Fatal Attraction:
Focality, Naivete, and Sophistication in Experimental "Hide-and-Seek" Games

Vincent P. Crawford and Nagore Iriberri[1]

University of California, San Diego

20 August 2004; this revision 30 January 2005

"Any government wanting to kill an opponent…would not try it at a meeting with government officials."
—comment on the poisoning of Ukrainian presidential candidate Viktor Yushchenko, quoted in Chivers (2004)

"…in Lake Wobegon, the correct answer is usually 'c'."
—Garrison Keillor (1997) on multiple-choice tests, quoted in Attali and Bar-Hillel (2003)

Abstract: "Hide-and-Seek" games are zero-sum two-person games in which one player wins by matching the other's decision and the other wins by mismatching. Although such games are often played on cultural or geographic "landscapes" that frame decisions non-neutrally, equilibrium ignores such framing. This paper reconsiders the results of experiments by Rubinstein, Tversky, and others whose designs model non-neutral landscapes, in which subjects deviated systematically from equilibrium in response to them. Comparing alternative explanations theoretically and econometrically suggests that the deviations are best explained by a structural non-equilibrium model of initial responses based on "level-*k*" thinking, suitably adapted to non-neutral landscapes.

Keywords: behavioral game theory, experiments, hide-and-seek games, framing effects, salience, bounded rationality, level-*k* thinking

JEL classification numbers: C70, C92

---

[1]Email: vcrawfor@weber.ucsd.edu and niriberr@weber.ucsd.edu. We are grateful to the National Science Foundation (Crawford) and the Centro de Formacion del Banco de España (Iriberri) for research support; to Miguel Costa-Gomes, Victor Ferreira, Barry Nalebuff, Steven Scroggin, Ricardo Serrano-Padial, Joel Sobel, David Swinney, Mark Voorneveld, Mark Walker, Joel Watson, and three anonymous referees for helpful comments or discussions; to Dale Stahl for helpful discussions and for providing a copy of Bacharach and Stahl (1997a); to Stahl and Daniel Zizzo for searching for Bacharach and Stahl (1997b); to Barry O'Neill and Amnon Rapoport (with the help of Ido Erev and Mark Walker, respectively) for providing data from their experiments; and to Ariel Rubinstein for providing a copy of Rubinstein and Tversky (1993), searching for additional data, and helpful discussions. Glenn Close and Michael Douglas (http://www.imdb.com/title/tt0093010/) were no help at all.

Game theorists have been intrigued by "Hide-and-Seek" games—zero-sum two-person games with two outcomes in which one player wins by matching the other's decision and the other wins by mismatching—for more than 50 years (von Neumann (1953)). These games cleanly model a strategic problem that is central to many economic, political, and social settings as well as the obvious military and security applications. Examples include entry games where entry requires a differentiated product and blocking it requires matching the entrant's design; election campaigns in which a challenger can win only by campaigning in different areas than the incumbent; and fashion games in which hoi polloi wish to mimic the elite but the elite prefer to distinguish themselves.

Although zero-sum two-person games are one of game theory's success stories, equilibrium analysis of Hide-and-Seek games is not very helpful as a guide to prediction or decision making in applications. There seem to be two main reasons for this, illustrated by our epigraphs: Hide-and-Seek games are often played without clear precedents, so equilibrium depends on strategic thinking rather than learning; but such thinking may not follow the fixed-point logic of equilibrium.[2] And Hide-and-Seek games are usually played on naturally occurring cultural or geographic "landscapes" that are non-neutral across locations in framing, payoffs, or both. Equilibrium ignores such landscapes except as they affect payoffs, but non-equilibrium thinking may respond to them.

Both reasons are also illustrated by the experimental results of Rubinstein and Tversky (1993; "RT") and Rubinstein, Tversky, and Heller (1996; "RTH"); see also Rubinstein (1999; "R"). RT, RTH, and R (collectively "RTH") elicited subjects' initial responses to Hide-and-Seek games. RTH explained the games in "stories," probably increasing comprehension. In a leading example, R told Seekers: "You and another student are playing the following game: Your opponent has hidden a prize in one of four boxes arranged in a row. The boxes are marked as follows: A, B, A, A. Your goal is, of course, to find the prize. His goal is that you will not find it. You are allowed to open only one box. Which box are you going to open?" Hiders were told an analogous story. Thus the entire structure, including the order and labeling of locations, was publicly announced.[3]

This story makes the framing of locations non-neutral in two ways. The "B" location is uniquely distinguished by its label, and is thus focal in one of Schelling's (1960) senses. And the

[2]A game theorist would reply to our first epigraph, "But if the investigators were expected to think this way, a meeting with government officials is precisely where a government *would* try to kill an opponent."
[3]RT's and RTH's subjects' payments appeared sufficient to motivate them, and the binary-lottery structure of the payoff function implies under standard assumptions that players maximize expected money payoffs, without regard to risk preferences. Note however that in R's experiments, subjects were not paid or screened for exposure to game theory.

two "end A" locations, though not distinguished by their labels, may be inherently focal, as RT and RTH argue, citing Christenfeld (1995).[4] As RT note, these two focalities interact to give the remaining location, "central A," its own brand of uniqueness as "the least salient location." This aspect of their designs is important as a tractable abstract model of a naturally occurring landscape.

Figure 1 translates RTH's story into a payoff matrix. Because the Seeker chooses without observing the Hider's choice, their choices are strategically simultaneous. If the Seeker chooses the same location as the Hider, he wins a payoff normalized here to one; if not, the Hider wins that payoff. This game has a unique equilibrium in mixed strategies, in which both players randomize uniformly across locations, independent of their framing. Because the Seeker can choose only one location, the Hider has an advantage in equilibrium, with expected payoff ¾ versus the Seeker's ¼.

Despite this clear equilibrium prediction, RTH's publicly announced order and labeling of locations create a potential for framing effects, and their subjects deviated systematically from equilibrium in ways that were sensitive to framing.[5] Table 1 gives the aggregate choice frequencies for the "RTH-4" treatment described above and RTH's most closely related treatments. In RTH-4 the "least salient" central A location was the modal choice for both Hiders and Seekers, and was even more prevalent for Seekers than Hiders. As a result, the frequency with which Seekers found the prize exceeded ¼ and Seekers had higher expected payoffs than in equilibrium. These patterns extend, properly interpreted, to the other five treatments in the top half of Table 1, which we argue in Section I are closely analogous to RTH-4. They also extend, with minor exceptions, to a large sample from more recent internet experiments (Ariel Rubinstein, private communication); and with variations explained below to the less closely analogous treatments in the bottom half of Table 1.

These deviations from equilibrium are particularly puzzling because they are asymmetric across player roles in games where equilibrium is symmetric: Why don't Hiders tempted to hide in

---

[4]See also Attali and Bar-Hillel (2003), who give a comprehensive discussion of this kind of focality and present a variety of evidence on it, including the locations of correct answers in multiple-choice questions as suggested in our second epigraph, which is borrowed from theirs. They also note a strong tendency for subjects in both player roles to favor the two central out of four locations when they are presented in linear order as in RTH, but with no focally labeled location. See also their and R's discussions of Ayton and Falk's (1995) two-dimensional experiment.

[5]Although in this game any strategy, pure or mixed, is a best response to equilibrium beliefs, *systematic* deviations of aggregate choice frequencies from equilibrium probabilities must (with high probability) have a cause that is partly common across players, and are therefore indicative of systematic deviations from equilibrium. Other studies of framing effects in different kinds of games include Scharlemann et al. (2001), who studied trust games in which otherwise anonymous players were "labeled" by photographs; and Mehta, Starmer, and Sugden (1994), who studied coordination games in which decisions had naturally occurring labels, as in Schelling's (1960) classic experiments.

central A realize that Seekers will be just as tempted to look there? And why do Hiders behave in a way that allows Seekers to find them more than 25% of the time, when they could hold it down to 25% via the equilibrium mixed strategy or even less by hiding anywhere but central A?

RTH took the deviations as prima facie evidence that the subjects did not think strategically, and with one exception (explained in Section I), they did not consider alternative explanations. RT: "The finding that both choosers and guessers selected the least salient alternative suggests little or no strategic thinking." RTH: "…the players employed a naïve strategy (avoiding the endpoints), that is not guided by valid strategic reasoning. In particular, the hiders in this experiment either did not expect that the seekers too, will tend to avoid the endpoints, or else did not appreciate the strategic consequences of this expectation." But in our view, patterns of behavior observed in so many settings are unlikely to lack a coherent explanation; and given the simplicity of the strategic question Hide-and-Seek games pose, the explanation is unlikely to be nonstrategic. On the contrary, deviations from equilibrium in games where its rationale is especially strong seem a promising "proving ground" for comparing alternative, non-equilibrium theories of initial responses to games.

This paper compares alternative explanations of RTH's results, and conducts an illustrative theoretical and econometric analysis that helps to discriminate among them. Our main goals are to resolve a long-standing behavioral puzzle and to learn more about the structure of strategic thinking in games played on non-neutral landscapes. Following RTH, we focus mainly on Hide-and-Seek games played on landscapes that are non-neutral in the framing of locations but neutral with regard to payoffs.[6] And although some applications allow learning, which may eliminate deviations from equilibrium over time, we also follow RTH in focusing on subjects' initial responses, which reveal their strategic thinking more clearly and shed more light on players' responses to novel situations.

Section I explains the other RTH treatments in Table 1 and their analogies to RTH-4, and reports tests for differences in choice frequencies, finding no significant differences across treatments. Section II considers the possible explanation of RTH's results that is perhaps closest to the mainstream, an equilibrium analysis of the Hide-and-Seek game with payoff perturbations that

[6]There is a complementary experimental literature on Hide-and-Seek games with landscapes that are not payoff-neutral; see Rosenthal, Shachat, and Walker (2003; "RSW") and the papers cited there. The explanation proposed here has the potential to elucidate subjects' responses to non-payoff-neutral landscapes as well, but we do not consider such settings here (see for example Camerer, Ho, and Chong (2002, Section 2.5) and Crawford (2004, pp. 20-21). Von Neumann (1953) characterized equilibria in Hide-and-Seek games with payoffs that vary with location, including a two-dimensional version in which a Hider hides in a matrix and a Seeker tries to guess either the Hider's Row or Column.

reflect "hard-wired" preferences about the salient focally labeled and end locations. Such a model can reproduce the patterns RTH observed, but only by postulating large unexplained differences between Hiders and Seekers in the magnitudes of the perturbations as well as their signs.

This equilibrium with perturbations model raises three important issues. Although the sign differences in the perturbations it uses to explain RTH's results are intuitive—Hiders averse to focality, Seekers attracted to it—reducing the explanation to an *assumed* asymmetry in behavior begs the question of why Hide-and-Seek elicits systematically different responses from Hiders and Seekers. The model's unrestricted perturbations also give it enough flexibility to explain virtually any pattern of choices, raising concerns about overfitting. Finally, tailoring behavioral assumptions so closely to the strategic structure of the Hide-and-Seek game may reduce the model's *portability*, the extent to which estimating its parameters from subjects' responses in one setting is useful in predicting or explaining behavior in other settings.[7] We return to these issues below.

Section III considers a generalization of the equilibrium with perturbations model based on McKelvey and Palfrey's (1995) notion of Quantal Response Equilibrium ("QRE"). QRE allows players to respond to each other's decision noise in a way that describes the patterns of deviations from equilibrium in some other experiments, and so has the potential to explain RTH's results without unexplained differences in the magnitudes of the payoff perturbations. However, QRE with perturbations does no better than equilibrium with perturbations in explaining RTH's results.

Section IV returns to the Hide-and-Seek game without payoff perturbations and considers explanations based on a structural non-equilibrium model of initial responses based on "level-*k*" thinking, which describes the patterns of deviation from equilibrium in a variety of other experiments (Stahl and Wilson (1994, 1995); Nagel (1995); Ho, Camerer, and Weigelt (1998); Costa-Gomes, Crawford, and Broseta (2001); Camerer, Ho, and Chong (2004); and Costa-Gomes and Crawford (2004)). Our level-*k* model builds on Bacharach and Stahl's (1997a) "level-*n* variable-frame" model of a simplified version of RTH's games.[8] Each player follows one of a hierarchy of general strategic decision rules or *types*, whose population frequencies are equal for Hiders and Seekers. Type *Lk*, for $k > 0$, anchors its beliefs in an *L0* type and then adjusts them via

[7]Although equilibrium is a general strategic decision rule, it is not clear how to parsimoniously extend the model's payoff perturbations to some games with strategic structures more complex than Hide-and-Seek (Section VI).
[8]Bacharach and Stahl's (1997a) analysis of Hide-and-Seek did not appear in the published version of their paper, Bacharach and Stahl (2000). Bacharach and Stahl (1997b), whose title suggests a more detailed version of their Hide-and-Seek analysis, is unavailable. The relation between their model and ours is discussed in Section IV.

thought-experiments involving iterated best responses: *L1* best responds to *L0*, *L2* to *L1*, and so on. The anchoring type *L0* is nonstrategic, but responds to the framing by probabilistically favoring the salient focally labeled and end locations, equally for Hiders and Seekers. *Lk* Hiders' and Seekers' asymmetric, heterogeneous reactions to this role-symmetric *L0* yield a simple explanation of the patterns RTH observed, without invoking unexplained differences in behavior across player roles.

