Notes for Syllabus Section I:

The Robinson Crusoe model; the Edgeworth Box in Consumption and Factor allocation

Overview: *General* equilibrium is the study of market clearing pricing and allocation taking into account the interaction among markets for distinct goods. The principal ideas can be illustrated in the two classic oversimplified models: the Edgeworth Box is a two-household pure exchange economy; the Robinson Crusoe model is a single-household two-commodity production economy. General equilibrium consists of prices so that simultaneously (taking into account interactions across markets) for each commodity supply and demand are equated. Pareto efficiency consists of allocation of factors and consumption so that utility is fully achieved, all opportunities for one household's utility increase without reduction in another's are fully utilized. General equilibrium pricing can be illustrated in these simple models. The First Fundamental Theorem of Welfare Economics — that the allocation of resources and consumption in a general equilibrium is Pareto efficient — can be illustrated here as well.

The demonstration typically takes the following form. Solve for Pareto efficiency and pricing that can support it. Solve for general equilibrium and prices that can support it. Show that general equilibrium prices fulfill the pricing conditions for Pareto efficiency.

PARTIAL EQUILIBRIUM

$S_k(p^{o_k}) = D_k(p^{o_k})$, with $p^{o_k} \geq 0$, or $p^{o_k} = 0$ if $S_k(p^{o_k}) > D_k(p^{o_k})$.

GENERAL EQUILIBRIUM

For all $i = 1, \ldots, N$, $D_i(p^{o_1}, p^{o_2}, \ldots, p^{o_N}) = S_i(p^o_1, .., p^{o_N}), p^{o_i} \geq 0$, and $p^{o_i} = 0$ for goods i such that $D_i(p^{o_1}, \ldots, p^{o_N}) < S_i(p^{o_1}, \ldots, p^{o_N})$.

Partial equilibrium assumes 'other things being equal', that the variations considered are all local and cross-market interactions are negligible.

What's wrong with partial equilibrium?

• There may be no consistent choice of $(p^o_1, .., p^{o_N})$. Then there would be (apparent) partial equilibrium viewing each market separately but no way to sustain it, because of cross-market interaction.

• Competitive equilibrium is supposed to make efficient use of resources through optimizing firm and consumer choice where prices indicate scarcity. But to verify this notion in partial equilibrium assumes that prices in other markets reflect underlying scarcity. If not, then apparently efficient equilibrium allocation may be wasteful. A valid notion of equilibrium and efficiency needs to take cross-market interaction into account.

Three big ideas:

• Equilibrium: $S(p) = D(p)$, where S, D, and p are all N-dimensional vectors

•Decentralization

•Efficiency

The Edgeworth Box and the Robinson Crusoe model are two oversimplied examples in which most principal observations of the general equilibrium theory can be illustrated.

The Edgeworth Box

See Chapter 3, Starr *General Equilibrium Theory: An Introduction* (second edition). References in these notes to figures are to the textbook.

2 person, 2 good, pure exchange economy.

Fixed positive quantities of X and Y, and two households, 1 and 2.

Household 1 is endowed with $\overline{X}^1$ of good X and $\overline{Y}^1$ of good Y, utility function $U^1(X^1, Y^1)$. Household 2 is endowed with $\overline{X}^2$ ofgood X and $\overline{Y}^2$ of good Y, utility function $U^2(X^2, Y^2)$

The economy's resource endowment is characterized as
$X^1 + X^2 = \overline{X}^1 + \overline{X}^2 \equiv \overline{X}, Y^1 + Y^2 = \overline{Y}^1 + \overline{Y}^2 \equiv \overline{Y}$.

Each point in the Edgeworth box represents an attainable allocation of X^1 and X^2, Y^1 and Y^2, consistent with the total resource endowment $(\overline{X}, \overline{Y})$.

$1'$s origin is at the southwest corner; $1'$s consumption increases as the allocation point moves in a northeast direction; $2'$s increases as the allocation point moves in a southwest direction. Superimpose indifference curves on the Edgeworth Box. See figure 3.1.

