2000-10

UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

A DUAL DUTCH AUCTION IN TAIPEI: THE CHOICE OF NUMERAIRE AND AUCTION FORM IN MULTI-OBJECT AUCTIONS WITH BUNDLING

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DISCUSSION PAPER 2000-10
JUNE 2000

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June 5, 2000

Abstract

In Taipei we observed a dual Dutch fish auction, like a conventional Dutch auction with bundling but with the roles of quantity and price reversed, and fish the numeraire rather than money. This paper uses a symmetric independent private values framework to study how duality interacts with auction form when agents' utility functions are linear in money but strictly concave in fish. With known buyers' values, conventional and dual auctions, English or Dutch, are equivalent. With values known to buyers but not the seller, the seller prefers conventional to dual auctions. With privately known values, the seller can prefer either a dual Dutch auction or a conventional English or Dutch auction, but he prefers all three to a dual English auction.

Keywords: English and Dutch auctions, revenue-equivalence, multi-object auctions, bundling
(JEL C72, D44, D82)

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We are especially grateful to Zo? Crawford and Pi-Hua Hsu for helping us to make and interpret our observations, and to Zo? Crawford for allowing us to use her photographs of the Hu-Lin Street evening market. Her photographs, which include the dual Dutch auctioneer and the numeraire, can be viewed at http://weber.ucsd.edu/~vcrawfor/Photos.html#EveMkt . We also thank Raffaella Giacomini, Mark Machina, Joel Sobel, and Lars Stole for helpful discussions and suggestions. Crawford thanks Academia Sinica for its generous hospitality during his visit to Taipei, and the National Science Foundation for research support.

A Dual Dutch Auction in Taipei:

The Choice of Numeraire and Auction Form in Multi-Object Auctions with Bundling

In June 1996, in the Hu-Lin (Tiger-Forest) Street evening market near Taipei City Hall in east downtown Taipei, we observed a fish auction whose form was remarkable. First the auctioneer held up a basket and announced a price, which was fixed throughout the auction at 100 New Taiwan dollars, about US$3.65 in 1996. He then put a series of approximately identical fish into the basket, one by one, until some buyer signaled that he was willing to pay the fixed price for the fish that the basket then held, at which point the sale was concluded. This process was repeated for as long as we watched, in a series of auctions with the same kind of fish as numeraire and the same fixed money price for the basket.[1]

This auction form, which we shall call the Taipei auction, is unusual in two respects. First, it involves bundling, in that the price of the basket is set at a level at which competition requires the winner to buy more than one fish to get any at all. Second, it is units of money rather than fish that are bundled, with the auctioneer increasing the quantity of fish to be exchanged for a fixed money price instead of decreasing the money price of a fixed quantity of fish.

Because increasing the quantity of fish is like decreasing their money price, and the first buyer to signal his willingness wins the auction, the Taipei auction is like a conventional Dutch descending-price auction with bundling, but with money bundled rather than fish, and fish as the numeraire rather than money. To put it another way, because the price of fish is the quantity of money exchanged for a unit of fish, the Taipei auction is dual to a conventional Dutch auction with bundling, with the conventional roles of the quantity and price of fish reversed.

[1]The Hu-Lin Street evening market is targeted at working women and is famous for its low prices and variable quality. The fish were one to two pounds each, and the basket when sold usually contained half a dozen or more. After closing the sale, the seller added one small fish of a different species as lagniappe, which we ignore in our analysis. That seller was the only one in the market using this auction form. The seller was still in the same location in January 2000, with a different auctioneer but using the same auction form. At that time the auctioneer also used some more complex auction forms, but in all of them the market was cleared by varying the quantity of seafood rather than its money price. In one case, for instance, she proposed that ten buyers share a given basket of shrimp for 300 New Taiwan dollars each, about US$9.75 in 2000. When only five buyers signaled their willingness to accept, she reduced the proposed number of buyers to nine, and the sale was concluded with eight buyers sharing the basket. In the analysis we focus on the auction form described in the text, which is similar but analytically simpler.

Bundling in auctions is unusual but not rare, and has been analyzed in the literature.[2] The duality of the Taipei auction, however, is rare; and to our knowledge, the choice of numeraire in auctions has not yet been studied. This paper studies the choice of numeraire and auction form in multi-object auctions with bundling, with the goals of understanding the effects of duality, how the choice of numeraire interacts with the choice of auction form—English or, equivalently, second-price; or Dutch or, equivalently, first-price—and why duality is so rarely observed.[3]

We assume, counterfactually but innocuously, that fish and money are homogeneous and perfectly divisible, and we represent the seller's and the buyers' preferences by indirect utility functions over fish and money left to spend on other commodities. Because a conventional auction and its dual counterpart are isomorphic except for the interchanged roles of fish and money, an analysis of the choice of numeraire must somehow break the symmetry between them. We do so by assuming that the seller and the buyers have von Neumann-Morgenstern indirect utility functions that are quasilinear: additively separable, linear in money, but strictly concave in fish. This is a plausible approximation for a seller of perishable fish whose cost is sunk at the time of the auction, or for buyers who spend only a small fraction of their budgets on fish.

The interaction between the choices of numeraire and auction form also depends on the seller's and the buyers' information about buyers' preferences. We make assumptions that allow us to represent each buyer's preferences by a scalar value parameter that describes his valuation of fish, relative to money; and we use an independent private values model, in which the buyers' values are independent draws from the same distribution, which is common knowledge. All other aspects of the environment are common knowledge. The independent private values model is a plausible model of an auction of a familiar commodity whose quality is easy to judge, and which we presume is seldom purchased for resale. Within this framework we consider three information conditions: the buyers' values are common knowledge; the buyers' values are common knowledge among buyers, but unknown to the seller; and the buyer's values are privately known.

