

Notes for Syllabus Section VII: A quasi-linear model; Externalities and Public Goods

Overview: External effects occur when one household or firm's actions affect another's utility or technology directly, outside of market and price interactions. Assuming the usual continuity and convexity properties, existence of general equilibrium continues but efficiency is compromised. To achieve a market based efficient allocation, a tax-subsidy scheme with correctly computed tax-subsidy levels is appropriate. In the case of an external diseconomy the principle is: tax per unit = marginal social cost = sum (over all firms and households) of individual marginal valuation of diseconomy. Similar statement for subsidy on external benefits.

Public goods are those for which it is impossible or impractical to exclude users. When they are provided to one user, they are provided universally, so it is impractical to use a price mechanism to determine the quantity provided, since most users will find it optimizing to accept the level provided without charge, the *free rider* problem. The efficiency question is how to determine, without a price mechanism, what is the efficient level of provision. The underlying principle is the quantity provided should generate marginal cost = marginal social benefit = sum (over all firms and households) of individual marginal valuation of benefit.

The (impractical) theoretical construct that demonstrates these principles is *Lindahl equilibrium*. The notion is to find prices, individualized over firms and households, for the externality or public good that would hypothetically clear markets if the externality or public good were traded successfully on private markets. Just as ordinary markets use *public prices for private goods*; the Lindahl equilibrium uses *private prices for public goods*.

These principles are most easily demonstrated in a class of models where the monetary valuation of externalities and public goods is easily determined. That is the quasi-linear model, where there is a variable utility for the externality or public good and a constant marginal utility for 'money', a shorthand for a composite of all other goods.

Externalities and Public Goods

Externality: One firm or household's actions affect another's utility or available technology directly through non-market interaction, rather than through prices and markets. Reflects missing markets.

Public good: Provided to one implies provided to all. "Inappropriable;" cannot exclude others from use, hence cannot charge a price.

Externalities and public goods are typically consistent with existence of equilibrium. Assuming continuity, convexity and Walras' Law are maintained, competitive general equilibrium will exist under the usual assumptions. The problem is with efficiency. In the presence of externalities or public goods, a general equilibrium allocation may not be Pareto efficient. Because externalities and public goods are typically allocated by non-market mechanisms, there is no reason to expect the equilibrium allocation to be Pareto efficient. The puzzle for us as economists is to model efficient levels of provision of external effects and public goods and how to determine and implement them.

If a good is not traded in the market, then we may expect, in general, that the resulting market allocation be inefficient: effective price of 0 implies underprovision, overuse.

Reasons for non-market allocation:

Institutional: the good is not regarded as private property but is rather treated as a common property resource - waterways, fisheries, parks, common grazing lands.

Cost of exclusion/enforcement/transaction is large relative to individual marginal benefits of use or marginal costs of provision --- national defense, flood control, roads.

External effect:

a) Two firms on a stream. Upstream firm dumps effluent into stream. Downstream firm's costs and technology affected by access to clean water (input to production). Upstream firm's decisions affect downstream firm's technology or input availability by non-market means. There is no available market on which the two firms can adjust their competing demands for use of the water.

b) Common fishery. Over-fishing. One firm's catch affects another's input availability or technology/output.

c) Public health, vaccination.

d) Hardin - Tragedy of the commons, overgrazing.

e) City kid embarrassing fallacies: bees pollinate corn (fallacy -- who does pollinate corn?); apiary (bee hive) produces external benefit to fruit ranch (fallacy -- bees are private property).

"Pecuniary externality"

Valid version - absence of markets (maybe futures markets) implies non-market interaction between firms. Mine and railroad. Two suppliers simultaneously enter and oversupply same market.

Fallacious version - Cross price effects of multiple participants in same market; all the other wine-lovers are driving up the price of Chateau d'Yquem. All those darn 'Zonies drive up the rents in La Jolla during the summer.

That's not an externality --- there's no inefficiency, that's just the market.

Network Externality

Additional users affect utility of all users. Windows vs. Mac for file sharing or providing ample market for software. Telephone; additional transactors add to market liquidity. Probably requires a scale economy (natural monopoly) correctly to model.

EXTERNALITY

Efficient allocation:

First order condition:

Marginal Social Benefit = Marginal Social Cost

Sum of Marginal Private Benefits = Sum of Marginal Private Costs

Lindahl Equilibrium, Internalizing the externality

Market vs. regulation

Effluent charges

equate marginal cost/benefits across multiple users

enforcement costs, Pigouvian tax

Regulation

enforcement

may not equate margins

but see Starrett "Fundamental NonConvexity" May need regulation to

achieve efficient corner solution.

