom extrapolate an *L0* specification across games. Here the definition of *L0* as a strategically naïve response is more than a convenient categorization: It is important for the model's portability.

8.3. *Coordination with Payoff Asymmetries and Salient Labels*

Crawford et al. (2008) and Bardsley et al. (2010) reported experiments on coordination via salient labels, revisiting Schelling's (1960) classic experiments. Crawford et al. randomly paired subjects to play payoff-asymmetric games like Battle of the Sexes and similar payoff-symmetric games. Unpaid pilots used naturally occurring labels like Schelling's, in this case the world-famous Sears Tower (now less famous as the "Willis Tower") and the little-known AT&T Building across the street. The salience of Sears Tower makes it obvious to coordinate on the "both-Sears" equilibrium (especially in Chicago where the experiments were run). In the symmetric version of the game, as in Schelling's experiments, most subjects did so (Figure 6).

Most researchers have assumed that this result is robust to slight payoff asymmetries, but games like the second and third games in Figure 6 pose a harder problem because both-Sears is one player's best way to coordinate but the other's worst way. Using Crawford et al.'s terms, there is a tension between the "label salience" of Sears and the "payoff salience" of a player's best way to coordinate: Payoff salience reinforces label salience for P2s but opposes it for P1s. In Crawford et al.'s pilots, the coordination rate crashed in the second and third games.

To investigate this phenomenon further, Crawford et al. ran paid treatments using abstract decision labels X and Y, with X presumed and shown to be more salient than Y; and with the same tension between label and payoff salience as in Chicago Skyscrapers (Figure 7). The expected coordination rate again crashed with only slight payoff differences. But unlike in Chicago Skyscrapers, the cause of miscoordination changed as the payoff differences grew: With slight differences, most subjects in both roles favored their partners' payoff-salient decisions; but with moderate or large differences they favored their own payoff-salient decisions.

**P2 (90% Sears)**

| **P1 (90% Sears)** | | **Sears** | **AT&T** |
|---|---|---|---|
| | **Sears** | **100,100** | **0,0** |
| | **AT&T** | **0,0** | **100,100** |

**Symmetric**

**P2 (58% Sears)**

| **P1 (61% Sears)** | | **Sears** | **AT&T** |
|---|---|---|---|
| | **Sears** | **100,101** | **0,0** |
| | **AT&T** | **0,0** | **101,100** |

**Slight Asymmetry**

**P2 (47% Sears)**

| **P1 (50% Sears)** | | **Sears** | **AT&T** |
|---|---|---|---|
| | **Sears** | **100,110** | **0,0** |
| | **AT&T** | **0,0** | **110,100** |

**Moderate Asymmetry**

**Figure 6. Chicago Skyscrapers**

**P2 (76% X)**

| **P1 (76% X)** | | **X** | **Y** |
|---|---|---|---|
| | **X** | **5,5** | **0,0** |
| | **Y** | **0,0** | **5,5** |

**Symmetric**

**P2 (28% X)**

| **P1 (78% X)** | | **X** | **Y** |
|---|---|---|---|
| | **X** | **5,5.1** | **0,0** |
| | **Y** | **0,0** | **5.1,5** |

**Slight Asymmetry**

**P2 (61% X)**

| **P1 (33% X)** | | **X** | **Y** |
|---|---|---|---|
| | **X** | **5,6** | **0,0** |
| | **Y** | **0,0** | **6,5** |

**Moderate Asymmetry**

**P2 (60% X)**

| **P1 (36% X)** | | **X** | **Y** |
|---|---|---|---|
| | **X** | **5,10** | **0,0** |
| | **Y** | **0,0** | **10,5** |

**Large Asymmetry**

**Figure 7. X-Y Treatments**

Crawford et al.'s games can be viewed as tests of notions like Bardsley et al.'s (2010, p. 40) team reasoning, whereby "each player chooses the decision rule which, if used by all players, would be optimal for each of them". Subjects who used team reasoning would note that only label salience provides a basis for coordination, and that at least with slight payoff differences, any increase in the probability of coordination swamps the payoff difference between ways to coordinate. Thus they would ignore payoff salience and use label salience to coordinate with high frequency. From this point of view Crawford et al.'s results for the asymmetric games pose two puzzles: Why didn't subjects ignore payoff and use label salience to coordinate? And why did the cause of miscoordination change as the payoff differences grew in the X-Y treatments?

To resolve these puzzles, Crawford et al. proposed a level-*k* model in which *L0* is nonstrategic and the same in both roles but responds to both kinds of salience, with a "payoffs bias" that favors payoff over label salience.[56] For intermediate bias strengths, with slight payoff differences players choose the strategy that maximizes the probability of coordination, but with larger differences they gamble on choices with lower probability of coordination but higher potential payoffs. This allows the model to track the reversal of the cause of miscoordination as the payoff differences grew in the X-Y treatments, and the other main patterns in the data.