Section IV's analysis shows that a level-*k* model is a promising alternative to an equilibrium with perturbations explanation of RTH's results. But both models' predictions depend on behavioral parameters, and the analysis does not yet provide a clear basis for preferring one over the other. To make the analysis more concrete and prepare for a more detailed comparison, Section V estimates the models econometrically, pooling the data from RTH's six main treatments. Both models' parameterizations are flexible enough to fit the data very well, with a small likelihood advantage for the equilibrium model. Section V addresses the concerns about overfitting this flexibility raises by computing separate parameter estimates for each treatment and using them to "predict" the results of the other five treatments. Here the level-*k* model has a modest advantage, with 18% lower mean squared error than the equilibrium model and better predictions in 20 of 30 comparisons.

Section VI takes up the issue of portability by using the equilibrium with perturbations and level-*k* models, estimated from RTH's pooled data, to "predict" subjects' initial responses to two games beyond the sample: O'Neill's (1987) famous card-matching game and the closely related card-matching game studied by Rapoport and Boebel (1992). Both games raise the same kinds of strategic issues as RTH's game, but with more complex patterns of wins and losses, different framing, and in the latter case five locations. These differences provide a good test of portability.

The equilibrium model's payoff perturbations are readily adapted to O'Neill's game, but the resulting model fits his subjects' initial responses worse than equilibrium without perturbations, which fits poorly. The level-*k* model adapts just as easily and fits O'Neill's data very well. By contrast, there is no plausible, parsimonious way to adapt the payoff perturbations model to the more complex structure of Rapoport and Boebel's game. The level-*k* model again adapts easily but fits their data only slightly better than equilibrium. Overall, the level-*k* model with our non-strategic, role-symmetric specification of *L0* has a modest but significant advantage in portability.

Section VII is the conclusion.

I. Analogies Across RTH's Treatments and Preliminary Statistical Tests

This section describes RTH's other treatments in Table 1 and explains the senses in which they are analogous to the RTH-4 treatment discussed in the Introduction. We argue that the patterns of deviations from equilibrium observed in RTH-4—the fact that central A is the modal location for both Hiders and Seekers, its greater prevalence for Seekers, and the fact that Seekers find the prize more often than in equilibrium—extend, properly interpreted, to the five RTH treatments that are most closely analogous to RTH-4 (Table I, top half). We also report preliminary statistical tests of whether the choice frequencies in those six treatments can be pooled in the econometric analysis.

In RTH-4 the Hider hid a desirable "prize," which we call a "treasure" as in RTH (1996). The five other treatments in the top half of Table 1 include three more "Treasure" treatments, RT-AABA-Treasure (with the B in the third position, perhaps because it was run in Hebrew), RT-1234-Treasure, and R-ABAA. They also include two "Mine" treatments, RT-AABA-Mine and RT-1234-Mine, identical to the corresponding Treasure treatments except that the hidden object is undesirable, so that Hiders' and Seekers' payoffs are interchanged. This yields an equivalent normal form with players' roles reversed, leaving equilibrium predictions otherwise unchanged. However, because Hiders inherently move first, even though Seekers do not observe their choices Mine treatments have different extensive forms than Treasure treatments with roles reversed. RTH, suspecting that this difference might make it easier for Seekers to mentally simulate Hiders' choices, used Mine treatments to test whether it explains the role-asymmetric patterns in their data; but the Mine treatments yielded results very close to the corresponding Treasure treatments with roles reversed.[9] This suggests that the role-asymmetries were somehow driven by subjects' responses to the normal-form structure, as in all of the theoretical explanations considered here.

In the three ABAA or AABA Treasure treatments and the AABA Mine treatment, central A—RT's "least salient" location—was the modal choice for both Hiders and Seekers. This pattern extends to the 1234 Treasure and Mine treatments if we follow RT's suggestion that "the least salient response…may correspond to 3, or perhaps 2" and take 2 as analogous to B and 3 to central A. Given this identification, central A was more prevalent for Seekers in all four Treasure treatments and more prevalent for Hiders in both Mine treatments; thus this pattern is also the same in all six treatments if Hiders in Treasure and Seekers in Mine treatments are identified. Further, the frequencies with which Seekers found a Treasure *or* a Mine exceeded ¼, so that Seekers (Hiders) had higher (lower) expected payoffs than in equilibrium in Treasure treatments, and vice versa in Mine treatments. Accordingly, our analysis "builds in" these analogies by identifying 2 with B, and Mine treatments with the corresponding Treasure treatments with reversed player roles. To avoid unnecessary repetition, we use "central A" ("B") to refer to either a central A (B) or a 3 (2) location; and we refer to Mine treatments as if they were Treasure treatments with reversed roles.

After transforming the data as these identifications suggest, chi-square tests for differences in subjects' aggregate choice frequencies across the six treatments in the top half of Table 1 reveal no significant difference for Seekers (p-value 0.4836) or Hiders (p-value 0.1635). Pairwise tests suggest that RTH-4 differs somewhat from the other treatments in having high frequencies of B (as well as central A) for both Hiders and Seekers. Although this difference is intriguing, we focus on explaining the prevalence of central A for Hiders and Seekers and its greater prevalence for Seekers in all six treatments, pooling them for the econometric analysis. The pooled sample includes 624 Hiders and 560 Seekers, with the aggregate choice frequencies in Table 3.[10]

II. Equilibrium with Payoff Perturbations

This section considers the candidate explanation of RTH's results that is perhaps closest to the mainstream, equilibrium analysis of a Hide-and-Seek game with payoff perturbations that reflect the salience of focally labeled and/or end locations and its strategic consequences. The perturbations themselves are "hard-wired" preferences, independent of strategic reasoning. Following RTH's discussions, we assume that Seekers receive an additional payoff of e for choosing an end location (equal for both ends for simplicity) or f for choosing the focally labeled B location; and that Hiders lose equal payoffs for such choices (Figure 2). We start by assuming e and

[9]In Weber, Camerer, and Knez's (2004) experiments with ultimatum and weak-link coordination games, timing without observability had only a weak effect on outcomes.

[10]We made minor corrections to the published data to reconcile the reported frequencies and sample sizes. Although our analysis is limited to the six treatments in the top half of Table 1, the five treatments in the bottom half give additional evidence of the robustness of the patterns in RTH's data. These are Treasure treatments with the same payoff structure as RTH-4, but labels with positive or negative connotations and/or focally labeled end locations. RTH-2 and RTH-5 are analogous to RTH-4 except for the connotations of the focal label. RTH-1 and RTH-3 are like RTH-4 except that the focal label is at an end position, and in RTH-3 it has a negative connotation. RTH-6 is like RTH-5 except that the focal label is in the third rather than second position; and is like RTH-2 and RTH-4 except for this difference in position and that the focal label has a positive connotation in RTH-6 but negative or neutral connotations in RTH-2 or RTH-4. The choice frequencies for these treatments echo those for the ones we analyze, with shifts in expected directions. It seems likely that Section IV's level-k explanation of RTH's results could be adapted to them by introducing and estimating payoff perturbations and, possibly, new $L0$ choice probabilities.

f have the same magnitudes for both player roles. We also focus on the leading case where $e, f > 0$, which Section V's estimates show to provide the best explanation of RTH's results for this model.

If $-1 < f - 2e$, $2e - 3f$, $2e + f < 3$, then the perturbed game has a unique, symmetric, totally mixed equilibrium. In this equilibrium Hiders and Seekers both play A, B, A, A with probabilities $¼ - e/2 + f/4$, $¼ + e/2 - 3f/4$, $¼ + e/2 + f/4$, and $¼ - e/2 + f/4$ respectively. A Hider's expected payoff is $¾ - e/2 - f/4$ and a Seeker's is $¼ + e/2 + f/4$. Thus, both Hiders and Seekers play central A with probability $¼ + e/2 + f/4$, which is greater than ¼ whenever $2e + f > 0$, but equal for Hiders and Seekers if e and f are equal. A Seeker finds the treasure with probability:

$$(1)\; m = \left(\frac{1}{4} - \frac{e}{2} + \frac{f}{4}\right)^2 + \left(\frac{1}{4} + \frac{e}{2} - \frac{3f}{4}\right)^2 + \left(\frac{1}{4} + \frac{e}{2} + \frac{f}{4}\right)^2 + \left(\frac{1}{4} - \frac{e}{2} + \frac{f}{4}\right)^2 = \frac{1}{4}\left(1 + (2e - \sqrt{3}f)^2 + 4(\sqrt{3} - 1)ef\right),$$

which is greater than ¼ for any $e, f > 0$, so equilibrium with payoff perturbations can explain why Seekers find the treasure more often than in a standard equilibrium. Equilibrium with perturbations of equal magnitudes but opposite signs across player roles can thus explain why central A is modal for both Hiders and Seekers, but not its greater prevalence for Seekers. The model can explain the role asymmetry in RTH's results only by invoking unexplained differences in the magnitudes of e and f as well as their signs, with $2e + f$ sufficiently larger for Hiders than Seekers.[11] It is clear, however, that such differences give the model enough flexibility to explain virtually any pattern.

III. Quantal Response Equilibrium with Payoff Perturbations

This section considers explanations based on a generalization of equilibrium called Quantal Response Equilibrium ("QRE"; McKelvey and Palfrey (1995)), which explains the patterns of deviations from equilibrium in some other experiments.[12] In a QRE players' choices are noisy, with the probability of each choice increasing in its expected payoff, given the distribution of others' choices; a QRE is thus a fixed point in the space of players' choice distributions. The specification is completed by a response distribution, whose noisiness is represented (inversely) by a precision parameter. Some of our results are independent of this distribution, but for others we adopt the standard assumption of logit responses and study the special case called "logit QRE".

[11] $2e+f$ must be larger for Hiders because larger values yield higher probabilities of central A, and Hiders' value determines Seekers' probability. In Section V we report estimates of $e_H = 0.2910$, $f_H = 0.2535$, and $e_S = f_S = 0.1539$ for the equilibrium with unrestricted perturbations model: all positive as expected, and nearly twice as large for Hiders.

[12] RSW find that QRE gives a reasonable explanation of the qualitative features of subjects' role-asymmetric deviations from equilibrium in 2x2 Hide-and-Seek games with neutral framing but varying payoffs. See however McKelvey, Palfrey, and Weber (2000), who find that QRE does less well in explaining behavior in other kinds of zero-sum games.

Because QRE responds only to the payoff structure, it ignores the framing of the Hide-and-Seek game without payoff perturbations. In that game, for any error distribution, there is a unique QRE, which yields the same choice probabilities as equilibrium. To see this, suppose to the contrary that in a QRE the most probable location for Hiders, call it P, has probability greater than ¼. Because QRE choice probabilities increase with expected payoffs and the game is constant-sum, P must then have the highest expected payoff for Seekers, thus probability greater than ¼ for them. But then some location other than P has higher expected payoff for Hiders, a contradiction.

This section accordingly considers explanations that combine logit QRE with payoff perturbations as in Figure 2, which make QRE sensitive to the framing and give it the potential to explain RTH's results. As is almost always the case, such models must be solved computationally.

Figure 3 illustrates logit QRE with payoff perturbations restricted to be equal in magnitude but opposite in sign across player roles, as a function of λ, with $e = 0.2187$ and $f = 0.2010$, the values that best fit RTH's data for Section II's equilibrium with restricted perturbations model (Section V). The maximum likelihood estimate of λ in the QRE with restricted perturbations model is effectively infinite, reducing the model to the analogous equilibrium model (footnote 18). As in Figure 3, for all combinations of $e, f = 0.1, 0.2, 0.3$, or 0.4 (all consistent with a totally mixed equilibrium), the logit QRE probability of central A dips below 0.25 for low values of λ for Seekers but never for Hiders; and it is always higher for Hiders, reversing the patterns in RTH's data.[13]

Thus, logit QRE can explain the prevalence of central A for Hiders and Seekers with perturbations of equal magnitudes but opposite signs across player roles, as in Section II's perturbed equilibrium model. But the main difficulty is explaining the greater prevalence of central A for Seekers, and in this case logit QRE robustly predicts that central A is more prevalent for *Hiders*.