Solutions

Turn common resource into private property

Establish property rights to external effects

Merge competing activities to "internalize the externality"

MasColell "A Simple Bilateral Externality" example

Two households; 1 engages in externality creating activity at level h (1's choice).

Quasi-linear separable utility

$$u^i(w^i, h) = \phi^i(h) + w^i, \quad i = 1, 2$$

Competitive (interior) solution with commodity h not priced: $\phi^{1'}(h^*) = 0$.

Efficient solution maximizes total surplus $[\phi^1(h^0) + \phi^2(h^0)]$

The first order condition for (interior) surplus maximization is $\phi^{1'}(h^0) = -\phi^{2'}(h^0)$

Quota: Regulator knows $\phi^1(h)$ and $\phi^2(h)$. Solves for h^0 . Enforces.

Informationally demanding, centralized, may not be incentive compatible.

Pigouvian taxation: Set $t = -\phi^{2'}(h^0)$. t = tax on h per unit.

Then agent 1 optimizes $[\phi^1(h) - th]$, FOC is $\phi^{1'}(h) = t$ resulting in $h^* = h^0$.

Informationally demanding, decentralized.

Coase Theorem (strong form): If property rights to all external effects are established then bargaining leads to an efficient allocation.

What's right with this picture: 1FTWE or core \subseteq Pareto efficient allocation.

What's wrong with this picture: assumes CE or nonempty core with externality.

Ignores cost of establishing and enforcing property rights (presumed reason for existence of externality to start with), cost of bargaining, monopoly, scale economy.

Coase Theorem (weak form): If property rights to all external effects are established, then allocative efficiency is independent of who receives the rights.

Interpretation: The upstream or downstream firm may own rights to the river. An efficient allocation method (e.g. competitive equilibrium) can achieve efficient allocation in either case, though the distribution of income, welfare, and wealth will differ.

MasColell, Multilateral Externalities

I households, J firms each firm j producing external effect h_j .

Cumulative external effect, $\sum_j h_j$.

$$\text{Marshallian surplus} = S = \sum_i \varphi^i(\sum_j h^j) + \sum_j \pi^j(h^j)$$

The first order condition for maximizing S with respect to choice of h^j , $j = 1, 2, \dots, J$, is

$$-\sum_i \varphi^i'(\sum_j h^{oj}) = \pi^j'(h^{oj})$$

Marginal Social Cost = Marginal Social Benefit

How to implement efficient allocation?

(1) firm-specific quotas, h^{oj} . Informationally demanding. Not decentralized.

(2) Pigouvian tax set at $\sum_i \varphi^i'(\sum_j h^{oj})$. Informationally demanding.

Decentralized implementation.

(3) Tradeable permit system with outstanding permit level of $\sum_j h^{oj}$.

Informationally demanding. Decentralized implementation.

(3') Cap and trade. Issue permits for outstanding volume of h . Decentralized implementation, not necessarily efficient.

Lindahl equilibrium

MasColell notation: I households, J firms

The Lindahl optimization for firms and households:

Each household i faces a price, t^i , at which it can (hypothetically) sell pollution rights, q^i . Firms wishing to pollute must buy pollution rights at a price t.

$$t = \sum_{i=1}^I t^i$$

Firm j's profits are $\pi^j(h^j) - t h^j$.

Household i 's utility is $\varphi^i(q^i) + w^i + t^i q^i$

First order conditions: $\pi^j = t$, implying j 's chosen h^{jo} , $\varphi^i = -t^i$, implying i 's chosen q^{io} .

A Lindahl equilibrium occurs where

t^i adjusts for all i to t^{io} , with $t^o = \sum_{i=1}^I t^{oi}$, so that

$$\sum_{j=1}^J h^{oj} = q = q^{oi} \text{ for all } i.$$

"Private prices for public goods."

That is, the individualized prices t^{io} equate each household's chosen level of q to the aggregate level of q actually prevailing based on the firms' separate optimizations subject to t^o .

This is an idealized (not practical) notion of the decentralized pricing system that achieves an efficient allocation of the common level of external effect. Note that

$$\sum_{i=1}^I t^{oi} = - \sum_{i=1}^I \varphi^i \left(\sum_{j=1}^J h^{oj} \right) = \pi^j(h^{oj}) = t$$

so the first order conditions for an efficient allocation are fulfilled.