Although this analysis suggests that a comprehensive model of coordination with payoff asymmetries and salient labels will include some level-*k* features, it is far from the end of the story. Both Crawford et al. and Bardsley et al. found some evidence of team reasoning in other treatments, in each case mixed with additional evidence for level-*k* thinking.

8.4. *Directions for Future Work*

One direction for future work involves further experiments to delineate the ranges of applicability of team reasoning and level-*k* thinking. The notion of team reasoning as defined by Bardsley et al. is readily accessible to strategic thinking in pure coordination games or games with negligible payoff differences; and despite Crawford et al.'s results for slight payoff differences, its empirical importance in such games is generally well established. But as defined it is limited to games with negligible payoff differences, because otherwise there is rarely a rule that would be optimal for all if used by all. (That lack of generalizability is why we don't treat team reasoning as a theory in Section 2.) The only similar notion of which we are aware that

[56] The payoffs bias is neutral in RT's and O'Neill's games, as it is in pure coordination games. Thus this assumption about *L0* is consistent with CI's assumptions. Notions like equilibrium and QRE ignore labeling, and so do not address these questions.

applies to more general games is Harsanyi and Selten's (1988) equilibrium refinement "payoff-dominance", whereby players play some equilibrium that is not Pareto-dominated by any other equilibrium. But unlike team reasoning in pure coordination games, which solves a nonstrategic optimization problem, payoff-dominance depends essentially on equilibrium fixed-point reasoning, and the mathematical devices Harsanyi and Selten use to make it well-defined for general games take it further from behavioral plausibility as a model of initial responses. Further experiments might explore whether team reasoning influences behavior in coordination games with non-negligible payoff differences and if so, how it coexists with other modes of thinking.

A second direction involves deeper investigation of the foundations of level-*k* and related models of people's responses to salient labels. CI and Crawford et al. impose psychologically plausible but largely untested restrictions on *L0* and the symmetry of rule distributions across player roles and then jointly econometrically estimate the distributions and *L0*'s parameters from decision data. Two alternative approaches may add to our understanding of such models.

Bardsley et al. used a design with three treatments, each with the same set of naturally occurring decision labels, most with non-neutral connotations, as in Crawford et al.'s Chicago Skyscrapers treatments. Those treatments were run within subjects, without feedback until the end. In the "picking" treatment subjects were asked to pick one of the labels and rewarded without regard to their choice. In the "guessing" treatment each subject was paired with a picker, asked to guess the picker's choice, and rewarded for correct guesses. And in the "coordination" treatment subjects were paired to play a pure coordination game as in Schelling's experiments.

Bardsley et al. argue (p. 45) that because picking elicits a subject's nonstrategic response to the labels in a given decision problem, it directly reveals the *L0* that is appropriate for a CH or level-*k* model of that subject's response to the coordination game with the same labels. Given their conclusion for *L0*, they argue that guessing directly reveals the appropriate *L1*. They then use those restrictions to conduct a data analysis that yields some support for team reasoning and some for a CH model, which for their games is essentially equivalent to a level-*k* model.

Although Bardsley et al.'s approach is appealing, it is questionable whether picking and guessing directly reveal the *L0* and *L1* that are appropriate for the coordination treatment. Even though an *L1* player thinks strategically naively about how others' responses to a coordination game influence his decisions' expected payoffs, he can distinguish a game with a given set of labels from the picking task with the same labels; otherwise he would lack the information to evaluate his expected payoffs in the game. Given that, there is no compelling reason that such a player's prediction of pickers' likely responses in the guessing task should be the same as his beliefs in the game with the same labels. Most of Bardsley et al.'s tasks confound salience with personal preference over labels with non-neutral connotations, and it seems unlikely that those factors have exactly the same relative influence in picking, guessing, and coordination tasks.[57]

We believe, nonetheless, that there is much to be learned from designs of this type. But we would favor an initial focus on labels whose patterns have implications for salience but neutral connotations (as, for example, in those of RT's hide-and-seek treatments that CI selected for analysis). We also favor much more within-subjects variation in the structures of the games played, with the goal of making it possible to infer the rules subjects are following more precisely and better assessing their ranges of applicability. This variation might well extend to coordination games that raise different issues, such as symmetry-breaking or assurance; and games that raise issues other than coordination, such as outguessing games.

Another promising alternative is proposed by Penczynski (2011), who reports new data on RT's games, including chats as in BP (2011; Section 3.5). In a sample of 47 (less than a tenth of the sample from RT's treatments CI 2007b analyzed, after pooling), Penczynski finds substantial but statistically insignificant differences in decisions from RT's results for hiders. He finds support for a level-*k* model in both decisions and chats, but one with a role-asymmetric *L0* in which thinking for both hiders and seekers starts with the initial responses of seekers, B is far more salient than the end locations, and seekers have higher levels on average than hiders.[58]

Some of these inferences are clear from the chat data, and the conclusion that B is more salient than the end locations seems more plausible than CI's estimate to the contrary. However, some of the inferences are ambiguous enough to make it worth investigating the extent of their consistency with CI's assumptions that the type distributions and the principles by which *L0* is determined are the same for hiders and seekers, which to some extent function as accounting conventions. In future chat designs, we would also favor more variation within subjects in the structures of the games, with the goal of identifying subjects' rules as much as possible via decision data alone, and then using that identification to tighten the inferences from the chat data.

