14.2, 14.6 --- Suggested Answers

14.2 *Suggested Answer*: Yes. The income tax scheme is equivalent to a revised endowment schedule where the new endowment of each household i,

$r^{i\prime} = .5r^{i} + ({}^{1}/_{\#H})\, .5\Sigma_{h\in H} r^{h}$. The rest of the model remains unchanged. Hence there is a competitive equilibrium here assuming all other sufficient conditions are fulfilled.

14.6. *Suggested Answer*: At a competitive general equilibrium p*, we have $\tilde{Z}(p^*) \leq 0$ co-ordinatewise, with $p^*_k=0$ for any k so that $\tilde{Z}_k(p^*) < 0$.

(i) Then if p^0 is a competitive equilibrium price vector, it follow for all i, that

$p^0{}_i \tilde{Z}_i(p^0) = 0$. Thus $Q_i(p^0) = \dfrac{\max[0, p^0{}_i + p_i{}^0 \tilde{Z}_i(p^0)]}{\sum_{j=1}^{N} \max[0, p^0{}_j + p_j{}^0 \tilde{Z}_j(p^0)]} = \dfrac{p^0{}_i}{1}$

(ii) The i^{th} vertex of the price simplex is a point of the form

v = (0, 0, ..., 1, 0, ..., 0) where $v_i = 1$. Thus

$$Q_i(v) = \frac{\max[0, v_i + v_i \tilde{Z}_i(v)]}{\max[0, v_i + v_i \tilde{Z}_i(v)] + \sum_{j=1, j\neq i}^{N} \max[0, v_j + v_j \tilde{Z}_j(v)]} = \frac{\max[0, v_i + v_i \tilde{Z}_i(v)]}{\max[0, v_i + v_i \tilde{Z}_i(v)]} = 1 = v_i$$

$Q_j(v) = 0$ for $j \neq i$,

and hence v is a fixed point of Q, for each vertex v.

(iii) A fixed point of Q could be a competitive equilibrium or it could be a vertex. Maybe there are other fixed points. The vertices are certainly not necessarily equilibria, so the existence of a fixed point of Q certainly does not prove the existence of a competitive general equilibrium.

Suggested Answer 14.20

14.20, Alpha (a)

$p_x > p_y$ implies there is an excess demand for y; A's demand for y is $\frac{5p_x+5p_y}{p_y} > 10$ and his demand for x is 0. B's demand is an interior solution. Hence there is an excess demand for y.

$p_x < p_y$ implies there is an excess demand for x;A's demand for x is $\frac{5p_x+5p_y}{p_x} > 10$ and his demand for y is 0. B's demand is an interior solution. Hence there is an excess demand for x.

$p_x = p_y$ implies there is an either an excess demand for x and an excess supply of y, or the opposite. A's demand is either of two corner solutions; either (0, 10) or (10, 0). B's demand is (5, 5). Hence there is no market-clearing and there is an excess demand for either x or for y.

(b) A does not fulfill C.VI(SC) nor C.VI(C) (strict or weak convexity). Hence the jumps in A's demand behavior leading to non-existence of equilibrium.

Beta

(a) $p_x > 3p_y$ implies there is an excess demand for y; in this case, both households demand y only.

$p_x < 3p_y$ implies there is an excess demand for x; in this case, both households demand x only.

$p_x = 3p_y$ implies there is an excess demand for x; in this case both households demand x only.

(b) Preferences P do not fulfill C.V, continuity. Hence the jumps in both households' demand behavior leading to non-existence of equilibrium.

24.7 The Arrow Corner is a failure of lower hemicontinuity of the budget correspondence and of upper hemicontinuity of the demand correspondence. It occurs when some prices are zero and when income is just sufficient to achieve the boundary of the consumption set X^i (in a typical example, this will occur at a zero income where X^i is the nonnegative orthant). Consider the following example. Let $N = 2, X^i = \mathbf{R}^2_+$, and

$$p^\nu = (1 - 1/\nu, 1/\nu), \nu = 1, 2, 3, \ldots.$$

Then we have $p^\nu \to p^\circ = (1, 0)$. Let c (the bound on the size of the demand vector) be chosen so that $100 < c < \infty$. Let household i's endowment vector r^i equal $(0, 100)$, with sale of r^i being i's sole source of income. Then we have

$$\tilde{B}^i(p) = \{x | x = (x_1, x_2), |x| \leq c, p \cdot x \leq p \cdot r^i\}.$$

Let i's utility function be $u^i(x_1, x_2)=x_1+x_2$ so that

$\tilde{D}^i(p)= \{x' | x' \in \tilde{B}^i(p) \cap \mathbf{R}^2_+,\ x'$ maximizes $u^i(x)$ for all $x \in \tilde{B}^i(p) \cap \mathbf{R}^2_+\}$. Demonstrate the following points:

(i) Show that $(0, c) \in \tilde{B}^i(p^\circ)$.
Suggested Answer: $p^\circ \cdot (0, c) = 0 = p^\circ \cdot (0, 100) = p^\circ \cdot r^i$ and $|(0, c)| = c \leq c$, so $(0, c) \in \tilde{B}^i(p^\circ)$.