An alternative way to think of this allocation is as a competitive (price-taking) equilibrium with production technology requiring strictly complementary inputs from the households. Each unit of h^j from firm j needs I different household-specific inputs q^i that are assumed to be competitively supplied. The quantities of q^i , $i = 1, \dots, I$, are necessarily identical in equilibrium. t^{oi} is the competitive equilibrium price of household i 's input.

Starrett's Fundamental Nonconvexity

Once the fish are dead, they're dead.

Consider an upstream and downstream firm. Upstream firm imposes external costs on downstream firm. That is, upstream firm output level is q , downstream firm production function is $F(L, q)$ with $F_L > 0$, $F_q < 0$, $F(L, q) \geq 0$. Zero is a lower bound on downstream firm output levels. F_q (or its absolute value) is the external cost imposed on the downstream firm by the upstream externality. Assume (weakly) increasing marginal external cost, that is, $F_{qq} \leq 0$. This is a necessary condition for convexity of the problem.

Then for q sufficiently large, the most profitable level of F will be $L = 0$, $F(L, q) = 0$. But then $F_q(0, q) = 0$. So marginal external cost is declining, that is, as q increases, eventually F_q goes to 0. A nonconvexity.

This is a scale economy argument on the externality: Once the level of externality is high enough to move the allocation of other goods to a corner solution, further increases in the external diseconomy have zero marginal cost. Once the neighborhood is noisy enough that all of the residents have moved out, increases in the noise level are at zero external cost. Once the water is so polluted nothing can live in it, increases in pollution levels are at zero external cost.

As usual with scale economies, an interior price-guided solution may not be efficient. Alternative: zoning. Concentrating externality-generating activities.

[Public goods continues next page.]

Public Goods

Impossibility of Exclusion

"Impossible" is a bit strong --- but it can be too difficult or costly:
flood control, national defense, local roads,

All households consume the same quantity; provided to one implies provided to all.

MasColell et al definition: 0 marginal cost provision to an additional user, non-rival. Implicit scale economy (or joint product)

Varian definition: non-rival, non-excludable

Note: "club", "local public goods"

Free Rider - if one or a few pay for provision, it is provided to all. There is little private incentive to help defray the cost. Cannot effectively use a market allocation.

Not all goods provided by government are public goods: schools, fire protection for example are private goods typically provided by government.

Efficient provision of public good --- following MasColell. Quasi-linear separable utility: $u^i = \phi^i(q) + m^i$

I households. $\phi^i(q)$ = household i's utility from q of public good denominated in terms of good m.

$c(q)$ = cost of provision of q, in terms of input of m.

Choose $q \geq 0$ to Max $S = \sum_i \phi^i(q) - c(q)$. This leads to the first order

condition (for an interior max), $\sum_i \phi^i'(q) = c'(q)$,

Marginal Social Benefit=Sum of marginal private benefits=Marginal social cost.

Under-provision of public goods in a private market

In a private market, household i buys x^i of the public good. He enjoys $\sum_{i=1}^I x^i$ of the public good. Household i treats $\sum_{k \neq i} x^k$ parametrically. Let x be available at a price of p^* per unit. Then individual optimization for i requires

$\max_{x^i \geq 0} \phi^i(x^i + \sum_{k \neq i} x^k) - p^* x^i$. The first order condition (including a corner

solution) is $\phi^i'(x^{*i} + \sum_{k \neq i} x^k) \leq p^*$ with equality if $x^{*i} > 0$, and with

inequality if $x^{*i} = 0$.

Implication: The household, \hat{i} , that most wants the public good buys it in

the quantity, q^* , so that $\phi^{\hat{i}}'(q^*) = p^*$. Typically, $p^* \ll \sum_{i=1}^I \phi^i'(q^*)$, so q^*

represents an under-supply relative to an efficient allocation. Households $i \neq \hat{i}$ are free riders.

Efficient quantity of public good

Private market will not efficiently provide, nor will any voluntary mechanism, due to the free rider problem. Hence government provision. (note, with excludable partly rival goods with bounded scale economy, form a club).

How to advise govt.? Seek q so that

Marginal social benefit $\equiv \sum$ marginal private benefits = Marginal social cost, maximize Marshallian surplus, this was DuPuit's problem.