[57] Bardsley et al. stress that their subjects were told in picking tasks simply to pick one of the labels. But they (p. 48) treat the paraphrases of their instructions "choose a label as if you were just picking" and "choose the label with the greatest immediate appeal to you" as synonymous. It seems likely that their subjects also blurred the distinction between salience and preference.

[58] By contrast, in CI's (2007b) analysis allowing such asymmetry yields only a modest likelihood advantage. And RT (see CI, p. 1736) tested for role-asymmetric behaviour via their "mine" treatments, and found no significant evidence for it.

## 9. *Communication in Outguessing and Coordination Games*

This section considers level-*k* models of strategic communication via natural language ("cheap talk") in outguessing, coordination, and other games. Equilibrium analysis misses some important aspects of how such communication functions in practice. The fact that the receiver must have rational expectations, for instance, implies that in two-person games of pure conflict with known preferences, cheap talk messages must be uninformative, and must be ignored, so that deception cannot occur. And the fact that messages do not directly affect payoffs precludes any role for their literal meanings. Yet in practice, deceptive messages are common and sometimes successful even in games of pure conflict, and literal meanings play a prominent role in how messages are interpreted. A level-*k* analysis yields a systematic way to think about these and related phenomena, and bring us closer to how communication appears to work in reality.

We illustrate these possibilities in several settings, including Crawford's (2003) level-*k* model of preplay communication in two-person games of pure conflict; Ellingsen and Östling's (2010) and Crawford's (2007) level-*k* analyses of communication of intentions in coordination and other games; and Wang et al.'s (2010) experimental analysis of communication of private information in sender-receiver games. We also discuss Malmendier and Shanthikumar's (2007, 2009) CH empirical analyses of the interaction between stock analysts and traders.

### 9.1. *Communication of Intentions in Outguessing Games*

> "Have you forgotten the tactic of 'letting weak points look weak and strong points look strong'?"
>
> — General Kongming, in Luo Guanzhong's (1991) historical novel, *Three Kingdoms*.

> "Don't you know what the military texts say? 'A show of force is best where you are weak. Where strong, feign weakness.'"
>
> — General Cao Cao, in *Three Kingdoms*.

In the Huarongdao story, set around 200 A.D., fleeing General Cao Cao, trying to avoid capture by pursuing General Kongming, chose between two escape routes, the easy Main Road and the rough Huarong Road (http://en.wikipedia.org/wiki/Battle_of_Red_Cliffs). The game is like *Far Pavilions* Escape (Section 4), but with communication, in that before Cao Cao's choice Kongming had an opportunity to send a message by lighting campfires along one of the roads. This message had an obvious literal meaning, but it was scarcely more costly to send a false message than a true one, so the message was approximately cheap talk. Kongming then chose which road on which to wait in ambush. In the story Kongming lit campfires along the Huarong Road and waited in ambush there, sending a deceptively truthful message. Cao Cao, misjudging Kongming's deviousness, inverted the message, took the Huarong Road, and was captured.

Huarongdao also resembles the organizing example in Crawford's (2003) level-*k* analysis of deceptive communication of intentions, Operation Fortitude South, the Allies' attempt to deceive the Germans regarding where they planned to invade Europe on D-Day (6 June 1944; http://en.wikipedia.org/wiki/Operation_Fortitude). The Allies' message is approximately cheap talk and the underlying game is an outguessing game with conflicting interests, made zero-sum in the analysis to sharpen the point.[59] There are two possible attack or defense locations, Calais and Normandy. The greater ease of Calais is reflected in payoffs that imply that attacking an undefended Calais is better for the Allies than attacking an undefended Normandy, hence better for the Allies if the Germans are equally likely to defend each place; and defending an unattacked Normandy is worse for the Germans than defending an unattacked Calais, hence worse for the Germans if the Allies are equally likely to attack each place.

In the event the Allies faked preparations for invasion at Calais, sending a deceptively *deceptive* message. The Germans, misjudging the Allies' deviousness, defended Calais and left Normandy lightly defended; and the Allies then invaded Normandy.

In each case the key strategic issue is how the sender—Kongming or the Allies—should choose his message and how the receiver—Cao Cao or the Germans—should interpret it, knowing that the sender is thinking about the message from the same point of view.