Competitive Equilibrium in the Edgeworth Box

$(\mathrm{p_x^o}\ ,\ \mathrm{p_y^o})$ so that $(\mathrm{X^{01}},\ \mathrm{Y^{01}})$ maximizes $\mathrm{U^1(X^1,\ Y^1)}$ subject to
$(\mathrm{p_x^o}\ ,\ \mathrm{p_y^o}) \cdot (\mathrm{X^1},\ \mathrm{Y^1}) \le (\mathrm{p_x^o}\ ,\ \mathrm{p_y^o}) \cdot (\overline{X}^1,\ \overline{Y}^1)$ and
$(\mathrm{X^{02}},\ \mathrm{Y^{\circ 2}})$ maximizes $\mathrm{U^2(X^2,\ Y^2)}$ subject to
$(\mathrm{p_x^o}\ ,\ \mathrm{p_y^o}) \cdot (\mathrm{X^1},\ \mathrm{Y^1}) \le (\mathrm{p_x^o}\ ,\ \mathrm{p_y^o}) \cdot (\overline{X}^2,\ \overline{Y}^2)$, and

$$(\mathrm{X},\ \mathrm{Y^{\circ 1}}) + (\mathrm{X^{\circ 2}},\ \mathrm{Y^{\circ 2}}) = (\overline{X}^1,\ \overline{Y}^1) + (\overline{X}^2,\ \overline{Y}^2)$$

or $(\mathrm{X^{\circ 1}},\ \mathrm{Y^{\circ 1}}) + (\mathrm{X^{\circ 2}},\ \mathrm{Y^{\circ 2}}) \le (\overline{X}^1,\ \overline{Y}^1) + (\overline{X}^2,\ \overline{Y}^2)$, where the inequality holds co-ordinatewise and any good for which there is a strict inequality has a price of 0.

Pareto efficiency in the Edgeworth Box

An allocation is Pareto efficient if all of the opportunities for mutually desirable reallocation have been fully used. The allocation is Pareto efficient if there is no available reallocation that can improve the utility level of one household while not reducing the utility of any household.

Assuming convexity and monotonicity of preferences (quasi-concavity and non-satiation of utility functions) and an interior solution, Pareto efficiency is achieved at an allocation where there is tangency of 1 and $2'$s indifference curves.

Pareto efficient allocation: $(\mathrm{X^{\circ 1}},\ \mathrm{Y^{\circ 1}}), (\mathrm{X^{\circ 2}},\ \mathrm{Y^{\circ 2}})$ maximizes $\mathrm{U^1(X^1,\ Y^1)}$ subject to $\mathrm{U^2(X^2,\ Y^2)} \ge \mathrm{U^{\circ 2}}$ (typically assuming non-satiation equality will hold and $\mathrm{U^{\circ 2} = U^2(X^{\circ 2}, Y^{\circ 2})}$) and subject to the resource constraints

$$\mathrm{X^1 + X^2} = \overline{X}^1 + \overline{X}^2 \equiv \overline{X}; \mathrm{Y^1 + Y^2} = \overline{Y}^1 + \overline{Y}^2 \equiv \overline{Y}\ .$$

Equivalently, $\mathrm{X^2} = \overline{X} - \mathrm{X^1}, \mathrm{Y^2} = \overline{Y} - \mathrm{Y^1}$.