$S = \sum_i \phi^i(q) - c(q)$. Optimizing choice of q (assuming an interior solution) occurs where $\sum_i \phi^{i'}(q) = c'(q)$.

Two solution concepts: Lindahl equilibrium, Groves (-Ledyard, -Vickrey, -Clarke) mechanism.

Lindahl Equilibrium

“Private prices for public goods, public prices for private goods.”

Firm produces the public good at cost $c(q)$. There are I households, $i = 1, 2, 3, \dots, I$.

Think of it this way: There are I joint products, x_1, x_2, \dots, x_I .

$c(x_1, x_2, \dots, x_I) = c[\max(x_1, x_2, \dots, x_I)]$. They are pure joint products, so they will be produced in the same quantity. What does a market clearing general equilibrium look like? (This is an exercise in mathematician-style boiling water; we're reducing it to the previous case. We're restating the public

goods problem as a joint product problem --- one we think we know how to solve).

Find market clearing: quantity q^{**} , prices $p^{**} = \sum_i p_i^{**} = c'(q^{**})$, so that

$$\phi_i'(x_i^{**}) = p_i^{**} \text{ and so that}$$

$$q^{**} = x_i^{**} \text{ for all } i.$$

This is a market clearing equilibrium for the joint product. It is a Lindahl equilibrium for the public good q . And it fulfills the first order conditions for efficiency

$$\sum_i \phi_i'(q) = c'(q).$$

"Private prices for public goods." This is not a practical proposal. It is a thought experiment describing how a market might determine q^{**} , p^{**} .

Suppose decision-makers just ask people how much they want of the public good. Contingent valuation (Prof. Richard Carson). Problem: if responses are tied to provision of public good and payment for it, then there'll be strategic manipulation of answers. How can we get a reliable answer?

Groves mechanism

Following Varian

Internalize the externality by incorporating others' tastes in each household's maximand.

Consider a special case: one indivisible public good G at cost c . The decision is $G = 0$ or 1 .

Let r_i = i 's true valuation of $G = 1$

s_i = i 's share of tax burden, $0 \leq s_i \leq 1$, $\sum_i s_i = 1$

$s_i c$ = i 's cost if $G = 1$

$v_i = r_i - s_i c$ = i 's true net valuation of $G = 1$.

Efficient allocation

$G = 1$ if $\sum_i r_i > c$, equivalently, if $\sum_i v_i = \sum_i (r_i - s_i c) > 0$.

Just ask: overvaluation by i so that $(r_i - s_i c) > 0$; undervaluation by i so that $(r_i - s_i c) < 0$.

How to induce truthful revelation (or sufficient revelation to inform efficient allocation)?

Groves mechanism

- (1) Each agent reports a bid, b_i .
- (2) If $\sum_i b_i > 0$, set $G = 1$
If $\sum_i b_i < 0$, set $G = 0$.

- (3) Sidepayment to i : $\sum_{j \neq i} b_j$ if $G = 1$
 0 if $G = 0$.

Net benefit to i : $v_i + \sum_{j \neq i} b_j$ when $b_i + \sum_{j \neq i} b_j \geq 0$; $G = 1$.

0 when $b_i + \sum_{j \neq i} b_j < 0$; $G = 0$.

i chooses b_i to maximize payoff to i , knowing $b_j, j \neq i$.

Claim: Truth telling is a dominant strategy, $\{b_i = v_i\}$ is a dominant strategy.

Case 1: $v_i + \sum_{j \neq i} b_j \geq 0$

if i announces $b_i = v_i$ then $G = 1$ and i 's benefit is

$$v_i + \sum_{j \neq i} b_j \geq 0 = \text{payoff with } G = 0.$$

Case 2: $v_i + \sum_{j \neq i} b_j < 0$

if i announces $b_i = v_i$ then $G = 0$ and i 's benefit is

$$0 \geq v_i + \sum_{j \neq i} b_j = \text{payoff with } G = 1.$$

In both Case 1 and Case 2, household i finds that reporting $b_i = v_i$ is optimizing.

Cost of the Groves mechanism: Total side payments = 0 if $G = 0$

but if $G=1$ total side payments = $\sum_i \sum_{j \neq i} b_j > 0$. This can be an expensive operation to run.

Fixing the cost: lump sum tax, Groves-Ledyard mechanism (too detailed for these notes) same principle --- setting incentives so that individuals internalize others' payoffs to the public good decision.