Like equilibrium with payoff perturbations, logit QRE can only explain RTH's results by postulating large differences across player roles in the magnitudes of the perturbations e and f as well as their signs. But this again yields an effectively infinite estimate of λ, reducing logit QRE, in terms of its substantive implications, to Section II's equilibrium with unrestricted perturbations model (footnote 18). Figure 4 illustrates logit QRE with $e_H = 0.2910$, $f_H = 0.2535$, and $e_S = f_S = 0.1539$, the values that give the best fit for the equilibrium with unrestricted perturbations model.

[13]There is enough structure to suggest that this result is symptomatic of a theorem, but we have been unable to prove it.

IV. A Structural Non-Equilibrium Model of Initial Responses Based on "Level-*k*" Thinking

Returning to the Hide-and-Seek game without payoff perturbations, this section considers a structural non-equilibrium model of initial responses based on level-*k* thinking. The model allows individual behavior to be heterogeneous; but each player, without regard to player role, is assumed to follow one of a hierarchy of five general strategic decision rules or types, *L0*, *L1*, *L2*, *L3*, or *L4*, drawn from the same distribution with given probabilities r, s, t, u, and v respectively.

Type *Lk* for $k > 0$ anchors its beliefs in an *L0* type and adjusts them via thought-experiments involving iterated best responses, given his player role: *L1* best responds to *L0*, *L2* to *L1*, and so on.[14] These best responses determine the normal choice for a player of type $k > 0$, with ties broken randomly with equal probabilities. But with probability ε, equal across roles and $k > 0$, a player makes an error, in which case he chooses each location with equal probability; these errors are independently and identically distributed across players.

Because type *Lk* for $k > 0$ ignores the framing except as it affects the anchoring type *L0*, *L0* is the key to the model's potential to explain RTH's subjects' responses to framing. We take *L0* to be nonstrategic, as is usual in such analyses; but we allow *L0* Hiders and Seekers to respond to the framing by probabilistically favoring the salient focally labeled and end locations, equally for each player role. We represent *L0*'s preferences directly as choice probabilities rather than payoff perturbations, taking the probabilities of the end locations to be equal for simplicity: Type *L0* Hiders and Seekers both choose A, B, A, A with probabilities $p/2$, q, $1-p-q$, $p/2$ respectively.[15] *L0*'s non-uniform choice probabilities depart from much of the literature, but in RTH's games a uniform *L0* would make *Lk* the same as equilibrium for all *k*, and there is ample precedent for adapting *L0* to the setting. See for example Ho, Camerer, and Weigelt's (1998) analysis of guessing games; or Crawford's (2003) analysis of strategic deception via cheap talk, where the Sender's and Receiver's anchoring types are based on truthfulness or credulity, as in the informal literature on

[14]Note that *L0* serves simultaneously as *L1*'s model of others, *L2*'s model of others' models of itself, and so on. This is a plausible simplification if the nonstrategic intuitions *L0* reflects are widely understood. It is less strained than the knowledge assumptions underlying equilibrium with payoff perturbations because it refers only to *L0* and there is no need for the knowledge about *L0* to be common. Even mutual knowledge of *L0* could be relaxed, with some effort. Costa-Gomes and Crawford (2004) summarize the experimental evidence for the level-*k* model, and give support for our assumptions that *L2* best responds to an *L1* without decision errors, and to *L1* alone rather than a mixture of *L1* and *L0*, etc., unlike in Stahl and Wilson (1994, 1995) or Camerer, Ho, and Chong (2004). In RTH's games *L5*'s choice probabilities (but not its expected payoffs) are the same as *L1*'s (Table 2), so that *L5* is equivalent to *L1*, *L6* to *L2*, and so on, and our types cycle. Thus our model with five types is the most general model of this kind, given our definitions.

deception. By contrast, the level-k model's other main component, the adjustment of beliefs via iterated best responses, appears to allow a satisfactory account of initial responses in many settings.

Lk types have accurate models of the game and are rational; they depart from equilibrium only in basing their beliefs on simplified models of other players. This yields a workable model of others' choices while avoiding the cognitive complexity of equilibrium analysis. In Selten's (1998) words: "Basic concepts in game theory are often circular in the sense that they are based on definitions by implicit properties…. Boundedly…rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found. Each step of the procedure is simple, even if many case distinctions by simple criteria may have to be made."

We focus on the leading case in which $p > ½$ and $q > ¼$, so that *L0* favors both B and end locations. Table 2 lists *L0*'s, *L1*'s, *L2*'s, *L3*'s, and *L4*'s normal choice probabilities in this case, in which $p + 2q > 1$ and $3p + 2q > 2$, distinguishing the regions $p < 2q$ and $p > 2q$. Table 2's bottom lines give the total choice probabilities for the entire population, including errors. Figure 5 summarizes types' normal choices for the other cases, where it is not assumed that $p > ½$ and $q > ¼$.

When $p > ½$ and $q > ¼$, *L1* Hiders choose central A to avoid *L0* Seekers and *L1* Seekers avoid central A in their searches for *L0* Hiders. For similar but more complex reasons, *L2* Hiders choose central A with probability between zero and one and *L2* Seekers choose it with probability one; *L3* Hiders avoid central A and *L3* Seekers choose it with probability between zero and one; and *L4* Hiders and Seekers avoid central A. Thus, although we include *L4* for completeness and to avoid biasing the econometric type frequency estimates, our explanation of the main patterns RTH observed does not require *L4*, which chooses central A only in error.

These types' choice patterns allow the model to explain RTH's results for plausible values of the population type frequencies. For central A to be modal for Hiders, its total probability for them,

(2) $$r(1-p-q)+(1-\varepsilon)[s+(t/2 \text{ if } p > 2q) + (t/3 \text{ if } p < 2q)]+(1-r)\varepsilon/4 > ¼.$$

This means that a weighted average of the frequencies of *L1* and *L2* (with weights depending on whether $p > 2q$, and on ε) exceeds the frequency of *L0* by enough to outweigh *L0* Hiders' tendency to avoid central A. For central A to be more prevalent for Seekers than Hiders,

---

[15]This departure from our equilibrium model is less important than it may seem because it does not affect the range of best responses for *Lk* for $k > 0$; and *L0*'s choices could be "purified" via privately observed payoff perturbations.

(3) $t + u > 2s$ if $p > 2q$ or $2t + u > 3s$ if $p < 2q$,

so that a weighted average of the frequencies of *L2* and *L3* (with weights depending on whether $p > 2q$) exceeds the frequency of *L1* by enough to outweigh *L1* Seekers' tendency to avoid central A.

It is noteworthy that RTH's results, viewed through the lens of the non-equilibrium level-*k* model via (2) and (3), actually imply *lower* bounds on the population's sophistication, in contrast to RTH's interpretation of their results as evidence of strategic naivete. These bounds show that the level-*k* model can explain why central A is the modal choice for both Hiders and Seekers and its greater prevalence for Seekers for values of the population type frequencies that are similar to, but possibly more sophisticated than, those that have been estimated for other settings (see Camerer, Ho, and Chong (2004), Costa-Gomes and Crawford (2004), and the papers discussed there). This is confirmed by Section V's econometric estimates of the population type frequencies (Table 3).

To put the level-*k* model's explanation of RTH's results into perspective, consider a simpler alternative that has been suggested to us: "Hiders feel safer avoiding focal locations, so they are most likely to choose central A; Seekers know this, so they are also most likely to choose central A." Although this sounds plausible, it has two weaknesses: It implicitly assumes that Hiders are systematically less sophisticated (*L1*, in our terminology) than Seekers (*L2*), and it does not even try to explain the greater prevalence of central A for Seekers.[16] Our level-*k* model, by contrast, explains RTH's results without invoking unexplained role differences in behavioral assumptions.

As noted in the Introduction, our level-*k* analysis builds on Bacharach and Stahl's (1997a) "level-*n* variable-frame" analysis of a simplified version of RTH's Hide-and-Seek game. Their game has payoffs like RTH's but only three locations, one clearly salient and one salient but less clearly so. The main difference between their model and ours is that their *L0* is asymmetric across player roles: *L0* Hiders probabilistically avoid salience and *L0* Seekers probabilistically favor it.[17] Their justification is evolutionary: "For early humans, 'looking' problems were more generic and 'hiding' problems more strategic" (their footnote 13). Their model predicts that the least salient

[16]The quotation in the text could be inverted to "Seekers are drawn to focal locations, so they are unlikely to choose central A; Hiders know this, so they are likely to choose central A." This version uses the same logic as the one in the text with player roles interchanged, but it does not fit the patterns in RTH's data.

[17]They also derive types' choices from more basic assumptions about players' perceptions of focality and their beliefs about others' perceptions, with *Lk* for all $k > 0$ directly sensitive to the framing and players' knowledge of it, while in our model only *L0* responds directly to the framing; and they assume that *L2* best responds to a mixture of *L1* and *L0* rather than to *L1* alone as in our model (footnote 14). These differences matter quantitatively, but are less important.

location is modal for both Hiders and Seekers and more prevalent for Seekers—the analog of RTH's results—with only types *L0*, *L1*, and *L2*, as long as *L2* outnumbers *L0* in the population.

This type distribution seems less sophisticated and correspondingly closer to previous estimates than our model's type distribution, which must also include *L3* to explain RTH's results. However, this difference is mainly semantic. Bacharach and Stahl's *L0* is strategic in that it distinguishes between the Hider's and Seeker's role. Its closest counterpart in our model is our *L1*, which best responds asymmetrically across player roles to our role-symmetric *L0* in much the same way that their *L0* is assumed to respond instinctively to the roles. Thus, re-numbering our types downward would make them comparable in sophistication, given *k*, to Bacharach and Stahl's types. This would allow us to explain RTH's results with a role-asymmetric *L0* and only types called *L1* and *L2*, as in our preliminary version, Crawford and Iriberri (2004, Tables II and III) (footnote 23).

Although Bacharach and Stahl's evolutionary argument for their role-asymmetric *L0* is quite intuitive for Hide-and-Seek, the implicit assumption that Seekers (who follow their attraction to salience) are less sophisticated than Hiders (who strategically anticipate Seekers' following their attraction) is troubling. Their *L0*, like the role-asymmetric payoff perturbations in Section II's equilibrium model, also begs the question of why RTH's game elicits systematically different responses from Hiders and Seekers. Our level-*k* model, by contrast, explains those responses via *Lk* types' asymmetric responses to a role-symmetric *L0*, as shaped by the game's payoff structure.

A role-asymmetric *L0* also makes the model's parameterization sufficiently more flexible to raise concerns about overfitting, as with Section II's equilibrium with unrestricted perturbations model. It also requires tailoring *L0* closely to the strategic structure of the game, which can make it difficult to adapt to games with strategic structures more complex than Hide-and-Seek's, reducing portability and weakening one of the level-*k* model's main advantages over equilibrium with perturbations. Sections V and VI show that that our level-*k* model's non-strategic, role-symmetric *L0* makes it less likely to overfit and gives it a significant advantage in portability over a model like Bacharach and Stahl's with a role-asymmetric *L0*, or an equilibrium with perturbations model.

V. Econometric Analysis and a Test for Overfitting

The models considered here all depend on behavioral parameters: payoff perturbations for equilibrium and QRE, a logit precision for QRE, and *L0* choice probabilities and population type frequencies for the level-*k* model. This section makes our analysis more concrete and prepares to consider overfitting and portability by using RTH's data to estimate the parameters. We focus on the equilibrium with perturbations and level-*k* models because the estimated QRE model reduces to the equilibrium model.[18] We then address the issue of overfitting by using estimates computed for each model, treatment by treatment, to "predict" the results of the other five treatments.

Our goal is to illustrate the possibilities of the level-*k* and equilibrium with perturbations models, not to take a definitive position on the behavioral parameters. In our view that would require more comprehensive experiments, perhaps with a design that tracks individual subjects' choices across different but related games as in Stahl and Wilson (1994, 1995); Costa-Gomes, Crawford, and Broseta (2001); or Costa-Gomes and Crawford (2004). Because both models have enough flexibility to fit the observed choice frequencies very well, our econometrics amount to little more than a calibration exercise. But using econometrics to calibrate the models constrains our discretion, and yields likelihoods that provide an objective criterion by which to compare them.

Our econometric model is a mixture-of-types model as in Stahl and Wilson (1994, 1995) or Costa-Gomes, Crawford, and Broseta (2001), with a single type for the equilibrium with perturbations model and five for the level-*k* model (Sections II and IV). Let X_{ij} denote the total numbers of Hiders or Seekers (with $i = H, S$) who choose location j (with A, B, A, A denoted 1, 2, 3, 4). Let π_k be the probability that a given subject is type k, and let p_{ikj} be the probability that a Hider or Seeker of type k chooses location j. The full-sample likelihood can then be written:

(4) $$L \equiv \prod_{i=H,S} \prod_{j=1,2,3,4} \left[\sum_k \pi_k p_{ijk} \right]^{X_{ij}}.$$

The p_{ijk} for the equilibrium model are $¼ – e/2 + f/4$, $¼ + e/2 – 3f/4$, $¼ + e/2 + f/4$, and $¼ – e/2 + f/4$ for locations A, B, A, A ≡ 1, 2, 3, 4, for $i = H, S$ (Section II). Because the equilibrium that best describes RTH's results will be totally mixed, we dispense with an error structure in this case.