Moreover, in each case essentially the same thing happened: In D-Day the message was literally deceptive but the Germans were fooled because they "believed" it—either because they were credulous or, more likely, because they inverted the message one too many times. Kongming's message was literally truthful but Cao Cao was fooled because he inverted it. Although the sender's and receiver's message strategies and beliefs were different, the outcome in the underlying game was the same: The sender won, but in the less beneficial of the two possible ways. Why did the receiver allow himself to be fooled by a costless (hence easily faked)

[59] Operation Fortitude differs from Huarongdao in the relation between payoffs and labeling, in that both Cao Cao and Kongming prefer the Main Road, holding the probability of being outguessed equal; while the Allies and the German have opposing preferences about the location of the invasion, other things equal. But in Crawford's level-*k* analysis this difference is inessential, because with *L0* anchored in truthfulness, players' responses to messages override any effects of labeling.

message from an *enemy*? And if the sender expected his message to fool the receiver, why didn't he reverse it and fool the receiver in the way that allows him to win in the *more* beneficial way?

Traditional equilibrium analysis cannot explain these puzzles. Not only does it preclude a role for the literal meanings of messages, with conflicting interests there is no equilibrium in which cheap talk messages conveys information or the receiver responds to them.[60] In such an equilibrium, if the receiver found it optimal to respond to the message the response would help the receiver and therefore hurt the sender, who would then prefer to send an uninformative message (Crawford and Sobel 1982). Communication is therefore irrelevant, and the underlying game must be played according to its unique mixed-strategy equilibrium. Yet in real interactions, a receiver's thinking often assigns a prominent role to the literal meanings of messages, without necessarily taking them at face value; a sender's message and action are part of an integrated strategy; and players' actions may differ from the ones chosen without communication.

These puzzles can be plausibly explained via a level-*k* analysis. In games with communication, it would be behaviorally odd if a player's strategically naïve assessment of a message, even from an enemy, did not initially favor its literal interpretation, even if he ends up not taking it at face value. Accordingly, Crawford (2003, Table 1) assumed that the *L0* that pertains to senders is truthful and the *L0* that pertains to receivers is credulous.[61] Given this, the types are defined by iterating best responses as in other level-*k* models: an *L1* receiver believes what he is told; an *L1* sender lies; an *L2* receiver inverts what he is told; an *L2* sender lies; an *L3* receiver inverts; an *L3* sender tells the truth (anticipating an *L2* receiver's inversion); and so on. In this categorization, Cao Cao was *L2*, while Kongming was *L3*.[62] Similarly, it appears that the Allies were *L2*, while the Germans were *L1*, or perhaps (inverting one too many times) *L4*.

It is instructive to analyze a more general model, in which some players in each role understand both equilibrium analysis and the likelihood that their partners' strategic thinking may be simpler than that. Accordingly, Crawford (2003) assumed that with positive probability, each player role is filled either by one of the possible level-*k* types, for which his generic term

[60] But see Joseph Farrell (1993), whose notion of neologism-proofness sometimes allows literal meanings influence, but not here.

[61] The literature has not converged on how types should be numbered, or on whether *L0* receivers should be defined as credulous or as uniform random—compare Ellingsen and Östling (2010)—but the issue is partly semantic because truthful *L0* senders imply credulous *L1* receivers. Here we rename the types to conform to later usage; taking *L0* receivers to be credulous; and given that, defining *Lk* in either role as the type that iterates best responses *k* times. Note that the definition of *L0* resolves the indeterminacy of the meaning of messages, which is not determined by equilibrium in cheap-talk analyses.

[62] Evidently Cao Cao had bought used, out-of-date editions of the texts. As the last possibility illustrates, in a level-*k* model, unlike a CH model, it can be just as costly to be too clever as to be not clever enough, which we view as a realistic feature.

was *Mortal*; or by a *Sophisticated* type. He assumes that the frequencies of *L0* senders and receivers are zero. Higher-level *Mortal* types are defined as above, avoiding fixed-point reasoning. *Sophisticated* types, by contrast, know everything about the game, including the distribution of *Mortal* types. They and their possibly *Sophisticated* partners play a (Bayesian) equilibrium in a "reduced game" between possible *Sophisticated* senders and receivers, obtained by plugging in the mechanically determined distributions of *Mortal* players' strategies.[63] *Sophisticated* subjects are rare in experiments, but presumably more common in field settings. Even though level-*k* types trivially allow for the possibility of deception, it is far from clear whether deception is possible with *Sophisticated* players, or how it would work.

The possibility of *Mortal* players completely changes the character of the game between *Sophisticated* players: Because their expected payoffs are influenced by *Mortal*s' decisions, the reduced game is no longer zero-sum and its messages are no longer cheap talk. Further, it no longer has symmetric information: In the reduced game a sender's message, ostensibly about his intentions, is read by a *Sophisticated* receiver as a signal of the sender's privately known type.