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[2]William Adams and Janet Yellen (1976), Eric Maskin and John Riley (1984a), and R. Preston McAfee, John McMillan, and Michael Whinston (1989) analyze tied sales of heterogeneous goods by a monopolist. Roger Myerson (1981) and Kala Krishna (1993) analyze tied sales of homogeneous goods in multi-object auctions.

Section I introduces our model and briefly discusses possible roles for bundling in a sequence of auctions like the Taipei auction, taking the choice of numeraire as given. Sections II-IV consider the choice of auction form and numeraire under the three information conditions. There we take bundling as given and assume that the seller chooses the bundle that is optimal in each auction considered in isolation, ignoring that it is part of a sequence. The possible relationships among the seller's equilibrium expected utilities can be summarized as follows:

Conventional Dutch	=	**Dual Dutch**	**Conventional Dutch**	>	**Dual Dutch**	**Conventional Dutch**	$\gtrless$	**Dual Dutch**
$\parallel$		$\parallel$	$\parallel$		$\parallel$	$\parallel$		$\vee$
Conventional English	=	**Dual English**	**Conventional English**	>	**Dual English**	**Conventional English**	>	**Dual English**
Values commonly known			**Values unknown to seller**			**Values privately known**		

When the buyers' values are common knowledge, English and Dutch auctions with a given numeraire yield identical equilibrium outcomes, in which the auction is won by the highest valuer of fish, who pays money for the bundle of fish or receives fish in exchange for the bundle of money in an amount that makes the second-highest valuer indifferent between winning and losing. In this case, a conventional auction, English or Dutch, and its dual counterpart yield the same outcome, so the choice of numeraire cannot be explained by the asymmetry in how fish and money enter preferences. Even with complete information, the seller's optimal choice of bundle causes a subtle inefficiency: The bundle is determined by the tradeoff between the seller's and the *second*-highest valuer's preferences, and is therefore too small to maximize the surplus generated by the exchange that actually takes place between the seller and the highest valuer.

When buyers' values are common knowledge among buyers but unknown to the seller, English and Dutch auctions with a given numeraire still yield identical equilibrium outcomes, in which the auction is won by the highest valuer, who again pays money or receives fish according to the second-highest value. Now, however, the choice of numeraire has real consequences, and conventional and dual auctions yield different expected utilities and volumes of trade. From the seller's point of view, a

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[3]We have heard of other dual auctions in Taipei and Tuscany—both of cloth!—but these are the only other examples we know of. Because the perishability of fish helps to motivate the assumptions on preferences we use to explain the possible occurrence of dual auctions, the durability of cloth is a cause for concern, which we do not address here.

conventional auction has an "insurance" advantage over a dual auction, because it induces uncertainty only about the allocation of money, which is costless under our assumptions on preferences, while a dual auction induces uncertainty about the allocation of fish, which is costly. This makes it possible for the seller to realize higher expected utility with a conventional auction. Because the seller normally chooses the auction form, this result yields a simple, plausible explanation of why dual auctions are so seldom observed.[4]

The welfare comparison between conventional and dual auctions is more complex for the buyers. The same buyer wins the auction in each case, and other buyers are indifferent between auction forms. In this version of our model, the buyers know all buyers' values, and the winning buyer's welfare is determined by the difference between his value and the second-highest value, which is the same in each case, and the size of the bundle. The seller's optimal fish bundle in a conventional auction is larger than the amount of fish received by a buyer in its dual counterpart who wins when the second-highest value is very high; the seller's optimal money bundle in a dual auction is smaller than the money paid by a buyer in its conventional counterpart who wins when the second-highest value is very high; and such a buyer always prefers the outcome of a conventional auction to that of its dual counterpart. Under a plausible additional restriction on preferences, the seller's optimal fish bundle in a conventional auction is larger than the amount of fish received by a buyer in its dual counterpart who wins when the second-highest value is above its ex ante mean; the seller's optimal money bundle in a dual auction is smaller than the money paid by a buyer in its conventional counterpart who wins when the second-highest value is above its ex ante mean; and such a buyer always prefers the outcome of a conventional auction to that of its dual counterpart. Without further restrictions on preferences and the distribution from which values are drawn, these comparisons appear to be ambiguous for a buyer who wins when the second-highest value is below its ex ante mean, who might prefer the outcome of a dual or a conventional auction. The welfare comparison for the buyers remains ambiguous ex ante.

When the buyers' values are privately known, an English auction, conventional or dual, yields the same outcome as when the buyers' values are common knowledge among buyers, because bidding

[4]As explained below, this result is not as straightforward as the insurance intuition suggests. Martin Weitzman's (1974) analysis suggests that an analysis with more general assumptions on preferences could relate the seller's preferences over auction forms to the relative concavities of the seller's and buyers' preferences over fish and money and the extent of the seller's uncertainty about buyers' preferences.

their true values is a dominant strategy. Thus, the results when the buyers' values are common knowledge among the buyers but unknown to the seller extend immediately to English auctions with privately known values.

Given the seller's fish bundle, a conventional Dutch auction with privately known values is a standard Dutch auction of a single indivisible object with risk-neutral buyers. The auction is still won by the highest valuer, who now pays a money price equal to the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest. From the seller's point of view, the expectation of this price is the *un*conditional expectation of the second-highest value. Thus, for any given bundle, the seller's expected revenue and utility are the same as in a conventional Dutch (or English) auction when the buyers' values are common knowledge among the buyers, and a conventional Dutch auction yields the seller the same expected revenue and utility as a conventional English auction, an instance of the Revenue Equivalence Theorem (William Vickrey (1961), McAfee and McMillan (1987, pp. 706-710)). The seller's optimal bundle is therefore also the same, so that a conventional Dutch auction is equivalent to a conventional English auction from the seller's point of view.