[18]With payoff perturbations restricted to have equal magnitudes across player roles, the estimated $\lambda\to\infty$. With perturbations allowed to differ in magnitude across roles, for any sufficiently large but finite λ QRE can adjust the perturbations to match the observed frequencies exactly. Thus λ and the perturbations are not identified, but all parameter values that maximize the likelihood are equivalent to those obtained when $\lambda\to\infty$. With finite λ the estimated perturbations for Hiders (Seekers) are higher (lower) than those estimated for equilibrium with perturbations.

The p_{ijk} for the level-k model can be read from Figure 5 (with equal choice probabilities for multiple listed locations for Hiders or Seekers in a given region) or from Table 2 when $p > ½$ and $q > ¼$. Here, to avoid specification bias, we specify a simple uniform error structure.[19]

Table 3 summarizes parameter estimates and likelihoods for the equilibrium with payoff perturbations and level-k models, the former with and without the restriction that the perturbations are equal in magnitude for Hiders and Seekers, and the latter estimated under the constraints $p > ½$ and $q > ¼$, which ensure that *L0* favors focally labeled and end locations.[20] Table 3 also gives the models' predicted choice frequencies, with RTH's observed frequencies for comparison.

For the equilibrium model, we strongly reject the restrictions that $e_H = e_S$ and $f_H = f_S$ (p-value 0.0022). Accordingly, we focus on the equilibrium model with unrestricted perturbations. Both it and the level-k model have enough flexibility to fit the observed choice frequencies very well, except that both restrict the two end locations to have equal probabilities, which is not quite true in the data. The unrestricted equilibrium model has a small likelihood advantage.[21] The signs of the estimated payoff perturbations are behaviorally plausible. We are unaware of any estimates with which to compare the magnitudes of the perturbations, but they are nearly twice as large for Hiders than Seekers, a difference for which it seems hard to find a plausible explanation.

The level-k model's estimates are generally behaviorally plausible, with $r = 0$, so that *L0* exists only in the minds of *L1* through *L4*; and $\varepsilon = 0$, so all choices are explained by *L1* through *L4*.[22] Because the estimated $r = 0$, *L0*'s choice probabilities p and q are not identified; but the region in which the likelihood is maximized subject to $p > ½$ and $q > ¼$ is identified as region 2 (Figure 5),

[19]With our error structure there are linear dependencies among types' choice probabilities for Hiders and Seekers; but they are different across roles, so the population type frequencies are still identified from the full sample (Table 2).
[20]These constraints are binding. Estimating the model without them yields estimates in region 5 of Figure 5, with $p < ½$, $q < ¼$, $2q > p$, and a fit nearly identical to that of equilibrium with unrestricted perturbations. We focus on the estimates in the text because such estimates allow the level-k model to overfit, creating the same "prediction" problems we identify below for equilibrium with unrestricted perturbations, and $p < ½$, $q < ¼$ is psychologically implausible for a nonstrategic *L0*; see however Attali and Bar-Hillel (2003), who argue that people are averse to end locations, based on evidence from settings including some that might be sufficiently non-strategic to be relevant to our *L0*.
[21]To put this difference in perspective, the maximum possible log-likelihood, associated with perfect prediction of the choice frequencies, is –1561.7 (strictly negative here because the model makes probabilistic predictions).
[22]The finding that there are no *L0* subjects is consistent with the common finding that people systematically underestimate others' sophistication relative to their own (see for instance Weizsäcker (2003)). In Hide-and-Seek without payoff perturbations, our uniform errors are perfectly confounded with the equilibrium mixed-strategy probabilities. Thus our finding that $\varepsilon = 0$ also suggests the absence of an *Equilibrium* type. Further, we can reject explanations in which a not-too-large part of the population choose locations with given probabilities (like *L0*) and the rest play equilibrium in a game among themselves, taking the first part's behavior into account, like the *Sophisticated* type in Costa-Gomes, Crawford, and Broseta (2001), Crawford (2003), or Costa-Gomes and Crawford (2004).

where $p > 2q$ so *L0* favors end more than focally labeled locations.[23] The estimated type frequencies seem somewhat more sophisticated than previous estimates (Stahl and Wilson (1994); Costa-Gomes, Crawford, and Broseta (2001); Costa-Gomes and Crawford (2004); Camerer, Ho, and Chong (2004)). As argued in Section IV, this difference is mainly semantic; but it could also reflect more sophisticated subject pools or the strategic transparency of Hide-and-Seek.

Given the flexibility of the equilibrium with unrestricted perturbations and level-*k* models' parameterizations, overfitting is a concern. We test for it by re-estimating the models separately for each of the six treatments and using the re-estimated models to "predict" the observed choice frequencies of the other five treatments. For the level-*k* model we restrict the estimates to region 2 ($p > 2q$, Figure 5) because the parameters are not identified in region 1 ($p < 2q$) without imposing $r = \varepsilon = 0$; and while the fit is slightly better in region 1 for RTH-4 (log-likelihood -143.0 versus -144.2 in region 2), RT-AABA-Treasure (-361.1 versus -363.6), and R-ABAA (-142.0 versus -142.1), region 2 yields more sensible estimates and better predictions. We evaluate goodness-of-fit by mean squared deviation ("MSD") between predicted and observed choice frequencies.

Tables 4-7 summarize the results of the overfitting test. Although the equilibrium with unrestricted perturbations model has a fit at least as good in each treatment (Table 7's diagonal), our favored specification of the level-*k* model has a modest advantage in prediction, with mean squared prediction error 18% lower and better predictions in 20 of 30 comparisons (Table 7).[24]

A level-*k* model with role-dependent *L0* in the style of Bacharach and Stahl (1997a) would yield predictions very close to the equilibrium with unrestricted perturbations model's, and so would probably have similar overfitting problems (Crawford and Iriberri (2004, Table III).

VI. Portability: Comparing the Equilibrium and Level-*k* Models in Related Games

As noted in the Introduction, an important issue in model selection is portability, the extent to which estimating a model's parameters from subjects' responses in one setting is useful in explaining behavior in other settings. In this section we compare the ability of the equilibrium with perturbations and level-*k* models, with parameters estimated from RTH's data, to "predict" subjects'

[23]The maximized log-likelihood in Region 1 is slightly lower, at -1567.9. In a sense the "best fit" is on the border between Regions 1 and 2, where $p = 2q$, which yields log-likelihood -1563.8; but here the parameters are not identified (in RTH's dataset) even if we restrict $r = \varepsilon = 0$. The parameters are identified if we restrict $r = s = v = 0$, allowing $\varepsilon > 0$, which yields the model with role-dependent *L0* of Crawford and Iriberri (2004, Tables II and III)). Due to these identification problems and the theoretical advantages of role-independent *L0*, we focus here on the interiors of regions.
[24]However, if we allow Region 1 estimates for RTH-4, RT-AABA-Treasure, and R-ABAA and assume r=ε=0 to avoid identification problems, the level-*k* and equilibrium with unrestricted perturbations models have the same overall MSD.

initial responses to O'Neill's (1987) famous card-matching game and Rapoport and Boebel's (1992) closely related game, the closest analogs of RTH's Hide-and-Seek experiments we are aware of.

O'Neill's and Rapoport and Boebel's games, like RTH's, are zero-sum two-person two-outcome games with non-neutral framing of locations. They differ from RTH's games in having different framing, in one case five locations, and win-loss patterns that vary across locations in more complex ways than in Hide-and-Seek. Figures 6 and 7 depict the payoff matrices, with the payoff of winning again normalized to one. The experimenters presented both games to subjects as stories, but Rapoport and Boebel's subjects were also given a matrix like that in Figure 7, but with Wins and Losses (from Player 1's, Row's, point of view) represented by W or L.[25] The stories and Rapoport and Boebel's matrix both ordered the locations as they are ordered in Figures 6 and 7.

O'Neill's and Rapoport and Boebel's subjects played the same game repeatedly in fixed pairs, with feedback after each play. O'Neill had only one treatment, while Rapoport and Boebel had two, which differed only in scaling and expected magnitude of payoffs in ways that do not affect the predictions of the theories considered here. We focus on subjects' first-round choices, interpreting them as initial responses. This is less straightforward here than for RTH's data, which were responses to a single play or a series played without feedback, because with fixed pairs and feedback subjects' first-round choices could have been made partly to influence future play.[26] Our interpretation reflects a judgment that future influences were not an important source of distortion in this case. Another difficulty is that Rapoport and Boebel's subjects played first in one player role, then the other. We deal with this by using only a subject's first response to the game, in either role. There were 25 subjects per role in O'Neill's experiment, and 10 per role in Rapoport and Boebel's.

It is clear from Figures 6 and 7 that in O'Neill's game A, 2, and 3 are strategically symmetric, as are F, I, and O in Rapoport and Boebel's game. Tables 8 and 10 give these games' unique equilibrium mixed strategies, which as in RTH's game are symmetric across player roles; and subjects' first-round responses. Although equilibrium reflects the strategic symmetries across

[25]O'Neill's story (p. 2708) was: "Each player has four cards—Ace, two, three, and a joker…. [Player 1] wins if there is a match of jokers (two jokers played) or a mismatch of number cards (two, three, for example). [Player 2] wins if there is a match of number cards (three, three, for example) or a mismatch of a joker (one joker, one number card). Although this wording leaves some room for doubt whether Ace was also a number card, the practice rounds would make it clear.

[26]Moreover, the first paid round was preceded by 10 or 15 unpaid practice rounds in the same fixed pairs, in which subjects could have learned such influences were possible. Such influences are theoretically possible via "contagion" even when subjects are randomly paired from large groups, with different partners each round, but they seem to be potentially empirically important only in small, fixed groups. Their potential behavioral importance is not eliminated by the fact that the only equilibrium of the repeated game is repeated play of the equilibrium of the one-shot game.

locations with equal equilibrium probabilities, both equilibrium with perturbations and the level-*k* model break the symmetries in response to differences in salience. We therefore keep strategically symmetric locations separate. Relative to equilibrium, O'Neill's subjects have a large, positive Joker effect for both players, with the Joker even more prevalent for player 2s.[27] Rapoport and Boebel's subjects have a large, positive I effect for player 1s in both treatments; a large, positive C effect for player 2s in treatment 1; and a large, positive L effect for player 2s in treatment 2. There is a statistically significant difference across their treatments for player 2s (*p*-value 0.0087) but not for player 1s (*p*-value 0.8557). We keep their treatments separate in both roles.

We begin by adapting Section II's equilibrium with perturbations model to O'Neill's game. O'Neill's game differs from RTH's in both framing and payoff structure. With regard to structure, O'Neill's game is not a Hide-and-Seek game, but player 1 (Row) can be viewed as a hider for the number cards A, 2, and 3 and a seeker for the Joker; and player 2 as reversing these roles. With regard to framing, the end positions of Ace and Joker reinforce the salience of their labels, and Joker's unique role in the payoff structure may make it more salient than Ace, despite its position. Extending Section II's analysis, we assume that a player choosing a salient card for which he is a seeker receives an additional payoff of $\alpha > 0$ for Ace and $\iota > 0$ for Joker, and a player choosing a salient card for which he is a hider loses equal payoffs for such choices, as in Figure 8.

If $3\alpha - \iota < 1$ and $\alpha + 3\iota < 2$, then the perturbed O'Neill game has a unique, symmetric, totally mixed equilibrium, in which Player 1 and 2 both play A, 2, 3, and J with respective probabilities $(1-3\alpha+\iota)/5$, $(1+2\alpha+\iota)/5$, $(1+2\alpha+\iota)/5$, and $(2-\alpha-3\iota)/5$. Thus, the model cannot explain the greater prevalence of J for player 2s without assuming that the magnitudes of α and ι differ across player roles. More importantly, the probability of J is maximized subject to $\alpha, \iota \geq 0$ when $\alpha = \iota = 0$, which yields only the equilibrium probability of 0.4, well below the observed frequency in each player role. Thus, if we assume that the Ace and Joker are salient, as seems compelling, then equilibrium with perturbations can do no better than unperturbed equilibrium in explaining O'Neill's initial responses.

[27]This Joker effect is something of a surprise because discussions of O'Neill's data have been dominated by a small, positive "Ace effect" when the data are aggregated over all rounds (player 1s and 2s played Ace 22.0% and 22.6% of the time, versus the equilibrium 20%). To our knowledge the Joker effect for initial responses has not been noted before, even though it is an order of magnitude larger than the Ace effect. Although O'Neill speculated that "players were attracted by the powerful connotations of an Ace," the fact that Ace is more prevalent only in the aggregate data suggests that this prevalence was mainly a product of the dynamics of subjects' choices rather than the Ace's salience.