A non-*L0 Mortal* sender's models of others always make it expect to fool receivers, which it does either by lying or telling the truth depending on whether it expects its message to be believed or inverted. Any given *Mortal* sender type therefore sends the message that maximizes its expected gain from fooling receivers—in D-Day, the one it expects to make the Germans think it will attack Normandy—and then chooses the strategy that successful deception makes optimal in the underlying game—always attacking Calais, in D-Day.

Given this, the equilibria of the reduced game are determined by the relative frequencies of *Mortal* and *Sophisticated* players. When *Sophisticated* players are common in both roles, the reduced game has a mixed-strategy equilibrium whose outcome mimics that of the game without communication. In that equilibrium *Sophisticated* players' mixed strategies offset *Mortal* players' deviations from equilibrium, eliminating *Sophisticated* senders' gains from fooling *Mortal* receivers, so *Sophisticated* and *Mortal* players in each role have equal expected payoffs.

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[63] Ricardo Serrano-Padial (2010) takes a similar approach to analyze the interaction between naïve and sophisticated traders in speculative markets. Naïve traders include any whose trading decisions can be expressed as functions of their information, without solving a fixed-point problem. Sophisticated traders play their part in a market equilibrium, but unlike equilibrium traders they take the frequency and behavior of naïve traders rationally into account. When there are enough sophisticated agents to counteract naive agents' deviations from equilibrium, the usual rational-expectations equilibrium ensues, even with a nonnegligible frequency of naïve traders. With an intermediate frequency of sophisticated traders, the market segments into intervals of the space of possible valuations in which sophisticated traders never bid; and disjoint intervals in which both naïve and sophisticated traders bid. In the "naïve" intervals, naïve traders have the pivotal influence on pricing, which deviates systematically from equilibrium. In the "sophisticated" intervals, pricing is just as predicted in the standard model.

When *Sophisticated* senders and receivers are rare—perhaps the more plausible case—the reduced game has an essentially unique equilibrium, in pure strategies.[64] *Sophisticated* Germans, for instance, always defend Calais because they know that *Mortal* Allies, who predominate when *Sophisticated* Allies are rare, will all attack Calais. *Sophisticated* Allies, knowing that they cannot influence *Sophisticated* Germans, send the message that fools the most common type of *Mortal* German (feinting at Calais or Normandy depending on whether more of them believe than invert messages) and then always attack Normandy. In this equilibrium, *Sophisticated* Germans allow themselves to be "fooled" by cheap talk messages from *Sophisticated* Allies because it is an unavoidable cost of exploiting the mistakes of far more common *Mortal* Allies.

It is surprising that when *Sophisticated* senders and receivers are rare there is a pure-strategy equilibrium, and perhaps more surprising that it has no pure-strategy counterpart in which *Sophisticated* Allies feint at Normandy and then attack Calais. In such an equilibrium, any deviation from *Sophisticated* Allies' equilibrium message would make *Sophisticated* Germans infer that the Allies were *Mortal*, making it optimal for them to defend Calais and suboptimal for *Sophisticated* Allies to attack there. If *Sophisticated* Allies feinted at Normandy and attacked Calais, their message would fool only the most common kind of *Mortal* German— in a pure-strategy equilibrium *Sophisticated* Germans can never be fooled, and a given message cannot fool both believers and inverters—with expected payoff gain equal to the frequency of the most common *Mortal* German type times the payoff of attacking an undefended Normandy. But such *Sophisticated* Allies could reverse their message and attack location, again fooling the most common *Mortal* German type, but now with expected payoff gain equal to the frequency of that type times the payoff of attacking an undefended Calais, which is higher than the payoff of attacking an undefended Normandy. This contradiction shows that in any pure-strategy equilibrium, *Sophisticated* Allies must feint at Calais and then attack Normandy.

Thus, the model explains the puzzling feature of our examples that the sender won, but in the less beneficial of the two possible ways. It also makes the sender's message and action part of an integrated strategy; gives the possibility of communication a genuine influence on the outcome.

*Sophisticated* players derive an advantage from their ability to avoid being fooled and/or to choose which *Mortal* type(s) to fool. This suggests that in an "evolutionary" analysis, the frequencies of *Sophisticated* types will grow. In this model, however, such growth will continue

[64] The game can then be solved via at most three steps of iterated conditional dominance, without fixed point reasoning.

only until the type frequencies enter the region of mixed-strategy equilibria, where types' expected payoffs are equal. Thus, *Sophisticated* and *Mortal* types can coexist in the long run.

9.2. *Communication of Intentions in Coordination Games*

> "After you, Alphonse." "No, you first, my dear Gaston!"
>
> —Frederick B. Opper's comic strip, *Alphonse and Gaston* (http://en.wikipedia.org/wiki/Alphonse_and_Gaston)

If level-*k* models allow preplay communication of intentions to affect the outcomes of zero-sum two-person games, it is no surprise that they also allow effective communication in coordination games. Here the stylized experimental facts (Crawford 1998) are that when coordination requires symmetry-breaking (Section 6), one-sided communication is more effective; that when coordination requires assurance (Section 7), two-sided communication is more effective; and that when coordination requires symmetry-breaking and communication is two-sided, more communication is better than less. These patterns have long resisted equilibrium explanations. Ellingsen and Östling (2010) and Crawford (2007) use similar level-*k* models to elucidate long-standing puzzles about how the effectiveness of communication varies with its structure and with the payoff structure in experiments and presumably in the field. In each case the power of the analysis stems from the use of a model that relaxes equilibrium in favor of a model that imposes realistic structure less agnostic than rationalizability or *k*-rationalizability.