With privately known values, a dual Dutch auction has a surprising advantage, which can outweigh the insurance advantage of conventional auctions. A dual auction effectively converts the buyers from risk-neutral (in money) to risk-averse (in fish), and in a dual Dutch auction with privately known values, risk-averse buyers' uncertainty about other buyers' bids induces them to bid more aggressively than if they were risk neutral (Charles Holt (1980), Maskin and Riley (1984b), McAfee and McMillan (1987), p. 719). Under our assumptions on preferences, all buyers are equally risk-averse, so in equilibrium the auction is still won by the highest valuer. However, the winning buyer receives less fish for a given money bundle than with risk-neutral buyers; and the seller's expected utility is higher, other things equal.

Other things are not equal, because the insurance advantage of conventional auctions persists with privately known values. A conventional English or Dutch auction can yield the seller higher or lower expected utility than a dual Dutch auction, depending on which advantage is more important. But with privately known values (or values common knowledge among the buyers but unknown to the seller), a dual English auction yields the seller lower expected utility than any of the other three auctions, because conventional auctions provide better insurance and a dual Dutch auction provides no worse insurance but elicits more aggressive bidding.

Our observations in the Hu-Lin Street evening market make it intriguing that the potential for improving on conventional auctions depends on the auction being both dual and Dutch. Our results for this case provide a possible rationale for the Taipei auction, and suggest that the conjunction of duality and Dutchness may not have been coincidental.[5]

I. The Model and Possible Roles for Bundling

This section introduces the model and discusses possible roles for bundling, taking the choice of numeraire as given for now. We assume that both fish and money are homogeneous and perfectly divisible. There is one seller, with an initial supply of fish F, and there are $n = 2$ buyers; we index the seller $i = 0$ and the buyers $i = 1,\ldots,n$. We assume that resale is impossible, and that the seller's and the buyers' preferences over auction outcomes can be represented by von Neumann-Morgenstern utility functions that are quasilinear: additively separable, linear in money, but strictly concave in fish. Agent i's utility is $u_i(f_i, m_i) = v_i g(f_i) + m_i$, $i = 0,\ldots n$, where f_i is his allocation of fish and m_i his allocation of money; and the function $g(\cdot)$ is increasing, strictly concave, and differentiable. Here we assume, for simplicity, that the seller's and the buyers' utilities for fish are all proportional to the same function, $g(\cdot)$. All of our results and proofs go through immediately when the seller has a different utility function for fish than the buyers, but the analysis would be significantly more complex if the utility function were allowed to differ among the buyers. Most of our results also hold without strict concavity for the seller. We normalize $v_0 = 1$ without loss of generality, so that $u_0(f_0, m_0) = g(f_0) + m_0$. The parameter v_i describes buyer i's marginal rate of substitution between fish and money, and determines both his reservation price in money for any bundle of fish and his reservation price in fish for any bundle of money. A higher v_i represents a higher money value of fish for any given level of f_i and m_i, and from now on we use "higher value" as a shorthand for "higher value of fish." For simplicity, we assume that $v_i > 1$, $i = 1,\ldots,n$, and we order the buyers so that (ignoring ties) $v_1 > v_2 > \ldots > v_n > 1$, so that efficiency requires an exchange between the buyer and the seller whose value is highest.

[5] David Lucking-Reiley (1999) gives a good summary of evidence from field data and laboratory experiments, which suggests that in practice, Dutch auctions may have other advantages over English auctions.

We assume that ex ante, the v_i, $i = 1,\ldots n$, are independently and identically distributed, with commonly known distribution function $H(\cdot)$ with bounded support $[v^{min}, v^{max}]$, where $v^{min} > 1$. Thus, the buyers are symmetric and have independent private values (for fish, measured in money, or for money, measured in fish). The structure of the environment, including the auction form, is otherwise common knowledge. We vary the informational assumptions within this framework, first allowing the buyers' values to be common knowledge to the seller as well as the buyers, then to be common knowledge among buyers but unknown to the seller, and finally to be privately known by each buyer. We focus on symmetric Nash or Bayesian equilibria throughout.

We close this section by considering possible roles for bundling, following Krishna's (1993) analysis of sequential versus bundled multi-object auctions. Krishna presents an example to show that even when buyers' values are common knowledge, bundling can increase the seller's utility and the efficiency of the allocation by eliminating the adverse effects of buyers' anticipations of pecuniary externalities across sequential auctions. Her example involves one seller and two buyers, whose preferences differ in a way that is inconsistent with our assumption that buyers' preferences have the common form $u_i(f_i, m_i) = v_i g(f_i) + m_i$. We therefore present an example like hers, but with buyers' preferences that are more compatible with our assumptions, in which bundling also increases the seller's utility and the efficiency of the allocation.