We now consider a level-*k* analysis of O'Neill's game. Our nonstrategic, role-independent specification of *L0* adapts to O'Neill's game just as naturally as the equilibrium model's payoff perturbations did.[28] Given the salience of A and J, we assume that *L0* player 1s and 2s both choose A, 2, 3, J with probabilities a, $(1–a–j)/2$, $(1–a–j)/2$, j respectively; and we take $a > ¼$ and $j > ¼$, so *L0* favors both A and J. Tables 9a and 9b lists *L0*'s, *L1*'s, *L2*'s, *L3*'s, and *L4*'s normal choice probabilities in the cases $3j – a > 1$ and $3j – a < 1$ respectively, in the latter case distinguishing the subcases $a + 2j < 1$ and $a + 2j > 1$. Types' choice patterns are determined just as for RTH's game, but differ due to the structure and framing of O'Neill's game. Tables 9a's and 9b's last lines give the total choice probabilities for the entire population, including errors. Table 8 gives the predicted frequencies of A, 2, 3, and J in the three cases, using Table 3's estimates of r, s, t, u, v, and ε.

Comparing the predicted and actual frequencies and the mean squared deviations in Table 8 shows that the level-*k* model easily outperforms the equilibrium model (with or without perturbations) in all regions. The level-*k* model also makes J modal for both player 1s and 2s in all regions, and when either $3j – a > 1$ or both $3j – a < 1$ and $a + 2j > 1$ it reproduces J's greater prevalence for player 2s. Thus the same principles we used to explain the prevalence of central A in RTH's experiments explain O'Neill's Joker effect. Region $3j – a < 1$ yields the best fits, better for player 1s when $a + 2j < 1$, better for player 2s when $a + 2j > 1$, and slightly better on average in the latter case. The overall fit is very close, given that the type frequencies were estimated from RTH's data and that adapting *L0*'s choice probabilities has limited influence on the model's predictions. As in RTH's game, explaining the observed patterns requires a fairly sophisticated type distribution.

We now consider adapting the equilibrium with perturbations model to Rapoport and Boebel's game. Although their game is very close in structure to RTH's and O'Neill's games, it no longer makes a player unambiguously a hider or seeker depending on which location he chooses, and therefore allows no straightforward parameterization of the payoff perturbations.[29] It would still be possible just to use a more flexible parameterization and "let the data speak". But we have already seen that even a parameterization as tight as in Section II's analysis of RTH's game tends to

---

[28]A role-asymmetric specification of *L0* like Bacharach and Stahl's (1997a) also adapts very naturally to O'Neill's game, and fits the data even better. The resulting model is close to the one in Crawford and Iriberri (2004); see footnote 23. As we have argued, this better fit must be weighed against the disadvantages of a role-asymmetric specification of *L0*.

[29]Player 1 (2) is a seeker (hider) for location C. When C is eliminated, player 1 (2) is a hider (seeker) for location L. But even when L and C are eliminated, the player roles for location F cannot be classified this way.

overfit, and a more flexible parameterization takes us further from portability. Thus, the prospects for a useful equilibrium with perturbations analysis of Rapoport and Boebel's results are dim.

Our level-*k* model's nonstrategic specification of *L0*, by contrast, adapts quite naturally to Rapoport and Boebel's game.[30] We assume that none of the labels C, L, F, I, O are focal per se; but it is natural to assume, extending our assumption that the end locations are inherently focal in RTH's games, that both the end and center locations are inherently focal in Rapoport and Boebel's game (Attali and Bar-Hillel (2004)). Accordingly, *L0* player 1s and 2s both choose C, L, F, I, O with probabilities $m/2$, $(1-m-n)/2$, n, $(1-m-n)/2$, and $m/2$ respectively, treating the end locations as equally salient for simplicity, with $m > 2/5$ and $n > 1/5$ so that *L0* favors both ends and the center.

Table 11 lists *L0*'s, *L1*'s, *L2*'s, *L3*'s, and *L4*'s normal choice probabilities, distinguishing the subcases $3m/2 + n < 1$ and $3m/2 + n > 1$; and its last lines give the total choice probabilities for the entire population, including errors. Table 10 gives the predicted frequencies of C, L, F, I, and O when $3m/2 + n <$ or > 1, using Table 3's estimates of r, s, t, u, v, and ε from RTH's data.

Comparing the predicted and actual frequencies and the mean squared deviations in Table 10 shows that the level-*k* model with $3m/2 + n > 1$ clearly outperforms the equilibrium model for treatment 1, but that the equilibrium model yields a better fit for player 2s in treatment 2. The level-*k* model with $3m/2 + n > 1$ reproduces a small fraction of the I effect for player 1s in both treatments and of the C effect for player 2s in treatment 1; but it completely misses the L effect for player 2s in treatment 2. Overall, the level-*k* model does better when $3m/2 + n > 1$, but it fits the data only slightly better than equilibrium without perturbations. We are therefore left with no truly convincing model of Rapoport and Boebel's subjects' initial responses. Given the large variation in player 2s' choice frequencies of C and L across the small samples of Rapoport and Boebel's treatments, further experiments with this game might be useful in resolving the puzzle.

This section's analysis shows that on balance, our level-*k* model has a significant advantage over an equilibrium with payoff perturbations model in portability. This advantage stems from two features of its specification. First, the types are general strategic decision rules meant to apply to any game; and if correctly specified their population frequencies should be stable across different games (there is considerable evidence for their specification, summarized in Costa-Gomes and

---

[30]Trying to specify a level-*k* model with a role-asymmetric *L0* in the style of Bacharach and Stahl (1997a) for Rapoport and Boebel's game leads to the same difficulties noted earlier for the equilibrium with perturbations model.

Crawford (2004)). Second, a nonstrategic, role-symmetric *L0* is easier to specify because it reflects only decision-theoretic considerations, and adapts more readily to new games because variations in framing are normally simpler than the variations in strategic structure that determine payoff perturbations.[31] Although those who use level-*k* models have long recognized the need to adapt *L0* to the setting, there has been little discussion of the principles its specification should follow. We hope our analysis makes clear the importance of a nonstrategic, role-symmetric specification of *L0*.

VII. Conclusion

This paper has compared alternative explanations of the systematic patterns of deviation from equilibrium observed in RTH's experiments with two-person constant-sum Hide-and-Seek games with unique mixed-strategy equilibria and non-neutral framing of locations, and in O'Neill's and Rapoport and Boebels' experiments with games that are not strictly Hide-and-Seek games but whose structures and framing are closely related to RTH's games. We focus on two models, an equilibrium model with payoff perturbations that reflect players' instinctive reactions to their roles, and a structural non-equilibrium model of initial responses based on "level-*k*" thinking. A QRE with payoff perturbations model reduces when estimated to equilibrium with perturbations.

Our level-*k* model explains the puzzling role asymmetries in RTH's results without assuming differences in behavior across roles, while equilibrium with perturbations requires large, unexplained differences in the magnitudes of Hiders' and Seekers' perturbations. It also dispenses with the strong coordination of beliefs assumption that underlies the equilibrium with perturbations model, which seems strained for initial responses to a game with subtle differences in payoffs.

When the models' behavioral parameters are estimated econometrically, both prove flexible enough to fit RTH's observed choice frequencies very well. Equilibrium with perturbations has a small likelihood advantage, but the level-*k* model has a modest advantage with regard to overfitting and within-sample "prediction". The level-*k* model also has a significant advantage in portability, in that it is readily adaptable to O'Neill's and Rapoport and Boebel's games, and that with parameters estimated from RTH's data, it does better in beyond-sample "predictions" for those games.

---

[31]We stress that the advantages of nonstrategic specification are confined to *L0*. The strategic thinking of *Lk* for $k > 0$ plays a major role in our level-*k* analyses, and for RTH's games we estimate a population frequency of 0 for *L0*.

Overall, our analysis helps to resolve a long-standing behavioral puzzle posed by RTH's and related results, and suggests that they can be interpreted as evidence on subjects' strategic thinking. It suggests that the level-*k* model better identifies the structure of initial responses to Hide-and-Seek and related games with non-neutral framing of locations than equilibrium or QRE models, which reinforces the conclusions suggested by evidence from more conventional experiments.

References

Attali, Yigal, and Maya Bar-Hillel (2003): "Guess Where: The Position of Correct Answers in Multiple-Choice Test Items as a Psychometric Variable," *Journal of Educational Measurement*, 40, 109-128

Ayton, P. and R. Falk (1995): "Subjective Randomness in Hide-and-Seek Games," Book of Abstracts of the 15th Bi-annual Conference on Subjective Probability, Utility and Decision-Making, p. 37; discussed at http://arielrubinstein.tau.ac.il/99/gt100.html.

Bacharach, Michael, and Dale Stahl (1997a): "Variable-Frame Level-*N* Theory," manuscript, Institute of Economics and Statistics, University of Oxford, http://www.eco.utexas.edu/~stahl/vflnt4a.pdf.

Bacharach, Michael, and Dale Stahl (1997b): "Variable-Frame Level-*n* Theory with Application to Hide and Seek," Working Paper, Institute of Economics and Statistics, University of Oxford.

Bacharach, Michael, and Dale Stahl (2000): "Variable-Frame Level-*n* Theory," *Games and Economic Behavior*, 32, 220–246.

Camerer, Colin, Teck-Hua Ho, and Juin Kuan Chong (2002): "A Cognitive Hierarchy Theory of One-Shot Games: Some Preliminary Results," manuscript at http://www.hss.caltech.edu/~camerer/thinking2002.pdf.

Camerer, Colin, Teck-Hua Ho, and Juin Kuan Chong (2004): "A Cognitive Hierarchy Model of Games," *Quarterly Journal of Economics*, 119, 861-898.

Chivers, C. J. (2004), "A Dinner in Ukraine Made for Agatha Christie," *The New York Times*, December 20, 2004, A1.

Christenfeld, N. (1995): "Choices from Identical Options," *Psychological Science*, 6, 50-55.

Costa-Gomes, Miguel, Vincent Crawford, and Bruno Broseta (2001): "Cognition and Behavior in Normal-Form Games: An Experimental Study," *Econometrica*, 69, 1193-1235.

Costa-Gomes, Miguel, and Vincent Crawford (2004): "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study," manuscript http://weber.ucsd.edu/~vcrawfor/#Guess.

Crawford, Vincent P. (2003): "Lying for Strategic Advantage: Rational and Boundedly Rational Misrepresentation of Intentions," *American Economic Review*, 93, 133-149.

Crawford, Vincent P. (2004): "Outguessing and Deception in Novel Strategic Situations," School of Economics and Social Sciences Distinguished Lecture, Singapore Management University, November 2004, slides at http://weber.ucsd.edu/~vcrawfor/SMUPubLecSlides.pdf.

Crawford, Vincent P., and Nagore Iriberri (2004): "Fatal Attraction: Focality, Naivete, and Sophistication in Experimental 'Hide-and-Seek' Games," preliminary version presented at the Theory Mini-Conference preceding the Far Eastern Meetings of the Econometric Society, Seoul, June 2004, http://econ.korea.ac.kr/MicroConf/Crawford.pdf.

Ho, Teck-Hua, Colin Camerer, and Keith Weigelt (1998): "Iterated Dominance and Iterated Best Response in Experimental '*p*-Beauty Contests," *American Economic Review*, 88**,** 947-969.

Keillor, Garrison (1997): *Wobegon Boy*. New York: Penguin.

McKelvey, Richard, and Thomas Palfrey (1995): "Quantal Response Equilibria for Normal-Form Games," *Games and Economic Behavior*, 10, 6-38.

McKelvey, Richard, Thomas Palfrey, and Roberto Weber (2000): "The Effects of Payoff Magnitude and Heterogeneity on Behavior in 2x2 Games with Unique Mixed Strategy Equilibria," *Journal of Economic Behavior and Organization*, 42, 523–548.

Mehta, Judith, Chris Starmer, and Robert Sugden (1994): "The Nature of Salience: An Experimental Investigation of Pure Coordination Games," *American Economic Review*, 84, 658-673.

Mookherjee, Dilip, and Barry Sopher (1997): "Learning and Decision Costs in Experimental Constant Sum Games," *Games and Economic Behavior*, 19, 97-132.

Nagel, Rosemarie (1995): "Unraveling in Guessing Games: An Experimental Study," *American Economic Review*, 85, 1313-1326.

O'Neill, Barry (1987): "Nonmetric Test of the Minimax Theory of Two-Person Zerosum Games," *Proceedings of the National Academy of Sciences of the United States of America*, 84, 2106-2109.