9.2.a. *Coordination via One Round of Communication*

Ellingsen and Östling (2010) adapt Crawford's (2003) level-*k* analysis to study the effectiveness of one round of one- or two-sided communication in games where communication of intentions plays various roles. Here the central puzzle turns on Farrell and Rabin's (1996) distinction between messages that are self-committing in the sense that if the message convinces the receiver, it's a best response for the sender to do as he said he would do; and those that are self-signaling, in that they are sent when and only when the sender intends to do as he said. In a two-person Stag Hunt game, each player does (weakly) better if his partner chooses high effort, without regard to his own intentions; "I intend to play High Effort" is self-committing but not self-signaling. Robert J. Aumann (1990) argued on this basis that such messages are not credible. But Charness (2000) and others have shown experimentally that messages that are self-committing but not self-signaling are effective in practice (but see Kenneth Clark et al. 2001).

Ellingsen and Östling's (2010) take a first step explaining the patterns of effectiveness of communication. They depart from Crawford (2003) by assuming that *L0* receivers are uniform random rather than credulous and that all types have a preference for honesty when they are otherwise indifferent about which message to send. In their model, one-sided communication solves the coordination problem in games where it requires symmetry-breaking, and is therefore more effective than two-sided communication, as is usually found in experiments. Their model can also explain why two-sided communication is more effective than one-sided communication in games where coordination requires assurance, as is also found in experiments. More generally, they show that in common interest games when both players are *L2* or higher, one- or two-way communication assures efficient coordination. But this tendency is not universal: In some games players have incentives to misrepresent that erode coordination.

9.2.b. *Coordination via Multiple Rounds of Communication*

Farrell (1987) and Rabin (1994) analyze the effectiveness of one or more rounds of simultaneous, two-sided communication about players' intentions. Their analyses assume equilibrium, sometimes weakened to rationalizability; and they further restrict attention to outcomes that satisfy plausible behavioral restrictions defining which combinations of messages create agreements, and whether and how agreements can be changed. They address two conjectures regarding symmetric-information games: that preplay communication will yield an effective agreement to play an equilibrium in the underlying game; and that the agreed-upon equilibrium will be Pareto-efficient within that game's set of equilibria (henceforth "efficient"). They show that rationalizable preplay communication need not assure equilibrium; and that, although communication enhances coordination, even equilibrium with "abundant" (Rabin's term for "unlimited") communication does not assure that the outcome will be efficient.

Equilibrium and rationalizability are natural places to start in analyses like Farrell's and Rabin's, but it seems worthwhile to reconsider their questions using a level-*k* model. Such a model relaxes equilibrium while counteracting the agnosticism of rationalizability in an evidence-based way. Crawford (2007) uses a level-*k* model anchored in truthfulness like Crawford's (2003) model to study the effectiveness of multiple rounds of simultaneous two-sided cheap-talk messages, focusing on Farrell's analysis of Battle of the Sexes. The level-*k* analysis provides a way to assess Farrell's and Rabin's assumptions about how players use language, and supports most but not all of them. The level-*k* analysis also gives a different take on how coordination rates relate to the game. In Farrell's equilibrium analysis of Battle of the Sexes, coordination rates are highly sensitive to the difference in players' preferences; but Crawford's analysis suggests that coordination rates will be largely independent of it. With one round of communication the level-*k* coordination rate is well above the rate without communication, and usually higher than the equilibrium rate. With abundant communication the level-*k* coordination rate is higher than the equilibrium rate unless preferences are fairly close. The level-*k* model's predictions with abundant communication are consistent with Rabin's results, but yield further insight into the causes and consequences of breakdowns in negotiations.

### 9.3. *Communication of Private Information in Outguessing Games*

> "…The news that day was the so-called 'October Surprise' broadcast by bin Laden. He hadn't shown himself in nearly a year, but now, four days before the [2004 presidential] election, his spectral presence echoed into every American home. It was a surprisingly complete statement by the al Qaeda leader about his motivations, his actions, and his view of the current American landscape. He praised Allah and, through most of the eighteen minutes, attacked Bush,… At the end, he managed to be dismissive of Kerry, but it was an afterthought in his 'anyone but Bush' treatise….
>
> Inside the CIA…the analysis moved on a different [than the presidential candidates' public] track. They had spent years, as had a similar bin Laden unit at FBI, parsing each expressed word of the al Qaeda leader…. What they'd learned over nearly a decade is that bin Laden speaks only for strategic reasons…. Today's conclusion: bin Laden's message was clearly designed to help the President's reelection."
>
> —Ron Suskind, *The One Percent Doctrine*, 2006, pp. 335-6 (quoted in Jazayerli 2008 http://www.fivethirtyeight.com/2008/10/guest-column-will-bin-laden-strike.html).