There are two buyers, one seller, and two identical fish. Let $v_0 = 0$, so that the seller values only money; this is inconsistent with our normalization but inessential, and could easily be relaxed. Let $g(1) = 2$, $g(2) = 3$, $v_1 = 5$, and $v_2 = 2$, so that buyer 1's reservation prices are \$10 for one fish and \$15 for two, and buyer 2's reservation prices are \$4 for one fish and \$6 for two. For simplicity, we respect the indivisibility of fish, comparing a sequence of two auctions of one fish each with a single, bundled auction of both fish. We also focus on conventional auctions.[6]

When the buyers' values are common knowledge, a conventional auction, English or Dutch, is always won by the highest valuer, at a price at least approximately equal to the second-highest value. In sequential auctions, however, the buyers' subgame-perfect equilibrium bidding strategies must reflect their rational anticipations of how the outcome of the current auction will influence the outcome of

[6]When values are common knowledge, the analogous dual auctions yield the same allocations (Proposition 1).

subsequent auctions. In our example, in a sequence of two auctions of one fish at a time, buyer 1 must win the second auction because his value is higher, but at a price that depends on whether buyer 2 won the first auction, due to the diminishing marginal value of fish. There is a unique subgame-perfect equilibrium, in which buyer 2 wins the first auction despite his lower value for fish, paying $3, and buyer 1 then wins the second auction, paying $2. In this equilibrium buyer 1's utility is 8; buyer 2's utility is 1; and the seller's utility is 5.[7] By contrast, in a single, bundled auction of both fish, buyer 1 wins and pays $6, and his utility is 9; buyer 2 loses, pays nothing, and his utility is 0; and the seller receives $6, and his utility is 6. Thus, the bundled auction both allocates the fish more efficiently and yields the seller higher revenue: Everyone but buyer 2 is better off, and compensation could yield a Pareto-improvement.

When buyers' values are not observable by the seller, bundling in a multi-object auction can also be useful in sorting buyers with independent private values. This role of bundling is present in Myerson's (1981) analysis of optimal auctions (but not the main focus), in Maskin and Riley's (1984a) analysis of optimal monopolistic quantity discounting, and in McAfee, McMillan, and Whinston's (1989) analysis of optimal monopolistic bundling of heterogeneous goods.[8]

II. Equivalence of Conventional and Dual Auctions with Commonly Known Values

In the rest of the paper, we take the occurrence of bundling as given and study the effect of auction form and choice of numeraire on the allocation generated by a single, bundled auction under alternative informational assumptions. To focus on these issues, we assume from now on that the seller and the buyers ignore strategic interactions with any subsequent auctions; that their preferences over outcomes of the current auction can be described by von Neumann-Morgenstern indirect utility functions that satisfy the assumptions in Section I; and the seller chooses the optimal bundle, given the auction form. For simplicity, we also assume that $g(\cdot)$ satisfies the Inada condition $\lim_{f \to \infty} g'(f) = \infty$, so that the seller's optimal bundle is always interior.

[7]If buyer 1 deviated, bidding just high enough to win both auctions, he would win fish worth 15 to him, paying 4 + 4 = 8 for them, for a utility of 7 rather than 8.

[8]In a common-values framework, Donald Hausch (1986) identifies a third possible role of bundling, eliminating buyers' incentives to underbid to avoid revealing private information that reduces their gain in subsequent auctions; but that role is not relevant in the independent private values model studied here.

In this section, we assume that buyers' values are commonly known to the seller as well as the buyers. Given the seller's bundle, an English or Dutch auction, conventional or dual, is a standard auction of a single, indivisible object. When the buyers' values are common knowledge, in equilibrium each auction is won by the highest valuer of fish, who pays money for the bundle of fish or receives fish in exchange for the bundle of money in an amount that makes the second-highest valuer indifferent between winning and losing. Given this, and recalling that $v_1 > v_2 > \ldots > v_n > 1$, in a conventional auction the seller's optimal fish bundle, f^c, solves the problem:

$$(1) \qquad \max\ g(F-f)+m \qquad \text{s. t.} \qquad m = v_2 g(f).$$

f^c then determines the money price, m^c, via the auction. In a dual auction the seller's optimal money bundle, m^d, also solves problem (1). m^d then determines the amount of fish received, f^d, via the auction.

With commonly known values, the constraint in problem (1) makes m a known, increasing function of f, or vice versa. Thus, it does not matter whether the seller chooses f^c, thereby determining m^c, or m^d, thereby determining f^d. This proves our first result:

PROPOSITION 1: *Suppose that the buyers' values are common knowledge. Then, in an English or Dutch auction, conventional or dual, the highest valuer wins the auction, paying money or receiving fish according to the second-highest value. The seller's optimal fish bundle in a conventional auction equals the amount of fish received by the winning buyer in its dual counterpart; the seller's optimal money bundle in a dual auction equals the money paid by the winning buyer in its conventional counterpart; and all four auctions yields the same outcome.*

Thus, when the buyers' values are common knowledge, the rarity of dual auctions cannot be explained by our assumed asymmetry in how fish and money enter agents' preferences.

Even with complete information, the seller's choice of bundle causes inefficiency:

PROPOSITION 2: *Suppose that the buyers' values are common knowledge. Then, in an English or Dutch auction, conventional or dual, the seller's optimal bundle is too small and the volume of trade is too low for efficiency.*

PROOF: By Proposition 1, all four auctions yield the same exchange of money for fish, and the seller's optimal bundle maximizes $g(F- f) + v_2\, g(f)$, in a conventional auction directly by choice of $f^{\,c}$, and in a dual auction indirectly by choice of m^d, with $f^{\,d}$ determined by the constraint of (1). Standard arguments then show that maximizing $g(F- f) + v_2\, g(f)$, the surplus from a hypothetical exchange between the seller and the second-highest valuer, yields a bundle too small to maximize $g(F- f) + v_1\, g(f)$, the surplus from the exchange between the seller and the highest valuer that actually takes place in equilibrium. This completes the proof.

This tendency for the seller's bundle to be inefficiently small plainly persists when the buyers' values are not commonly known, but we do not discuss this issue further below.

III. Conventional versus Dual Auctions with Values Unknown to the Seller

In this section, we assume that the buyers' values are common knowledge among the buyers but unknown to the seller. In this case, for a given choice of numeraire, English and Dutch auctions are both won by the highest valuer, who pays money or receives fish according to the second-highest value. Thus, we need only distinguish between conventional and dual auctions.