Rapoport, Amnon, and Richard Boebel (1992): "Mixed Strategies in Strictly Competitive Games: A Further Test of the Minimax Hypothesis," *Games and Economic Behavior*, 4, 261-283.

Rosenthal, Robert, Jason Shachat, and Mark Walker (2003): "Hide and Seek in Arizona," *International Journal of Game Theory*, 32, 273–293.

Rubinstein, Ariel, and Amos Tversky (1993): "Naïve Strategies in Zero-Sum Games," Sackler Institute of Economic Studies, Tel Aviv University, Working Paper 17-93.

Rubinstein, Ariel, Amos Tversky, and Dana Heller (1996): "Naive Strategies in Competitive Games," *Understanding Strategic Interaction - Essays in Honor of Reinhard Selten*, ed. Werner Guth et al., Berlin: Springer-Verlag, pp. 394-402.

Rubinstein, Ariel (1999): "Experience from a Course in Game Theory: Pre and Post-Class Problem Sets as a Didactic Device," *Games and Economic Behavior*, *28*, 155-170; see especially "Second Edition" posted at http://arielrubinstein.tau.ac.il/99/gt100.html.

Scharlemann, Jörn, Catherine Eckel, Alex Kacelnik, and Rick Wilson (2001): "The Value of a Smile: Game Theory with a Human Face," *Journal of Economic Psychology*, 22, 617-640.

Schelling, Thomas (1960): *The Strategy of Conflict*. Cambridge: Harvard University Press.

Selten, Reinhard (1998): "Features of Experimentally Observed Bounded Rationality," *European Economic Review*, 42, 413-436.

Stahl, Dale, and Paul Wilson (1994): "Experimental Evidence on Players' Models of Other Players," *Journal of Economic Behavior and Organization*, 25, 309-327.

Stahl, Dale, and Paul Wilson (1995): "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior*, 10, 218-254.

Von Neumann, John (1953): "A Certain Zero-Sum Two-Person Game Equivalent to the Optimal Assignment Problem," *Contributions to the Theory of Games, Vol. II*, ed. H.W. Kuhn and A.W. Tucker, *Annals of Mathematics Studies*, 28, Princeton, N.J.: Princeton Univ. Press, pp. 5-12.

Walker, Mark, and John Wooders (2001): "Minimax Play at Wimbledon," *American Economic Review*, 91, 1521-1538.

Weber, Roberto, Colin Camerer, and Marc Knez (2004): "Timing and Virtual Observability in Ultimatum Bargaining and 'Weak Link' Coordination Games," *Experimental Economics*, 7, 25–48.

Weizsäcker, Georg (2003): "Ignoring the Rationality of Others: Evidence from Experimental Normal-Form Games," *Games and Economic Behavior*, 44, 145-171.

Table 1. Aggregate Choice Frequencies in RTH's Experiments

RTH-4	**A**	***B***	**A**	**A**
Hider (53)	9%	36%	40%	15%
Seeker (62)	13%	31%	45%	11%
RT-AABA-Treasure	**A**	**A**	***B***	**A**
Hider (189)	22%	35%	19%	25%
Seeker (85)	13%	51%	21%	15%
RT-AABA-Mine	**A**	**A**	***B***	**A**
Hider (132)	24%	39%	18%	18%
Seeker (73)	29%	36%	14%	22%
RT-1234-Treasure	**1**	***2***	**3**	**4**
Hider (187)	25%	22%	36%	18%
Seeker (84)	20%	18%	48%	14%
RT-1234-Mine	**1**	***2***	**3**	**4**
Hider (133)	18%	20%	44%	17%
Seeker (72)	19%	25%	36%	19%
R-ABAA	**A**	***B***	**A**	**A**
Hider (50)	16%	18%	44%	22%
Seeker (64)	16%	19%	54%	11%
RTH-1	***Triangle***	**Circle**	**Circle**	**Circle**
Hider (53)	23%	23%	43%	11%
Seeker (62)	29%	24%	42%	5%
RTH-2	**Polite**	***Rude***	**Honest**	**Friendly**
Hider (53)	15%	26%	51%	8%
Seeker (62)	8%	40%	40%	11%
RTH-3	**Smile**	**Smile**	**Smile**	***Frown***
Hider (53)	21%	26%	34%	19%
Seeker (62)	7%	25%	34%	34%
RTH-5	**Frown**	***Smile***	**Frown**	**Frown**
Hider (53)	15%	40%	34%	11%
Seeker (62)	16%	55%	21%	8%
RTH-6	**Hate**	**Detest**	***Love***	**Dislike**
Hider (53)	11%	23%	38%	28%
Seeker (62)	20%	21%	55%	14%

Sample sizes in parentheses; focal labels in italics; order of presentation of locations to subjects as shown.

Table 2. Types' Expected Payoffs and Choice Probabilities in RTH's Game when $p + 2q > 1$ and $3p + 2q > 2$

Hider	Exp. Payoff $p < 2q$	Choice Pr. $p < 2q$	Exp. Payoff $p > 2q$	Choice Pr. $p > 2q$	Seeker	Exp. Payoff $p < 2q$	Choice Pr. $p < 2q$	Exp. Payoff $p > 2q$	Choice Pr. $p > 2q$
***L0* (Pr. *r*)**					***L0* (Pr. *r*)**				
A	-	$p/2$	-	$p/2$	**A**	-	$p/2$	-	$p/2$
B	-	q	-	q	**B**	-	q	-	q
A	-	$1\text{-}p\text{-}q$	-	$1\text{-}p\text{-}q$	**A**	-	$1\text{-}p\text{-}q$	-	$1\text{-}p\text{-}q$
A	-	$p/2$	-	$p/2$	**A**	-	$p/2$	-	$p/2$
***L1* (Pr. *s*)**					***L1* (Pr. *s*)**				
A	$1 - p/2 < ¾$	0	$1 - p/2 < ¾$	0	**A**	$p/2 > ¼$	0	$p/2 > ¼$	1/2
B	$1 - q < ¾$	0	$1 - q < ¾$	0	**B**	$q > ¼$	1	$q > ¼$	0
A	$p + q > ¾$	1	$p + q > ¾$	1	**A**	$1–p–q < ¼$	0	$1–p–q < ¼$	0
A	$1 - p/2 < ¾$	0	$1 - p/2 < ¾$	0	**A**	$p/2 > ¼$	0	$p/2 > ¼$	1/2
***L2* (Pr. *t*)**					***L2* (Pr. *t*)**				
A	1	1/3	1/2	0	**A**	0	0	0	0
B	0	0	1	1/2	**B**	0	0	0	0
A	1	1/3	1	1/2	**A**	1	1	1	1
A	1	1/3	1/2	0	**A**	0	0	0	0
***L3* (Pr. *u*)**					***L3* (Pr. *u*)**				
A	1	1/3	1	1/3	**A**	1/3	1/3	0	0
B	1	1/3	1	1/3	**B**	0	0	1/2	1/2
A	0	0	0	0	**A**	1/3	1/3	1/2	1/2
A	1	1/3	1	1/3	**A**	1/3	1/3	0	0
***L4* (Pr. *v*)**					***L4* (Pr. *v*)**				
A	2/3	0	1	1/2	**A**	1/3	1/3	1/3	1/3
B	1	1	1/2	0	**B**	1/3	1/3	1/3	1/3
A	2/3	0	1/2	0	**A**	0	0	0	0
A	2/3	0	1	1/2	**A**	1/3	1/3	1/3	1/3
Total	$p < 2q$		$p > 2q$		**Total**	$p < 2q$		$p > 2q$	
A	$rp/2+(1\text{-}\varepsilon)[t/3+u/3]+(1\text{-}r)\varepsilon/4$		$rp/2+(1\text{-}\varepsilon)[u/3+v/2]+(1\text{-}r)\varepsilon/4$		**A**	$rp/2+(1\text{-}\varepsilon)[u/3+v/3]+(1\text{-}r)\varepsilon/4$		$rp/2+(1\text{-}\varepsilon)[s/2+v/3]+(1\text{-}r)\varepsilon/4$	
B	$rq+(1\text{-}\varepsilon)[u/3+v]+(1\text{-}r)\varepsilon/4$		$rq+(1\text{-}\varepsilon)[t/2+u/3]+(1\text{-}r)\varepsilon/4$		**B**	$rq+(1\text{-}\varepsilon)[s+v/3]+(1\text{-}r)\varepsilon/4$		$rq+(1\text{-}\varepsilon)[u/2+v/3]+(1\text{-}r)\varepsilon/4$	
A	$r(1\text{-}p\text{-}q)+(1\text{-}\varepsilon)[s+t/3]+(1\text{-}r)\varepsilon/4$		$r(1\text{-}p\text{-}q)+(1\text{-}\varepsilon)[s+t/2]+(1\text{-}r)\varepsilon/4$		**A**	$r(1\text{-}p\text{-}q)+(1\text{-}\varepsilon)[t+u/3]+(1\text{-}r)\varepsilon/4$		$r(1\text{-}p\text{-}q)+(1\text{-}\varepsilon)[t+u/2]+(1\text{-}r)\varepsilon/4$	
A	$rp/2+(1\text{-}\varepsilon)[t/3+u/3]+(1\text{-}r)\varepsilon/4$		$rp/2+(1\text{-}\varepsilon)[u/3+v/2]+(1\text{-}r)\varepsilon/4$		**A**	$rp/2+(1\text{-}\varepsilon)[u/3+v/3]+(1\text{-}r)\varepsilon/4$		$rp/2+(1\text{-}\varepsilon)[s/2+v/3]+(1\text{-}r)\varepsilon/4$	

Table 3. Parameter Estimates and Likelihoods for the Leading Models

Model	Ln L	Parameter Estimates		Actual and Predicted Choice Frequencies for Hiders and Seekers			
				A	B	A	A
Observed choice frequencies			H	0.2163	0.2115	0.3654	0.2067
			S	0.1821	0.2054	0.4589	0.1536
Equilibrium without perturbations/random	-1641.4		H	0.2500	0.2500	0.2500	0.2500
			S	0.2500	0.2500	0.2500	0.2500
Equilibrium with perturbations of equal magnitudes across player roles	-1568.5	$e_H = e_S = 0.2187$	H	0.1897	0.2085	0.4122	0.1897
		$f_H = f_S = 0.2010$	S	0.1897	0.2085	0.4122	0.1897
Equilibrium with perturbations of unrestricted magnitudes across player roles	-1562.4	$e_H = 0.2910, f_H = 0.2535$	H	0.2115	0.2115	0.3654	0.2115
		$e_S = 0.1539, f_S = 0.1539$	S	0.1679	0.2054	0.4590	0.1679
Level-*k* with $p > ½, q > ¼$, and $p > 2q$	-1564.4	$r = 0, s = 0.1896, t = 0.3185, u = 0.2446, v = 0.2473, \varepsilon = 0$	H	0.2052	0.2408	0.3488	0.2052
			S	0.1772	0.2047	0.4408	0.1772

Table 4. Treatment by Treatment Parameter Estimates

Treatment	Level-*k*						Equilibrium with Perturbations			
	r	s	t	u	v	ε	e_H	f_H	e_S	f_S
RTH-4	0	0.2499	0.2643	0.4858	0.0000	0	0.3307	0.1451	0.2736	0.0377
RT-AABA-Treasure	0	0.1577	0.3265	0.3226	0.1932	0	0.3648	0.2941	0.1164	0.1640
RT-AABA-Mine	0	0.1566	0.3393	0.0686	0.4355	0	0.1818	0.2121	0.1028	0.2192
RT-1234-Treasure	0	0.1572	0.3810	0.1421	0.3197	0	0.3035	0.2976	0.1471	0.1390
RT-1234-Mine	0	0.2066	0.3153	0.2603	0.2178	0	0.2669	0.2406	0.1667	0.1111
R-ABAA	0	0.1933	0.3743	0.2683	0.1641	0	0.4141	0.3594	0.2500	0.2600

Table 5. Level-*k* MSDs Treatment by Treatment

Overall MSD 0.00343	RTH-4	RT-AABA-Treasure	RT-AABA-Mine	RT-1234-Treasure	RT-1234-Mine	R-ABAA
RTH-4	0.0020	0.0032	0.0098	0.0031	0.0019	0.0032
RT-AABA-Treasure	0.0047	0.0014	0.0061	0.0016	0.0009	0.0039
RT-AABA-Mine	0.0132	0.0042	0.0011	0.0029	0.0023	0.0085
RT-1234-Treasure	0.0072	0.0016	0.0029	0.0007	0.0002	0.0037
RT-1234-Mine	0.0054	0.0017	0.0035	0.0009	0.0000	0.0034
R-ABAA	0.0040	0.0023	0.0073	0.0016	0.0010	0.0023