#### 9.3.a. *October Surprise*

The situation described in the quotation can be modeled as a zero-sum two-person game of asymmetric information between bin Laden and a representative American voter. The American knows that he wants whichever candidate bin Laden doesn't want, but only bin Laden knows which candidate he wants. Bin Laden can send a cheap-talk message about what he wants. The key strategic issues are how bin Laden should relate his message to what he wants and how the American should interpret the message, knowing that bin Laden is choosing it strategically.

Once again, the literal meanings of messages are likely to play a prominent role in applications, but equilibrium analysis precludes such a role. There is again no equilibrium in which cheap talk conveys information or the receiver responds to the sender's message. However, Crawford's (2003) analysis is easily adapted (see also Kartik et al. 2007) to model the

CIA's conclusion that bin Laden's attack on Bush was intended to aid Bush's reelection. Let *L0* again be anchored on truthfulness for the sender (bin Laden) and credulity for the receiver (American). An *L0* or *L1* American believes bin Laden's message, and therefore votes for whichever candidate bin Laden attacks. An *L0* bin Laden who wants Bush to win attacks Kerry, but an *L1* (*L2*) bin Laden who wants Bush to win attacks Bush to induce *L0* (*L1*) Americans to vote for him via "reverse psychology". Given bin Laden's attack on Bush, an *L0* or *L1* American ends up voting for Bush, and an *L2* American ends up voting for Kerry. A *Sophisticated* bin Laden, recognizing that he cannot fool *Sophisticated* Americans, would choose his message to fool the most prevalent kind of *Mortal* American—believer or inverter—as in Crawford (2003).

9.3.b. *Experiments on communication of private information*

Wang et al. (2010), building on the experiments of Hongbin Cai and Wang (2006), studied communication of private information via cheap talk in discretized versions of Crawford and Sobel's (1982) sender-receiver games. In Wang et al.'s design, the sender observes a state, $S$ = 1, 2, 3, 4, or 5; and sends a message, $M$ = 1, 2, 3, 4, or 5. The receiver then observes the message and chooses an action, $A$ = 1, 2, 3, 4, or 5. The receiver's choice of A determines the welfare of both: The receiver's ideal outcome is $A = S$ and his von Neumann-Morgenstern utility function is $110 – 20|S–A|^{1.4}$; and the sender's ideal outcome is $A = S + b$ and his von Neumann-Morgenstern utility function is $110 – 20|S+b–A|^{1.4}$. Wang et al. varied the parameter representing the difference in preferences across treatments: $b$ = 0, 1, or 2.

The key issue is how much information can be transmitted in equilibrium, and how the amount is influenced by the difference between sender's and receiver's preferences. Crawford and Sobel characterized the possible equilibrium relationships between sender's observed $S$ and receiver's choice of $A$, which determine the informativeness of communication. They showed, for a class of models that generalizes Wang et al.'s (except for its discreteness), that all equilibria are "partition equilibria", in which the sender partitions the set of states into contiguous groups and tells the receiver, in effect, only which group his observation lies in. Crawford and Sobel also showed that for any given difference in sender's and receiver's preferences ($b$), there is a range of equilibria, from a "babbling" equilibrium with one partition element to equilibria with finer partitions that exist when $b$ is small enough. Under reasonable assumptions there is a most informative equilibrium, which has the most partition elements and gives the receiver the highest ex ante (before the sender observes the state) expected payoff. As the preference difference decreases, the amount of information transmitted in the most informative equilibrium increases.

In equilibrium, the receiver's beliefs on hearing the sender's message *M* are an unbiased—though noisy—estimate of *S*: Thus Crawford and Sobel's analysis of strategic communication has the puzzling feature that it cannot explain lying or deception, only intentional vagueness. Further, previous experiments (see Crawford 1998) with this model have consistently revealed systematic deviations from equilibrium, in the direction of excessive truthfulness (from the point of view of the equilibrium in a model where there are no lying costs!) and excessive credulity. But despite these deviations from equilibrium, experiments have consistently confirmed the comparative statics result that closer preferences allow more informative communication. It is a natural conjecture that the comparative statics result continues to hold even when equilibrium fails because it holds for a class of models that describe subjects' deviations from equilibrium.