(ii) Show that $x \in \tilde{B}^i(p^\nu), x = (x_1, x_2)$, implies $x_2 \leq 100$.
Suggested Answer: $p^\nu \cdot r^i = (1 - 1/\nu, 1/\nu) \cdot (0, 100) = 100 \cdot (1/\nu) \geq p^\nu \cdot (x_1, x_2)$ for $(x_1, x_2) \in B^i(p^\nu)$. So $(1 - 1/\nu, 1/\nu) \cdot (x_1, x_2) = (1 - 1/\nu) \cdot x_1 + (1/\nu) \cdot x_2 \leq 100 \cdot (1/\nu)$. Hence, $x_2 \leq 100$.

(iii) Show that $\tilde{D}^i(p^\circ) = \{(0, c)\}$
Suggested Answer: At (p°), $(x_1, x_2) \in \tilde{B}^i(p^\circ)$ implies $x_1 = 0$, $x_2 \in [0, c]$. $x_2 = c$ then maximizes $u^i(x_1, x_2)=x_1+x_2$ subject to these constraints.

(iv) Show that $\tilde{B}^i(p)$ is not lower hemicontinuous at $p = p^\circ$.
Suggested Answer: Lower hemicontinuity requires that there be a sequence $x^\nu, \nu = 1, 2, \ldots,$ so that $x^\nu \in B^i(p^\nu)$ so that $x^\nu \to (0, c)$. But by assumption, $c > 100$ and by part (ii) $x^\nu = (x_1^\nu, x_2^\nu)$ implies $x_2^\nu \leq 100$. Hence there is no $x^\nu \in B^i(p^\nu)$ so that $x^\nu \to (0, c)$. Thus $B^i(p)$ is not lower hemicontinuous at p°.

(v) Show that $\tilde{D}^i(p)$ is not upper hemicontinuous at $p = p^\circ$.
Suggested Answer: Upper hemicontinuity of $\tilde{D}^i(p)$ at p° requires that if there is $x^\nu \in \tilde{D}^i(p^\nu)$ so that $x^\nu \to x^\circ$ then $x^\circ \in \tilde{D}^i(p^\circ)$. For $\nu \geq 3, x^\nu = (x_1^\nu, x_2^\nu)$ implies $(x_1^\nu, x_2^\nu) = (0, 100)$ so $(x_1^\nu, x_2^\nu) \to (0, 100)$. Thus upper hemicontinuity requires that $(0, 100) \in \tilde{D}^i(p^\circ)$,

> but $(0, 100) \notin \tilde{D}^i(p^\circ) = \{(0, c)\}$ since $c > 100$. So $\tilde{D}^i(p)$ is not upper hemicontinuous at $p = p^\circ$.

Discuss this example with regard to the Maximum Theorem (Theorem 23.2).

Suggested Answer: One of the conditions of the Maximum Theorem is not fulfilled in this example. The Maximum Theorem says that if the opportunity set (in this case $B^i(p)$) is both upper and lower hemicontinuous then the set of maximizers of a continuous function on the opportunity set (in this case $D^i(p)$) will be upper hemicontinuous. As demonstrated in part (iv), $B^i(p)$ is not lower hemicontinuous at p°, so the Theorem cannot validly be applied in this case. And indeed the conclusion of the Theorem would be false in this case, as shown in part (v).

June 2011, # 2

(a) No, the 2nd order conditions specified are inconsistent with the depiction in 5.D.3. The cost and production function curves there indicate scale economy (a bounded scale economy, but scale economy nevertheless), increasing marginal product over a range, a nonconvexity in the technology. The conditions

$\frac{\partial^2 f}{\partial x^2} \leq 0$, $\frac{\partial^2 f}{\partial y^2} \leq 0$ formalize diminishing marginal product, convexity of the technology.

(b) No, not generally. We usually posit convexity of technology for existence of general equilibrium, P.I. There are two reasons for this. Convexity of technology helps to ensure boundedness of the attainable set — very large scale economies may lead to unbounded output. Convexity of the technology generates convexity of the supply correspondence, needed for application of the Kakutani fixed point theorem.

June 2014, #3

(a) Continuity of the utility function and sufficient income to stay off the boundary of the consumption set would be helpful. Best guess is that the domain of the utility function is intended to be R^m_+ so strict positivity of endowment is a sufficient, but not necessary condition. Indeed since utility is taken to be strictly increasing, equilibrium prices are likely to be strictly positive, so endowment ≥ 0 co-ordinatewise but $\neq 0$, is sufficient.

(b) For all households, i, h $e^i = \lambda e^j, \lambda > 0$. All endowments are linear multiples of each other. Under identical homothetic preferences, all household MRS's at endowment are identical, so no trade occurs.

(c) Same conditions for existence of equilibrium as in (a). This is just a redistribution of endowment prior to trade, so existence of equilibrium occurs under the usual conditions.

(d) This is complicated. Prof. Starr doesn't know. Not sure there is a general well-defined answer. Sounds like it depends on elasiticity of demand for each good i and on whether there is subsequent (untaxed) retrade.