Table 6. Equilibrium with Perturbations MSDs Treatment by Treatment

Overall MSD 0.00418	**RTH-4**	**RT-AABA-Treasure**	**RT-AABA-Mine**	**RT-1234-Treasure**	**RT-1234-Mine**	**R-ABAA**
RTH-4	0.0002	0.0088	0.0156	0.0079	0.0050	0.0087
RT-AABA-Treasure	0.0089	0.0001	0.0039	0.0013	0.0017	0.0022
RT-AABA-Mine	0.0153	0.0034	0.0005	0.0031	0.0032	0.0070
RT-1234-Treasure	0.0076	0.0009	0.0031	0.0005	0.0004	0.0025
REq: **T-1234-Mine**	0.0053	0.0018	0.0037	0.0009	0.0000	0.0036
R-ABAA	0.0085	0.0019	0.0071	0.0027	0.0032	0.0004

Table 7. Difference in MSDs (Equilibrium – Level-*k*) Treatment by Treatment

Overall MSD Difference 0.00075	**RTH-4**	**RT-AABA-Treasure**	**RT-AABA-Mine**	**RT-1234-Treasure**	**RT-1234-Mine**	**R-ABAA**
RTH-4	-0.0018	0.0056	0.0058	0.0048	0.0031	0.0055
RT-AABA-Treasure	0.0042	-0.0013	-0.0022	-0.0003	0.0009	-0.0017
RT-AABA-Mine	0.0021	-0.0008	-0.0006	0.0001	0.0008	-0.0015
RT-1234-Treasure	0.0003	-0.0007	0.0001	-0.0002	0.0002	-0.0012
RT-1234-Mine	-0.0002	0.0001	0.0001	0.0001	-0.0000	0.0002
R-ABAA	0.0045	-0.0004	-0.0001	0.0010	0.0023	-0.0020

Table 8. Initial Choice Frequencies and Model's Predictions in O'Neill's Game

Model	**Region**	**Actual and Predicted Choice Frequencies for Players 1 and 2**					**MSD**
			A	**2**	**3**	**J**	
Observed Choice Frequencies (25)		**1**	0.0800	0.2400	0.1200	0.5600	-
		2	0.2400	0.1200	0.0800	0.6400	-
Equilibrium (without Perturbations)		**1**	0.2000	0.2000	0.2000	0.4000	0.0120
		2	0.2000	0.2000	0.2000	0.4000	0.0200
Level-*k*	$3j - a > 1$	**1**	0.0815	0.2408	0.2408	0.4369	0.0074
		2	0.2958	0.1062	0.1062	0.4919	0.0065
Level-*k*	$3j - a < 1$, $a + 2j < 1$	**1**	0.0824	0.1772	0.1772	0.5631	0.0018
		2	0.1640	0.1640	0.1640	0.5081	0.0080
Level-*k*	$3j - a < 1$, $a + 2j > 1$	**1**	0.0000	0.2541	0.2541	0.4919	0.0073
		2	0.2720	0.0824	0.0824	0.5631	0.0021

Sample sizes in parentheses; order of presentation of locations to subjects as shown.

Table 9a. Types' Expected Payoffs and Choice Probabilities in O'Neill's Game when 3*j* – *a* > 1

Player 1	Exp. Payoff	Choice Pr.	Player 2	Exp. Payoff	Choice Pr.
***L0* (Pr. *r*)**			***L0* (Pr. *r*)**		
A	-	a	**A**	-	a
2	-	$(1-a-j)/2$	**2**	-	$(1-a-j)/2$
3	-	$(1-a-j)/2$	**3**	-	$(1-a-j)/2$
J	-	j	**J**	-	j
***L1* (Pr. *s*)**			***L1* (Pr. *s*)**		
A	$1-a-j$	0	**A**	$a+j$	1
2	$(1+a-j)/2$	0	**2**	$(1-a+j)/2$	0
3	$(1+a-j)/2$	0	**3**	$(1-a+j)/2$	0
J	j	1	**J**	$1-j$	0
***L2* (Pr. *t*)**			***L2* (Pr. *t*)**		
A	0	0	**A**	1	1/3
2	1	1/2	**2**	1	1/3
3	1	1/2	**3**	1	1/3
J	0	0	**J**	0	0
***L3* (Pr. *u*)**			***L3* (Pr. *u*)**		
A	2/3	1/3	**A**	0	0
2	2/3	1/3	**2**	1/2	0
3	2/3	1/3	**3**	1/2	0
J	0	0	**J**	1	1
***L4* (Pr. *v*)**			***L4* (Pr. *v*)**		
A	0	0	**A**	1/3	0
2	0	0	**2**	1/3	0
3	0	0	**3**	1/3	0
J	1	1	**J**	1	1
Total			**Total**		
A	$ra+(1-\varepsilon)[u/3]+(1-r)\,\varepsilon/4$		**A**	$ra+(1-\varepsilon)[s+t/3]+(1-r)\,\varepsilon/4$	
2	$r(1-a-j)/2+(1-\varepsilon)[t/2+u/3]+(1-r)\,\varepsilon/4$		**2**	$r(1-a-j)/2+(1-\varepsilon)[t/3]+(1-r)\,\varepsilon/4$	
3	$r(1-a-j)/2+(1-\varepsilon)[t/2+u/3]+(1-r)\,\varepsilon/4$		**3**	$r(1-a-j)/2+(1-\varepsilon)[t/3]+(1-r)\,\varepsilon/4$	
J	$rj+(1-\varepsilon)[s+v]+(1-r)\,\varepsilon/4$		**J**	$rj+(1-\varepsilon)[u+v]+(1-r)\,\varepsilon/4$	

Table 9b. Types' Expected Payoffs and Choice Probabilities in O'Neill's Game when $3j - a < 1$

Player 1	Exp. Payoff $a+2j<1$	Choice Pr. $a+2j<1$	Exp. Payoff $a+2j>1$	Choice Pr. $a+2j>1$
***L0* (Pr. *r*)**				
A	-	a	-	a
2	-	$(1-a-j)/2$	-	$(1-a-j)/2$
3	-	$(1-a-j)/2$	-	$(1-a-j)/2$
J	-	j	-	j
***L1* (Pr. *s*)**				
A	$1-a-j$	0	$1-a-j$	0
2	$(1+a-j)/2$	1/2	$(1+a-j)/2$	1/2
3	$(1+a-j)/2$	1/2	$(1+a-j)/2$	1/2
J	j	0	j	0
***L2* (Pr. *t*)**				
A	0	0	0	0
2	0	0	1	1/2
3	0	0	1	1/2
J	1	1	0	0
***L3* (Pr. *u*)**				
A	0	0	0	0
2	0	0	0	0
3	0	0	0	0
J	1	1	1	1
***L4* (Pr. *v*)**				
A	2/3	1/3	0	0
2	2/3	1/3	0	0
3	2/3	1/3	0	0
J	0	0	1	1
Total	$a+2j<1$		$a+2j>1$	
A	$ra+(1-\varepsilon)[v/3] + (1-r)\,\varepsilon/4$		$ra+ (1-r)\,\varepsilon/4$	
2	$r(1-a-j)/2+ (1-\varepsilon)\,[s/2+v/3]+ (1-r)\,\varepsilon/4$		$r(1-a-j)/2+ (1-\varepsilon)\,[s/2+t/2]+ (1-r)\,\varepsilon/4$	
3	$r(1-a-j)/2+(1-\varepsilon)\,[s/3+v/3]+ (1-r)\,\varepsilon/4$		$r(1-a-j)/2+ (1-\varepsilon)\,[s/2+t/2]+ (1-r)\,\varepsilon/4$	
J	$rj+(1-\varepsilon)\,[t+u]+ (1-r)\,\varepsilon/4$		$rj+(1-\varepsilon)\,[u+v]+ (1-r)\,\varepsilon/4$	

Player 2	Exp. Payoff $a+2j<1$	Choice Pr. $a+2j<1$	Exp. Payoff $a+2j>1$	Choice Pr. $a+2j>1$
***L0* (Pr. *r*)**				
A	-	a	-	a
2	-	$(1-a-j)/2$	-	$(1-a-j)/2$
3	-	$(1-a-j)/2$	-	$(1-a-j)/2$
J	-	j	-	j
***L1* (Pr. *s*)**				
A	$a+j$	0	$a+j$	1
2	$(1-a+j)/2$	0	$(1-a+j)/2$	0
3	$(1-a+j)/2$	0	$(1-a+j)/2$	0
J	$1-j$	1	$1-j$	0
***L2* (Pr. *t*)**				
A	0	0	0	0
2	1/2	0	1/2	0
3	1/2	0	1/2	0
J	1	1	1	1
***L3* (Pr. *u*)**				
A	1	1/3	0	0
2	1	1/3	1/2	0
3	1	1/3	1/2	0
J	0	0	1	1
***L4* (Pr. *v*)**				
A	1	1/3	1	1/3
2	1	1/3	1	1/3
3	1	1/3	1	1/3
J	0	0	0	0
Total	$a+2j<1$		$a+2j>1$	
A	$ra+(1-\varepsilon)\,[u/3+v/3]+ (1-r)\,\varepsilon/4$		$ra+(1-\varepsilon)\,[s+v/3]+ (1-r)\,\varepsilon/4$	
2	$r(1-a-j)/2+(1-\varepsilon)\,[u/3+v/3]+ (1-r)\,\varepsilon/4$		$r(1-a-j)/2+(1-\varepsilon)\,[v/3]+ (1-r)\,\varepsilon/4$	
3	$r(1-a-j)/2+(1-\varepsilon)\,[u/3+v/3]+ (1-r)\,\varepsilon/4$		$r(1-a-j)/2+(1-\varepsilon)\,[v/3]+ (1-r)\,\varepsilon/4$	
J	$rj+(1-\varepsilon)\,[s+t]+ (1-r)\,\varepsilon/4$		$rj+(1-\varepsilon)\,[t+u]+ (1-r)\,\varepsilon/4$	

Table 10. Initial Choice Frequencies and Models' Predictions in Rapoport and Boebel's Game

Model	Region		Actual and Predicted Choice Frequencies for Players 1 and 2					MSD 1	MSD 2
			C	L	F	I	O		
Observed Choice Frequencies, Treatment 1 (10)		1	0.1000	0.0000	0.2000	0.6000	0.1000	-	-
		2	0.8000	0.0000	0.0000	0.1000	0.1000	-	-
Observed Choice Frequencies, Treatment 2 (10)		1	0.1000	0.1000	0.1000	0.6000	0.1000	-	-
		2	0.2000	0.6000	0.2000	0.0000	0.0000	-	-
Equilibrium (without Perturbations)		1	0.3750	0.2500	0.1250	0.1250	0.1250	0.0740	0.0650
		2	0.3750	0.2500	0.1250	0.1250	0.1250	0.0520	0.0380
Level-*k*	$3m/2 + n > 1$	1	0.3085	0.3488	0.0612	0.2204	0.0612	0.0660	0.0505
		2	0.4657	0.1593	0.0618	0.2514	0.0618	0.0331	0.0702
Level-*k*	$3m/2 + n < 1$	1	0.3796	0.4369	0.0612	0.0612	0.0612	0.1160	0.0970
		2	0.4107	0.2204	0.1230	0.1230	0.1230	0.0433	0.0449

Sample sizes in parentheses; order of presentation of locations to subjects as shown.