Lagrangian

$$\mathrm{L} \equiv \mathrm{U^1(X^1,\ Y^1)} + \lambda[\mathrm{U^2}(\overline{X} - \mathrm{X^1}, \overline{Y} - \mathrm{Y^1}) - \mathrm{U^{\circ 2}}]$$

$$\frac{\partial L}{\partial X^1} = \frac{\partial U^1}{\partial X^1} - \lambda \frac{\partial U^2}{\partial X^2} = 0, \text{ equivalently } \frac{\partial U^1}{\partial X^1} = \lambda \frac{\partial U^2}{\partial X^2}$$

$\frac{\partial L}{\partial Y^1} = \frac{\partial U^1}{\partial Y^1} - \lambda \frac{\partial U^2}{\partial Y^2} = 0$, equivalently $\frac{\partial U^1}{\partial Y^1} = \lambda \frac{\partial U^2}{\partial Y^2}$

$\frac{\partial L}{\partial \lambda} = \mathrm{U}^2(\mathrm{X}^2,\ \mathrm{Y}^2) - \mathrm{U}^{\circ 2} = 0$

This gives us then the condition

$\mathrm{MRS}^1_{\mathrm{xy}} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \mathrm{MRS}^2_{\mathrm{xy}}$or equivalently

$\mathrm{MRS}^1_{\mathrm{xy}} = \frac{\partial Y^1}{\partial X^1}|_{U^1=\text{constant}} = \frac{\partial Y^2}{\partial X^2}|_{U^2=\text{constant}} = \mathrm{MRS}^2_{\mathrm{xy}}$

Pareto efficient allocation in the Edgeworth box: the slope of $2'$s indifference curve at an efficient allocation will equal the slope of $1'$s indifference curve; the points of tangency of the two curves. Exception: corner solutions, non-convex preferences (utility functions not quasi-concave).

Pareto efficient set =locus of tangencies of indifference curves

contract curve=individually rational Pareto efficient points

See figure 3.3.

Market allocation in the Edgeworth Box: general equilibrium $\mathrm{p_x}, \mathrm{p_y}$

Household 1: Choose X^1, Y^1, to maximize $\mathrm{U}^1(\mathrm{X}^1, \mathrm{Y}^1)$ subject to

$\mathrm{p_x X^1 + p_y Y^1 = p_x}\overline{X}^1 + \mathrm{p_y}\overline{Y}^1 = \mathrm{B}^1$

budget constraint is a straight line passing through the endowment point $(\overline{X}^1,\ \overline{Y}^1)$ with slope $-\frac{p_x}{p_y}$. Similarly for Household 2.

Lagrangian for Household $1'$s demand determination

$\mathrm{L} = \mathrm{U}^1(\mathrm{X}^1, \mathrm{Y}^1) - \lambda[\mathrm{p_x X^1 + p_y Y^1 - B^1}]$

$\frac{\partial L}{\partial X} = \frac{\partial U^1}{\partial X^1} - \lambda p_x = 0$

$\frac{\partial L}{\partial Y} = \frac{\partial U^1}{\partial Y^1} - \lambda p_y = 0$

Therefore, at the utility optimum subject to budget constraint we have

$\mathrm{MRS}^1_{\mathrm{xy}} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{p_x}{p_y}$;

Similarly for household 2, $\mathrm{MRS}^2_{\mathrm{xy}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \frac{p_x}{p_y}$;

Equilibrium prices: $\mathrm{p_x^*}$ and $\mathrm{p_y^*}$ so that

$$\mathrm{X}^{*1} + \mathrm{X}^{*2} = \overline{X}^1 + \overline{X}^2 \equiv \overline{X}$$

$$\mathrm{Y}^{*1} + \mathrm{Y}^{*2} = \overline{Y}^1 + \overline{Y}^2 \equiv \overline{Y},$$

(market clearing) where $\mathrm{X}^{*\mathrm{i}}$ and $\mathrm{Y}^{*\mathrm{i}}, \mathrm{i} = 1, 2$, are utility maximizing mix of X and Y at prices $\mathrm{p_x^*}$ and $\mathrm{p_y^*}$.

$$\frac{p_x^*}{p_y^*} = \mathrm{MRS}^1_{\mathrm{xy}} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \mathrm{MRS}^2_{\mathrm{xy}} = \frac{p_x^*}{p_y^*}$$

$$-\frac{\partial \mathrm{Y}^1}{\partial X^1}|_{U^1=U^{1*}} = \frac{p_x}{p_y} = -\frac{\partial \mathrm{Y}^2}{\partial X^2}|_{U^2=U^{2*}}$$

The price system decentralizes the efficient allocation decision.