In a conventional auction, the seller's optimal fish bundle, $f^{\,c}$, is the value of f that solves:

$$\text{(2)} \qquad \max\ E[g(F-f)+m] \qquad \text{s. t.} \qquad m = v_2\, g(f).$$

The seller's choice of $f^{\,c}$ and the realization of the random variable v_2 together determine the money price, m^c, via the auction. The expectation is taken with respect to the unconditional distribution of v_2, the second-highest value in n independent draws from the distribution $H(\cdot)$. In a dual auction, the seller's optimal money bundle, m^d, is the value of m that solves problem (2), but now m^d and the realization of v_2 together determine the amount of fish received, $f^{\,d}$, via the auction. The expectation is taken over the distribution of $f^{\,d}$ induced by the distribution of v_2.

We stress that when the buyers' values are unknown to the seller, in a conventional auction $f^{\,c}$ is deterministic and m^c is random, while in a dual auction m^d is deterministic and $f^{\,d}$ is random. As a result, even though f^c and m^d solve the "same" problem with different forms of uncertainty, and in equilibrium each auction yields an exchange between the buyer and the highest valuer, who pays money or receives fish according to the second-highest value, conventional and dual auctions yield different volumes of trade and expected utilities.

Given our assumptions that $g(\cdot)$ is concave and satisfies an Inada condition, the second-order conditions of problem (2) are always satisfied, and its solutions are always interior. f^c is therefore determined by the first-order condition

$$(3) \qquad y\,(f)\frac{1}{Ev_2} = 1, \text{ where } \; y\,(f) \equiv \frac{g'(F-f)}{g'(f)},$$

and m^d is determined by the first-order condition

$$(4) \qquad E\left[y\,(g^{-1}[m/v_2])\frac{1}{v_2} \right] = 1.$$

Because the seller is bound by his limited supply of fish, $f^c = F$ and, for all realizations of v_2, $f^d = F$, so that $m^d = v^{min}g(F)$. Given the Inada condition, (3) rules out violations of the first constraint; and (4) rules out violations of the second constraint with positive probability and therefore, given the continuity of $H(\cdot)$, rules out any violations of the second constraint at all.

The function $g^{-1}(\cdot)$ is positive, increasing, and convex, and the function $?(\cdot)$ is positive and increasing. Our assumptions on $g(\cdot)$ do not determine the curvature of $?(\cdot)$, but $?(\cdot)$ is convex for many common parameterizations of utility functions, and this appears to be the normal case.

Our first result for the case where the buyers' values are common knowledge among the buyers but unknown to the seller establishes the seller's preference for conventional auctions:

PROPOSITION 3: *Suppose that the buyers' values are common knowledge among the buyers but unknown to the seller. Then, in an English or Dutch auction, conventional or dual, the highest valuer wins the auction, paying money or receiving fish according to the second-highest value. A conventional English and a conventional Dutch auction yield the same outcome, and a dual*

English and a dual Dutch auction yield the same outcome. However, a conventional auction always yields the seller higher expected utility than its dual counterpart.

PROOF: The proofs of the first parts are immediate. To prove the last part, note that in a conventional auction it is feasible for the seller to set $f = Ef^d$, the expected amount of fish received for the seller's optimal money bundle, m^d, in a dual auction. f^c must therefore yield him an expected utility at least as high as Ef^d and the distribution of $m = v_2 g(Ef^d)$ it induces. Thus,

$$(5) \qquad E[g(F-f^c)+m^c] \geq g(F-Ef^d)+E[v_2 g(Ef^d)] = g(F-Ef^d)+[Ev_2][g(Ef^d)]$$
$$> Eg(F-f^d)+[Ev_2][Eg(f^d)] > Eg(F-f^d)+E[v_2 g(f^d)] = Eg(F-f^d)+m^d,$$

where the inequalities follow from revealed preference, the strict concavity of $g(\cdot)$ and Jensen's inequality, and the fact that v_2 and f^d are negatively correlated. This completes the proof.

REMARK: The seller's preference for conventional over dual auctions stems from the fact that from his point of view, a conventional auction induces uncertainty only about the allocation of money, which is costless, while a dual auction induces uncertainty about the allocation of fish, which is costly. However, the proof is not a direct translation of this insurance intuition, and it shows that the seller's preference requires only that either the seller or the winning buyer is strictly risk averse in the relevant range, even though the buyers bear no uncertainty. In fact the seller's preference extends to the case where both he and the buyers are risk-neutral. There, in each case, his welfare increases with the volume of trade. In a conventional auction he can set $f^c = F$, realizing expected utility Ev_2F. In a dual auction, because $g(\cdot)$ no longer satisfies the Inada condition we must impose the constraint $m^d = v^{min}F$ to ensure that $f^d = m^d/v_2 = F$. He therefore sets $m^d = v^{min}F$, realizing expected utility less than Ev_2F. Thus, with risk-neutrality Proposition 3's conclusion remains valid because the first inequality in (5) is strict.