Table 11. Types' Expected Payoffs and Choice Probabilities in Rapoport and Boebel's Game

Player 1	Exp. Payoff $3m/2+n>1$	Choice Pr. $3m/2+n>1$	Exp. Payoff $3m/2+n<1$	Choice Pr. $3m/2+n<1$	Player 2	Exp. Payoff $3m/2+n>1$	Choice Pr. $3m/2+n>1$	Exp. Payoff $3m/2+n<1$	Choice Pr. $3m/2+n<1$
***L0* (Pr. *r*)**					***L0* (Pr. *r*)**				
C	-	$m/2$	-	$m/2$	**C**	-	$m/2$	-	$m/2$
L	-	$(1-m-n)/2$	-	$(1-m-n)/2$	**L**	-	$(1-m-n)/2$	-	$(1-m-n)/2$
F	-	n	-	n	**F**	-	n	-	n
I	-	$(1-m-n)/2$	-	$(1-m-n)/2$	**I**	-	$(1-m-n)/2$	-	$(1-m-n)/2$
O		$m/2$		$m/2$	***O***		$m/2$		$m/2$
***L1* (Pr. *s*)**					***L1* (Pr. *s*)**				
C	$m/2$	0	$m/2$	0	**C**	$1-m/2$	0	$1-m/2$	1
L	$1/2+n/2$	1	$1/2+n/2$	1	**L**	$1/2-n/2$	0	$1/2-n/2$	0
F	$1/2-n/2$	0	$1/2-n/2$	0	**F**	$1/2+n/2$	0	$1/2+n/2$	0
I	$1-m-n$	0	$1-m-n$	0	**I**	$m+n$	1	$m+n$	0
O	$(1-m+n)/2$	0	$(1-m+n)/2$	0	**O**	$(1+m-n)/2$	0	$(1+m-n)/2$	0
***L2* (Pr. *t*)**					***L2* (Pr. *t*)**				
C	0	0	1	1	**C**	1	½	1	1/2
L	1	½	0	0	**L**	1	½	1	1/2
F	0	0	0	0	**F**	0	0	0	0
I	1	½	0	0	**I**	0	0	0	0
O	0	0	0	0	**O**	0	0	0	0
***L3* (Pr. *u*)**					***L3* (Pr. *u*)**				
C	1/2	1/4	1/2	1/4	**C**	1	1	0	0
L	0	0	0	0	**L**	1/2	0	1	1/4
F	1/2	1/4	1/2	1/4	**F**	1/2	0	1	1/4
I	1/2	1/4	1/2	1/4	**I**	0	0	1	1/4
O	1/2	1/4	1/2	1/4	**O**	1/2	0	1	1/4
***L4* (Pr. *v*)**					***L4* (Pr. *v*)**				
C	1	1	0	0	**C**	3/4	1/4	3/4	1/4
L	0	0	3/4	1	**L**	0	0	1/4	0
F	0	0	1/2	0	**F**	3/4	1/4	3/4	1/4
I	0	0	1/2	0	**I**	3/4	1/4	3/4	1/4
O	0	0	1/2	0	**O**	3/4	1/4	3/4	1/4

Total	$3m/2+n>1$	$3m/2+n<1$	**Total**	$3m/2+n>1$	$3m/2+n<1$
C	$rm/2+(1-\varepsilon)[u/4+v]+(1-r)\varepsilon/5$	$rm/2+(1-\varepsilon)[t+u/4]+(1-r)\varepsilon/5$	**C**	$rm/2+(1-\varepsilon)[t/2+u+v/4]+(1-r)\varepsilon/5$	$rm/2+(1-\varepsilon)[s+t/2+v/4]+(1-r)\varepsilon/5$
L	$r(1-m-n)/2+(1-\varepsilon)[s+t/2]+(1-r)\varepsilon/5$	$r(1-m-n)/2+(1-\varepsilon)[s+v]+(1-r)\varepsilon/5$	**L**	$r(1-m-n)/2+(1-\varepsilon)[t/2]+(1-r)\varepsilon/5$	$r(1-m-n)/2+(1-\varepsilon)[t/2+u/4]+(1-r)\varepsilon/5$
F	$rn+(1-\varepsilon)[u/4]+(1-r)\varepsilon/5$	$rn+(1-\varepsilon)[u/4]+(1-r)\varepsilon/5$	**F**	$rn+(1-\varepsilon)[v/4]+(1-r)\varepsilon/5$	$rn+(1-\varepsilon)[u/4+v/4]+(1-r)\varepsilon/5$
I	$r(1-m-n)/2+(1-\varepsilon)[t/2+u/4](1-r)\varepsilon/5$	$r(1-m-n)/2+(1-\varepsilon)[u/4](1-r)\varepsilon/5$	**I**	$r(1-m-n)/2+(1-\varepsilon)[s+v/4]+(1-r)\varepsilon/5$	$r(1-m-n)/2+(1-\varepsilon)u/4+v/4]+(1-r)\varepsilon/5$
O	$rm/2+(1-\varepsilon)[u/4]+(1-r)\varepsilon/5$	$rm/2+(1-\varepsilon)[u/4]+(1-r)\varepsilon/5$	**O**	$rm/2+(1-\varepsilon)[v/4]+(1-r)\varepsilon/5$	$rm/2+(1-\varepsilon)[u/4+v/4]+(1-r)\varepsilon/5$

Hider/Seeker	A	B	A	A
A	0, 1	1, 0	1, 0	1, 0
B	1, 0	0, 1	1, 0	1, 0
A	1, 0	1, 0	0, 1	1, 0
A	1, 0	1, 0	1, 0	0, 1

Figure 1. RTH's Hide-and-Seek Game

Hider/Seeker	A	B	A	A
A	0–*e*, 1+*e*	1–*e*, 0+*f*	1–*e*, 0	1–*e*, 0+*e*
B	1–*f*, 0+*e*	0–*f*, 1+*f*	1–*f*, 0	1–*f*, 0+*e*
A	1, 0+*e*	1, 0+*f*	0, 1	1, 0+*e*
A	1–*e*, 0+*e*	1–*e*, 0+*f*	1–*e*, 0	0–*e*, 1+*e*

Figure 2. RTH's Hide-and-Seek Game with Payoff Perturbations

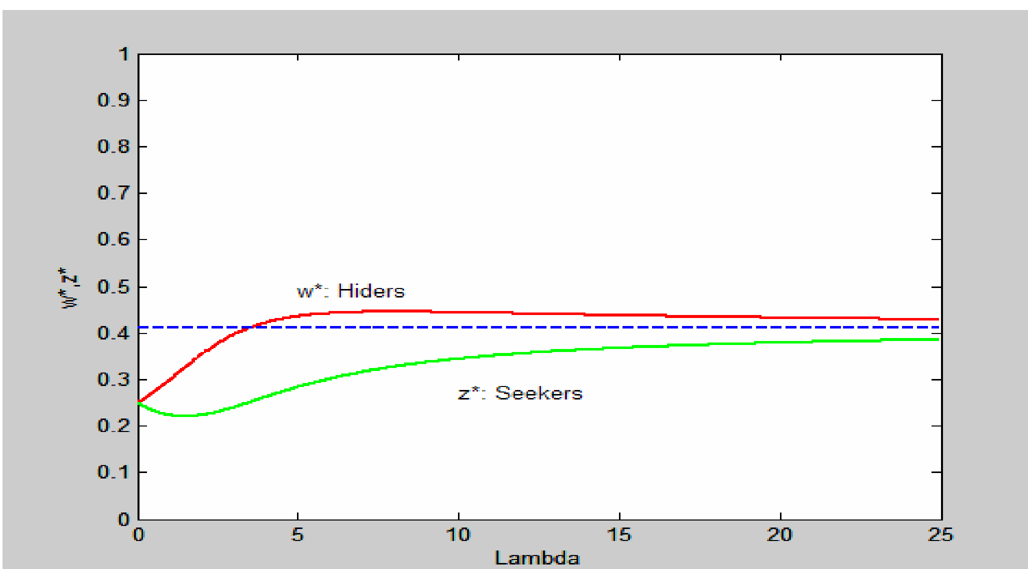

Figure 3. QRE with Payoff Perturbations of Equal Magnitudes Across Player Roles

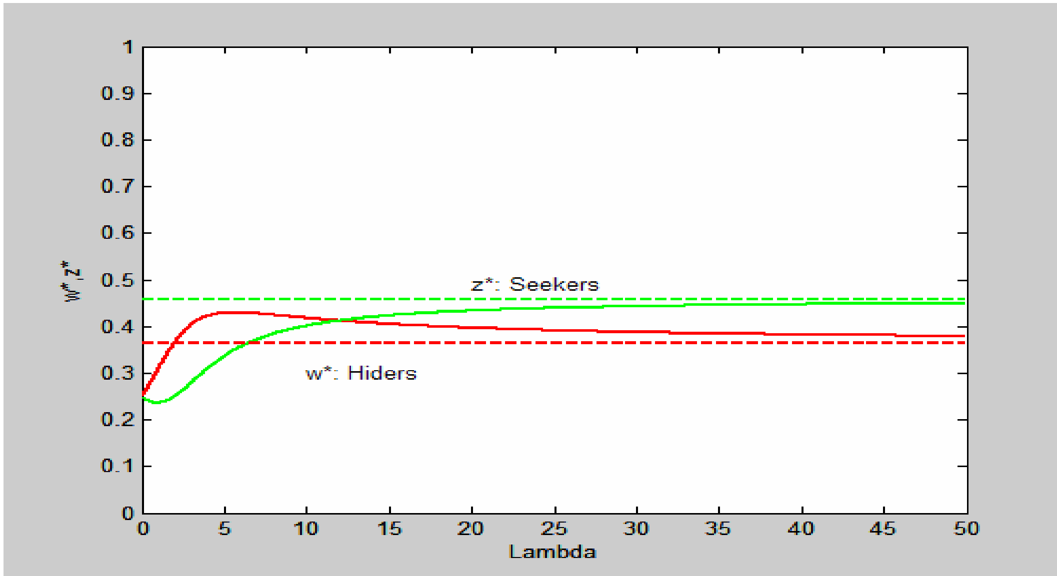

Figure 4. QRE with Payoff Perturbations of Differing Magnitudes Across Player Roles

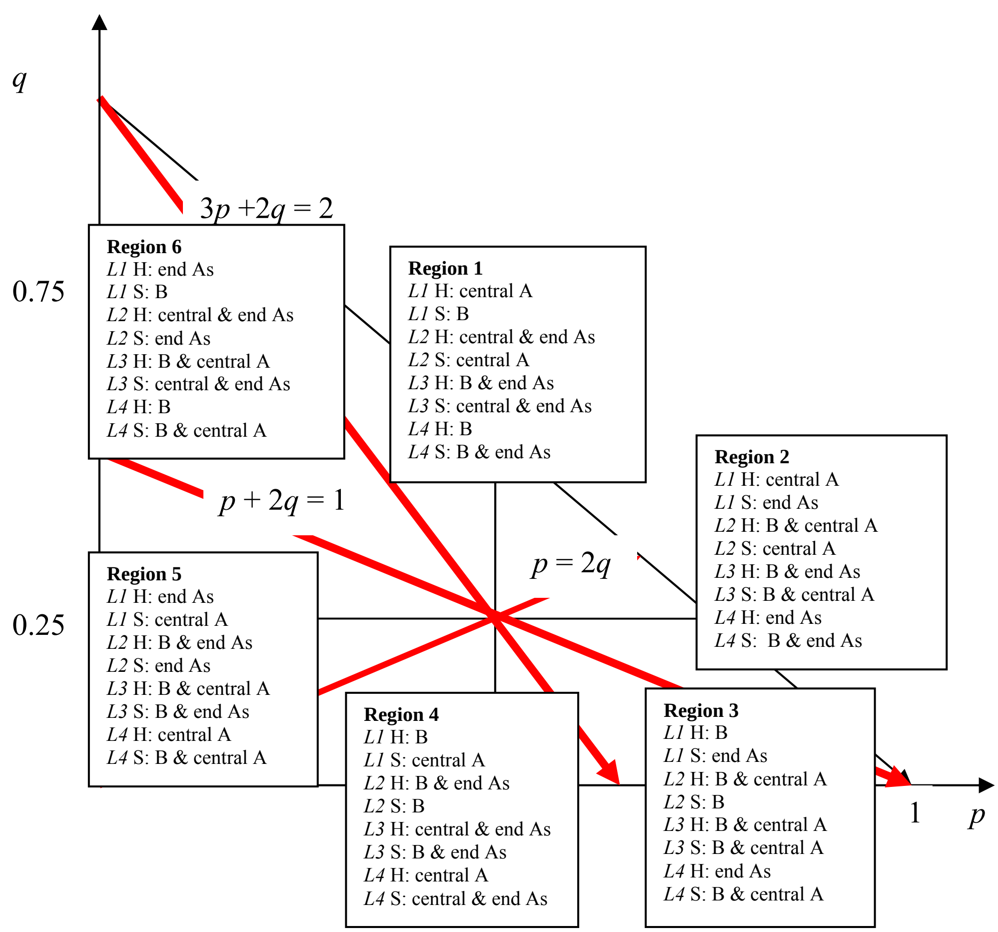

Figure 5. *L1*'s Through *L4*'s Choices as Functions of *L0*'s Choice Probabilities

Player 1/Player 2	A	2	3	J
A	1 0	0 1	0 1	1 0
2	0 1	1 0	0 1	1 0
3	0 1	0 1	1 0	1 0
J	1 0	1 0	1 0	0 1

Figure 6. O'Neill's Card-Matching Game

Player 1/Player 2	C	L	F	I	O
C	0 1	1 0	1 0	1 0	1 0
L	1 0	1 0	0 1	0 1	0 1
F	1 0	0 1	1 0	1 0	0 1
I	1 0	0 1	1 0	0 1	1 0
O	1 0	0 1	0 1	1 0	1 0

Figure 7. Rapoport and Boebel's Card-Matching Game

Player 1/Player 2	A	2	3	J
A	1+ α 0–α	0 1–α	0 1–α	1–ι 0–α
2	0+α 1	1 0	0 1	1–ι 0
3	0+α 1	0 1	1 0	1–ι 0
J	1+ α 0+ι	1 0+ι	1 0+ι	0-ι 1+ι

Figure 8. O'Neill's Card-Matching Game with Payoff Perturbations