The Robinson Crusoe Model

See Chapter 2.

q =oyster production

c =oyster consumption

168 (hours per week) endowment

L =labor demanded

R =leisure demanded

168-R =labor supplied

$$\mathrm{q} = \mathrm{F(L)} \qquad (2.1)$$

$$\mathrm{R} = 168 - \mathrm{L} \qquad (2.2)$$

Centralized Allocation of the Robinson Crusoe Model

We assume second order conditions (convexity=concavity of production and utility functions=diminishing marginal product (in production)+diminishing marginal rate of substitution(in consumption)) so that local maxima are global maxima:

$F'' < 0, \dfrac{\partial^2 u}{\partial c^2} < 0, \dfrac{\partial^2 u}{\partial R^2} < 0, \dfrac{\partial^2 u}{\partial c \partial R} > 0$.(Concavity, 2nd order conditions)

$u(c, R) = \mathrm{u(F(L),\ 168 - L)} \qquad (2.3)$

$\max u(F(L),\ 168 - L)$ (2.4), $L \geq 0$

$\frac{d}{dL}u(F(L), 168 - L) = 0$ (2.5)

$u_c F' - u_R = 0$ (2.6)

$[-\frac{dq}{dR}]_{u=u\max} = \frac{u_R}{u_c} = F'$ (2.7)

Pareto efficient
$MRS_{R,c} = MRT_{R,q} (= RPT_{R,q})$

Decentralized Market Allocation in the Robinson Crusoe Model

$\Pi = F(L) - wL = q - wL$ (2.8)

Income: $Y = w \cdot 168 + \Pi$ (2.9)

Budget constraint: $\mathrm{Y = wR + c}$ (2.10)

Equivalently, $\mathrm{c = Y - wR = \Pi + wL = \Pi + w(168 - R)}$, a more conventional definition of a household budget constraint.

Firm profit maximization in the market economy:
$\Pi = q - wL$ (2.11)

$\frac{d\Pi}{dL} = F' - w = 0$, so $F'(L^\circ) = w$ (2.14)

Household budget constraint:
$wR + c = Y = \Pi^\circ + w168$ (2.15)

Household utility optimization in the market economy:
Choose $\mathrm{c, R}$ to maximize $\mathrm{u(c,\ R)}$ subject to (2.15). The Lagrangian is
$\mathrm{V = u(c, R)} - \lambda$ (Y-wR-c)

$\frac{\partial V}{\partial c} = \frac{\partial u}{\partial c} + \lambda = 0$

$\frac{\partial V}{\partial R} = \frac{\partial u}{\partial R} + \lambda w = 0$

Dividing through, we have

$$\mathrm{MRS_{R,c}} = [-\frac{dc}{dR}]_{u=u^*} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w \qquad (2.19)$$

Equilibrium consists of a wage rate w° so that at $w^\circ, q = c$ and $L = 168 - R$, where $\mathrm{q, L}$ are determined by firm profit maximizing decisions and $\mathrm{c, R}$ are determined by household utility maximization. In a centralized solution $\mathrm{L} = 168 - \mathrm{R}$ by definition; in a market allocation wages and prices should adjust so that as an equilibrium condition L will be equated to 168-R.

Profit maximization at w° implies $w^\circ = F'(L^\circ)$. (Recall (2.14)) Utility maximization at $\mathrm{w^o}$ implies
$\dfrac{u_R(c^\circ, R^\circ)}{u_c(c^\circ, R^\circ)} = w^\circ$ (Recall (2.19))

Market-clearing implies $R^\circ = 168 - L^\circ, c^\circ = F(L^\circ)$.

So combining (2.14) and (2.19), we have

$$F' = \frac{u_R}{u_c} \qquad (2.25)$$

which implies Pareto efficiency. The market general equilibrium decentralizes the efficient allocation.