The buyers' welfare comparison is also more complex than the insurance intuition:

PROPOSITION 4: *Suppose that the buyers' values are common knowledge among the buyers but unknown to the seller. Then the same buyer wins the auction, whether it is conventional or dual, and losing buyers are indifferent between conventional and dual auctions.* f^c*, the seller's optimal fish bundle in a conventional auction, is larger than* $g^{-1}(m^d/v^{max})$*, and therefore larger than the amount of fish received by a buyer in its dual counterpart who wins when* v_2 ˜ v^{max}*;* m^d*, the seller's optimal money bundle in a dual auction, is smaller than* $m^c = g(f^c)v^{max}$*, and therefore smaller than the amount of money paid by a buyer in its conventional counterpart who wins when* v_2 ˜ v^{max}*; and such a buyer prefers the outcome of a conventional auction to that of its dual counterpart. If, in addition, the function ?(·) is convex, then for any* $v_2 = Ev_2$*, in a conventional auction* $f^c > g^{-1}(m^d/v_2)$*, the amount of fish received by a buyer in its dual counterpart who wins when the second-highest value is* v_2*; in a dual auction* $m^d < m^c = v_2\, g(f^c)$*, the money paid by a buyer in its conventional counterpart who wins when the second-highest value is* v_2*; and any buyer who wins when* $v_2 = Ev_2$ *prefers the outcome of a conventional auction to that of its dual counterpart.*

PROOF: It is clear that the highest valuer still wins the auction in each case, paying money or receiving fish according to the second-highest value, and that losing buyers are indifferent between conventional and dual auctions. In this version of our model the buyers know all buyers' values, and therefore bear no uncertainty in equilibrium. In a conventional auction, the winning buyer pays money price $m^c = v_2 g(f^c)$ for the fish bundle f^c, realizing utility $(v_1 - v_2)g(f^c)$. In a dual auction, the winning buyer receives $f^d = g^{-1}(m^d/v_2)$ units of fish in exchange for the money bundle m^d, realizing utility $(v_1-v_2)g(f^d)$. Thus, to show that a buyer who wins when v_2 ˜ v^{max} realizes higher utility in a conventional auction, and to justify the comparisons of the amounts exchanged in this case, we need only show that $g(f^c) > m^d/v^{max}$ ($= g(f^d)$ when $v_2 = v^{max}$). Suppose, per contra, that $g(f^c) = m^d/v^{max}$, so that $f^c = g^{-1}(m^d/v^{max})$. Then

$$(6)\ 1 = y(f^c)\frac{1}{Ev_2} \leq y(g^{-1}[m^d/v^{\max}])\frac{1}{Ev_2} < y(g^{-1}[m^d/v^{\max}])E\left[\frac{1}{v_2}\right] < E\left[y(g^{-1}[m^d/v_2])\frac{1}{v_2}\right] = 1,$$

where the equalities are from the first-order conditions (3) and (4) and the inequalities follow from the facts that $?(\cdot)$ and $g^{-1}(\cdot)$ are increasing, and from Jensen's inequality. The contradiction in (6) establishes the results for v_2 ˜ v^{max}. To show that if $?(\cdot)$ is convex, a buyer who wins when $v_2 = Ev_2$ realizes higher utility in a conventional auction, and to justify the comparisons of the amounts exchanged in this case, it suffices to show that $g(f^c) > m^d/Ev_2$, because if $v_2 = Ev_2$, $g(f^d) = m^d/Ev_2$. Suppose, per contra, that $g(f^c) = m^d/Ev_2$. Then

$$(7) \qquad 1 = E\left[y\,(g^{-1}[m^d/v_2])\frac{1}{v_2}\right] \geq E\left[y\,(g^{-1}[g(f^c)Ev_2/v_2])\frac{1}{v_2}\right]$$

$$> E[y(g^{-1}[g(f^c)Ev_2/v_2))]E\left|\frac{1}{v_2}\right| > y(f^c)\frac{1}{Ev_2} = 1,$$

where the inequalities follow from the facts that $?(\cdot)$ and $g^{-1}(\cdot)$ are increasing, that the random variables in brackets at the end of the first line are positively correlated, from Jensen's inequality, and from the convexity of $?(\cdot)$. This contradiction completes the proof of Proposition 4.

REMARK: Without further restrictions, Proposition 4's comparisons appear to be ambiguous for a buyer who wins when $v_2 < Ev_2$, who might prefer the outcome of a conventional auction or its dual counterpart. The welfare comparison for buyers is also ambiguous ex ante, where comparing $E[(v_1 - v_2)g(f^c)]$ and $E[(v_1 - v_2)g(f^d)]$ is further complicated by the correlation between $(v_1 - v_2)$ and f^d. The ambiguity also extends to the case where the buyers' values are privately known, but we do not discuss this issue further below.

IV. Dual Dutch versus other Auction Forms with Privately Known Values

In this section, we assume that each buyer's value is privately known. In an English auction, conventional or dual, the buyers' uncertainty about each other's values has no effect:

PROPOSITION 5: *Suppose that the buyers' values are privately known. Then, in an English auction, conventional or dual, the seller's optimal bundle and the auction outcome are the same as when the buyers' values are common knowledge among the buyers but unknown to the seller. Thus, the highest valuer wins the auction, paying money or receiving fish according to the* *second-highest value; a conventional English auction always yields the seller higher expected utility than a dual English auction; and Proposition 4's comparisons of the buyers' welfares and the amounts exchanged remain valid for English auctions.*

PROOF: When the buyers' values are privately known, in an English auction it is a dominant strategy for a buyer to bid his true value. The seller's optimal bundle is therefore still determined by problem (2), the outcome is the same as when buyers' values are common knowledge among the buyers but unknown to the seller, and the conclusions of Propositions 3 and 4 remain valid. This completes the proof.

By contrast, in a Dutch auction with privately known values, the buyers bear uncertainty about each other's bids, which makes it possible for the outcome to differ from the outcome when the buyers' values are common knowledge among the buyers but unknown to the seller.

