11.2

***Suggested Answer*:** The production technology is a segment along the x-axis from the origin to (-S, 0) and a cone expanding from (-S, 0), clearly a nonconvex set. To prove this more directly, consider two points in the technology set $\mathcal{Y}^j = \{(-L, y) \mid y = 0 \text{ if } L \leq S, y \leq a(L - S) \text{ if } L > S\}$. Let 0 denote the 0 vector. $0 \in \mathcal{Y}^j$ and $(-2S, aS) \in \mathcal{Y}^j$. But for any α, so that $0 < \alpha < 1$, the point $[\alpha 0 + (1-\alpha)(-2S, aS)] \notin \mathcal{Y}^j$, hence failing P.I. This reflects the scale economy embodied in the production function. Running the technology at fractional scale will not succeed.

12.7 $A^i (x^o, y^o)$ is not a closed set. To demonstrate this consider a sequence of points superior to (x^o, y^o) in $A^i (x^o, y^o)$, $(x^o - 1 + {}^1/\nu,\ y^o + 1 - {}^1/\nu) \succeq (x^o, y^o)$. Each element of the sequence is in $A^i (x^o, y^o)$ but the limit point $(x^o - 1, y^o + 1)$ is inferior to $A^i (x^o, y^o)$ under the ordering $\succeq$ and is not in $A^i (x^o, y^o)$. The implication for household demand behavior is that preferences cannot be represented as a continuous utility function and that at some prices demand may respond discontinuously to price changes.

At prices (p_x, p_y) where $p_x > 2 p_y$, only y is demanded. At prices (p_x, p_y) where $p_x \leq 2p_y$, only x is demanded. There is a discontinuous change at $p_x = 2p_y$.

12.8 The obvious candidate equilibrium price vector is $({}^2/_3 , {}^1/_3)$. But at that price, there's an excess demand for x and an excess supply of y. But raising the price of x doesn't help. At $({}^2/_3 + \varepsilon, {}^1/_3 - \varepsilon)$, for any $\varepsilon > 0$, there's an excess demand for y. No this is not a counterexample to Theorem 5.2, because the assumptions of 5.2 are not fulfilled. The preferences here, $\succsim$, though otherwise fulfilling the assumptions of Chapter 5, do not fulfill C.V; they are discontinuous resulting in a discontinuous excess demand function. The observation that there is no equilibrium does not contradict Theorem 5.2.

23.4 $T = S = R^2$. Let $(x_1, x_2) \in T; (y_1, y_2) \in S$.
$\varphi(x_1, x_2) = \{(y_1, y_2) | (y_1)^2 + (y_2)^2 \leq |x_1| + |x_2|\}$.
$f(y_1, y_2) = |y_1| + 2|y_2|$.
$\mu(x_1, x_2) = \{(y_1^\circ, y_2^\circ) \in R^2 | (y_1^\circ, y_2^\circ)$ maximizes $f(y_1, y_2)$ subject to $(y_1, y_2) \in \varphi(x_1, x_2)\}$.

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Firm j has a scale economy so P.V does not apply. The technology set is not convex.

Part (a) Nonemptiness still holds, since $\tilde{S}^j(p)$ represents maximization of a continuous function over a compact set. Continuity and point-valuedness can fail due to the nonconvexity. More important,

Part(b) can fail completely, since the argument for this property is based on convexity.

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(i) At $(\frac{1}{2}+\epsilon, \frac{1}{2}-\epsilon)$ three households demand approximately $(0, 20+20\epsilon)$ each, creating an unsatisfied demand for y. At $(\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon)$ three households demand approximately $(20+20\epsilon, 0)$ each, creating an unsatisfied demand for x.

At $(\frac{1}{2}, \frac{1}{2})$ zero, one, two, or three households demand $(0, 20)$ and the remaining zero, one, two, or three households demand $(20, 0)$. Total supply is $(30, 30)$. In any of the several cases there is unsatisfied excess demand.

(ii) Demand behavior in this class of examples is not convex-valued. It pivots between extremes without touching the middle. That is contrary to the assumption of convexity of preferences in the usual Arrow-Debreu models, C.VI(C) in Starr's *General Equilibrium Theory*. The assumptions for existence of equilibrium in an Arrow-Debreu model are not fulfilled.