As Wang et al.'s Figures 1-3 show, unless sender's and receiver's preferences are identical ($b = 0$), most senders exaggerate the truth, in the direction that would, if believed, move receivers toward senders' ideal action. Despite senders' exaggeration, their messages contain some information, measured by the correlation between *S* and *A*; and most receivers are credulous, responding to the sender's message more than they should. Despite the widespread deviations from equilibrium, the results reaffirm Crawford and Sobel's equilibrium-based comparative-statics prediction, with the amount of information transmitted increasing as the preference difference decreases from $b = 2$ to $b = 1$ to $b = 0$. Wang et al. go beyond previous work by showing in a detailed data analysis, including eyetracking measures of information search as well as conventional decisions, that their results are well explained by a level-*k* model anchored in truthfulness, following Crawford's (2003) analysis. In Wang et al.'s analysis, the preference difference and a sender's level determine how much he inflates his message (in the direction in which he would like to move the receiver), and a receiver's level determines how much he discounts the sender's message. Econometric type estimates are broadly consistent with earlier results.[65] The model gives a unified explanation of subjects' excessive truthfulness and credulity, and of the affirmation of predictions based on the equilibrium-based comparative statics result.

[65] See also Kartik et al. (2007). Wang et al. focus on sender subjects because they, but not receiver subjects, were eye-tracked. For comparison, Hongbin Cai and Wang (2006) in a closely related non-eye-tracking design classified 6% of senders and 9% of receivers as *L0*, 25% of senders and 9% of receivers as *L1*, 31% of senders and 34% of receivers as *L2* or *Equilibrium*, and 13% of senders and 28% of receivers as *Sophisticated*. They also state that an agent LQRE model fits their data well.

### 9.3.c. *Field Studies*

Malmendier and Shanthikumar (2007, 2009) discuss the interaction between stock analysts and traders. Analysts issue recommendations on individual stocks that range from "strong sell" and "sell" to "hold", "buy", and "strong buy"; and they also issue earnings forecast. In managing their portfolios, traders are presumed to use all the information available on the market, of which analysts' recommendations are a major source.

An analyst's recommendation or forecast is like a message in a sender-receiver game. Particularly when an analyst is affiliated with the underwriter of a particular stock, he has an incentive to distort such messages. Malmendier and Shanthikumar (2007) find that analysts tend to bias their stock recommendations upward, the more so when they are affiliated with the underwriter of the stock. They also find two main patterns of responses to recommendations among receivers: Large investors tend to buy following "strong buy" recommendations, but not to sell following "hold" recommendations, thus discounting recommendations somewhat. Small traders, by contrast, are credulous enough to follow recommendations almost literally. Malmendier and Shanthikumar (2009) find somewhat different patterns of responses to earnings forecasts. Large investors tend to react strongly and in the direction suggested by forecast updates, without regard to whether the forecast came from an affiliated analyst. Small investors, by contrast, react insignificantly to the forecasts of unaffiliated analysts and significantly negatively to the forecasts of affiliated analysts.

Malmendier and Shanthikumar (2007, 2009) use these and other patterns in the data to distinguish between explanations of the bias in recommendations based on optimism-driven selection effects and those based on strategic distortion. They conclude that strategic distortion is the more important factor. Their analyses, which rest mainly on qualitatively patterns in the data, might be sharpened and refined by an explicit model of strategic distortion and its effects along the lines of a multidimensional generalization of the level-*k* analyses discussed in this section.

## *10. Conclusion*

This paper has surveyed theoretical, experimental, and empirical work on models of strategic thinking and their applications in economics. Better models of strategic thinking are plainly important in applications to games without clear precedents. But such models can also help by making more precise predictions of equilibrium selection when it is plausible that learning has converged to equilibrium, or when initial responses are important for their own sake.

Although Nash equilibrium can be and has been viewed as a model of strategic thinking, experimental research shows with increasing clarity that subjects' initial responses to games often deviate systematically from equilibrium, and that the deviations have a large structural component that can be modeled in a simple way. Subjects' thinking tends to avoid the fixed-point reasoning or indefinitely iterated dominance reasoning that equilibrium sometimes requires, in favor of level-*k* rules of thumb that anchor beliefs in an instinctive reaction to the game and then adjust them via a small number of iterated best responses. The resulting level-*k* or CH models share the generality and much of the tractability of equilibrium analysis, but can in many settings systematically out-predict equilibrium. Importantly, level-*k* models not only predict that deviations will sometimes occur; they also predict which settings will evoke them; the forms they will take; and, given estimates of the type frequencies, their likely frequencies.

In simple games where the low-level types that describe most people's behavior often mimic equilibrium decisions, a level-*k* analysis may establish the robustness of equilibrium predictions. In more complex games where level-*k* types deviate from equilibrium, a level-*k* analysis can resolve empirical puzzles by explaining the systematic part of the deviations. We have illustrated those possibilities in applications ranging from zero-sum betting and auctions with private information; to coordination via symmetry-breaking or assurance; outguessing and coordination games played on non-neutral salience landscapes; and strategic communication in outguessing and coordination games. We hope that this survey has shown that structural nonequilibrium models of strategic thinking deserve a place in the analyst's toolkit.

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