In a conventional Dutch auction, the effect of this difference is limited:

PROPOSITION 6: *Suppose that the buyers' values are privately known. Then, in a conventional Dutch auction, the highest valuer wins the auction, at a money price equal to the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest. For any given bundle, the seller's expected revenue and utility are the same as in a conventional English auction. A conventional Dutch auction therefore has the same optimal bundle as a conventional English auction and yields the seller the same expected utility, which is higher than his expected utility in a dual English auction.*

PROOF: Given the seller's fish bundle, a conventional Dutch auction with privately known values is equivalent to a standard single-object auction with risk-neutral seller and buyers. Thus, in symmetric equilibrium the bundle is always won by the highest valuer, who pays a money price equal to the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest (which is the appropriate assumption because a buyer's bid influences the outcome only when his value is the highest). From the point of view of the seller, who does not know v_1, the expectation of this price is $E(E[v_2 g(f^c)|v_1]) = Ev_2 g(f^c)$ by the law of iterated expectations, where the first expectation is taken with respect to the unconditional distribution of v_1. Thus, although conventional English and Dutch auctions yield different equilibrium relationships between the buyers' values and the auction price, for any given bundle the seller's expected revenue (and utility) are the same, an instance of the Revenue Equivalence Theorem (William Vickrey (1961), McAfee and McMillan (1987, pp. 706-710)). Conventional English and Dutch auctions therefore have the same optimal bundles and yield the seller the same expected utility. By Propositions 3 and 5, this expected utility is the same as in a conventional English or Dutch auction when the buyers' values are common knowledge among the buyers but unknown to the seller, and it is higher than the seller's expected utility in a dual English auction when the buyers' values are either common knowledge among the buyers or privately known. This completes the proof.

Proposition 6 shows that with privately known values, the seller is still indifferent between a conventional Dutch and a conventional English auction, and that he prefers both to a dual English auction. These relationships are simple consequences of revenue equivalence and the insurance advantage of conventional over dual auctions identified in Proposition 3.

It remains to consider the seller's welfare in a dual Dutch auction. There, the buyers' uncertainty about each other's bids has a significant effect on the outcome, which gives a dual Dutch auction a potential advantage over a conventional English or Dutch auction or, a fortiori, a dual English auction. A dual auction effectively converts the buyers from risk-neutral (in money) to risk-averse (in fish), and in a dual Dutch auction with privately known values, risk-averse buyers' uncertainty about other buyers' bids induces them to bid more aggressively than if they were risk neutral (Charles Holt (1980), Maskin and Riley (1984b), McAfee and McMillan (1987), p. 719). The winning buyer therefore receives less fish for a given money bundle than with risk-neutral buyers; and the seller's expected utility is higher, other things equal. However, the insurance advantage of conventional auctions persists with privately known values. The seller always prefers a dual Dutch auction to a dual English auction, but he can prefer either a conventional English or Dutch auction or a dual Dutch auction, depending on whether the benefits of more aggressive bidding outweigh the benefits of insurance.

It is intriguing that the potential for improving on conventional auctions depends on the auction being both dual and Dutch. Our results for this case provide a possible rationale for the Taipei auction, and suggests that its conjunction of duality and Dutchness was not coincidental.[9]

PROPOSITION 7: *Suppose that the buyers' values are privately known. Then, in a dual Dutch auction, the highest valuer wins the auction; but the bidding is more aggressive than if each buyer's bid were directly determined by the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest, with the result that the winning buyer receives less fish for a given money bundle. This more aggressive bidding benefits the seller, other things equal, and can outweigh the insurance advantage of conventional auctions. The seller's expected utility is always higher in a dual Dutch auction than in a dual English auction, but it can be either higher or lower than in a conventional English or Dutch auction.*

PROOF: Given the seller's money bundle, a dual Dutch auction with privately known values is a standard single-object auction with risk-averse seller and buyers. Under our assumptions all of the buyers are equally risk averse, so in symmetric equilibrium the highest valuer still wins the auction (Holt (1980), pp. 436-439). Holt's results also imply that in a dual Dutch auction with money bundle m, the winning (lowest) fish bid can be written $f(v_1) = g^{-1}(m/b(v_1))$, where $b(\cdot)$ is an increasing, continuous, and differentiable function. Further, the bidding is strictly more aggressive than with risk-neutral buyers, so that there exists a $b^{\min} > 0$ such that

(8) $$b(v_1) \geq b^{\min} + E(v_2 \mid v_1) \text{ for all } v_1.$$

The seller's optimal money bundle, m^d, is then the value of m that solves the problem:

(9) $$\max E[g(F - g^{-1}[m / b(v_1)]) + m],$$

[9]The dual Dutch auction is still not optimal, but the optimal auction may be difficult to implement because it involves complex subsidization of high bidders who lose and penalization of low bidders (Maskin and Riley (1984b), McAfee and McMillan (1987), pp. 718-720)). With correlated values, the information about buyers' values revealed during an English auction may result in higher revenue to the seller than in a Dutch auction, despite the advantage of the Dutch auction noted here (Paul Milgrom and Robert Weber (1982)).

where the expectation is taken with respect to the unconditional distribution of v_1. Because $g(\cdot)$ satisfies an Inada condition, the solution of (9) must be interior; and given the fact that $?(\cdot)$ and $g^{-1}(\cdot)$ are increasing, m^d is uniquely determined by the first-order condition

$$(10) \qquad E\left[y\,(g^{-1}[m/b(v_1)])\frac{1}{b(v_1)}\right] = 1.$$

To see that the seller's expected utility in a dual Dutch auction can be lower than in a conventional English or Dutch auction, suppose that $g(\cdot)$ is linear, so that the seller and buyers are risk-neutral. This violates our assumptions, but can be smoothed so that $g(\cdot)$ is slightly strictly concave everywhere but near 0, where it is concave enough to satisfy our Inada condition; a continuity argument will then yield the desired conclusion.

When $g(\cdot)$ is linear, the seller's welfare increases with the volume of trade. Because $g(\cdot)$ no longer satisfies the Inada condition we must impose the constraints $f^c = F$ and $m^d = v^{min}F$ to ensure that $f^d = F$. In a conventional auction, the seller has a boundary maximum at $f^c = F$, receiving expected revenue Ev_2F for the fish bundle F, and realizing expected utility Ev_2F. In a dual Dutch auction, the seller again has a boundary maximum at $m^d = v^{min}F$. Each buyer's fish bid is directly determined by the expectation of the second-highest value conditional on his own value, on the assumption that it is the highest, so the winning buyer receives $f^d = m^d/E(v_2|v_1)$ units of fish and the seller's expected utility is

$$(11) \qquad \begin{aligned} &E(F - v^{\min}F/E(v_2\,|v_1)) + v^{\min}F < E(F - Ev_2F/E(v_2\,|\,v_1)) + Ev_2F \\ &< E(F - Ev_2F/E[E(v_2\,|\,v_1)]) + Ev_2F = Ev_2F, \end{aligned}$$

where the inequalities follow from the fact that his expected utility is increasing in m^d, and from Jensen's inequality. Thus, with risk-neutrality, the seller strictly prefers a conventional English or Dutch auction to a dual Dutch auction, and smoothing and continuity yield the conclusion.

To see that the seller's expected utility in a dual Dutch auction can be higher than in a conventional auction, suppose that $g(\cdot)$ is strictly concave, and that the support of the distribution $H(\cdot)$, $[v^{min}, v^{max}]$, satisfies $v^{min} = v^{max} - 1$. Now imagine that the distribution $H(\cdot)$ shifts rightwards, preserving its shape, with v^{min}, v^{max} ? 8. Suppressing the dependence of variables on v^{min} and v^{max}, it is clear from Proposition 6 and (3) that in a conventional Dutch auction, f^c ? F, so that the seller's equilibrium expected utility approaches $g(0) + Ev_2\, g(F)$ in the limit. In a dual Dutch auction, as v^{min}, v^{max} ? 8, it is clear that $b(v_1)$? 8 for all v_1. It then follows from (10) that $y\,(g^{-1}[m^d / b(v_1)]) \to \infty$ for all v_1, because, by the monotonicity of ?$(\cdot)$, $g(\cdot)$, and $b(\cdot)$,

$$(12) \qquad y\,(g^{-1}[m / b(v^{\max})])\frac{1}{b(v^{\max})} < y\,(g^{-1}[m / b(v_1)])\frac{1}{b(v_1)} < y\,(g^{-1}[m / b(v^{\min})])\frac{1}{b(v^{\min})},$$

and $y\,(g^{-1}[m / b(v^{\max})]) \to \infty$ if and only if $y\,(g^{-1}[m / b(v^{\min})]) \to \infty$. This implies that $f^d = g^{-1}(m^d/b(v_1))$? F for all v_1, so m^d ? $Eb(v_1)g(F)$. Because the winning buyer's bid maximizes the probability of winning times his utility when he wins, $v_1 g(f) - m$, as m^d ? $Eb(v_1)g(F)$, $b(\cdot) - v^{max}$, $b^{min} - v^{max}$, and $E(v_2|v_1) - v^{max}$ all converge to limits for which (8) holds. Taking expectations in (8), the seller's equilibrium expected utility in a dual Dutch auction approaches $g(0) + Eb(v_1)g(F) = g(0) + [b^{min} + Ev_2]\, g(F)$ in the limit, which is strictly greater than the seller's equilibrium expected utility in a conventional auction, completing the proof.

REMARKS: It may seem puzzling that in the last part of the proof, the support of the distribution of $f^d = g^{-1}(m^d/b(v_1))$ collapses on F, while the winning fish bid has a nonnegligible effect on the seller's welfare. The reason is that in the limit, the buyers compete by tiny variations in their bid amounts of extremely valuable fish, which despite their small size have nonnegligible effects on the seller's money revenue and expected utility.

Finally, we stress that our limiting argument in the second part of the proof is just a device to show that it is possible for the seller to prefer a dual Dutch auction; there is no reason to suppose that for low values, the seller must prefer a conventional auction. When $H(\cdot)$ is uniform, with $v^{min} = v^{max} - 1$;

$F = 100$; and $g(f) \equiv f^{1/k}$, numerical solutions yield expected utilities for the seller in a conventional English or Dutch auction, a dual English auction, and a dual Dutch auction, respectively, as follows:

k / v^{min}	2	3	4
2	25.39, 24.65, 24.92	12.77, 12.34, 12.51	9.10, 8.75, 8.86
3	33.33, 33.96, 34.19	17.12, 16.68, 16.84	12.10, 11.74, 11.85
4	43.33, 43.50, 43.71	21.57, 21.12, 21.26	15.13, 14.78, 14.89
5	53.33, 53.18, 53.37	26.08, 25.60, 25.74	18.21, 17.85, 17.95
6	63.33, 62.93, 63.10	29.40, 30.12, 30.25	20.03, 20.93, 21.03
7	73.33, 72.73, 72.90	34.04, 34.66, 34.79	23.19, 24.03, 24.12

Table 1. Seller's expected utilities in conventional, dual English, and dual Dutch auctions

Thus, the seller can prefer a dual Dutch auction to a conventional English or Dutch auction (or, a fortiori, a dual English auction) for low to moderate values of v^{min} as well as high ones. It is apparent, however, that even in this simple example the comparison varies in a complex and nonmonotonic way: When $k = 2$, for instance, the seller prefers a dual Dutch auction when $v^{min} = 3$, 4, or 5, and a conventional English or Dutch auction when $v^{min} = 2$, 6, or 7.

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