Your Name:

Suggested Answers

Please answer all questions. Each of the eight questions marked with a big number counts equally. Designate your answers clearly.

Short correct answers are sufficient and get full credit. Including irrelevant (though correct) information in an answer will not increase the score.

All notation not otherwise defined is taken from Starr's *General Equilibrium Theory: An Introduction (2nd edition).*

If you need additional space to answer a question, write "(over)" at the end of text on the first page and continue on the back of the page. If you still need additional space, use an additional sheet of paper, making sure that it has your name on it and is attached to your exam when submitted.

This examination is open book, open notes (hard copy only). Calculators, cell phones, computers, iPads, etc., advice of classmates, are not allowed.

1 Social Choice

MasColell, Whinston, & Green's Proposition 21.D.2 (page 803) says that in the case of single-peaked preferences, majority voting on pairwise alternatives results in transitive group decision-making fulfilling the Pareto Principle. The Arrow Possibility Theorem (Proposition 21.C.1, page 796) says THERE IS NO non-dictatorial, Pareto Principle, Unrestricted Domain, Transitive, Independent of Irrelevant Alternatives, group decision-making procedure. These results are, or appear to be, contradictory. Explain.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. The Arrow Possibility Theorem includes the condition of Independence of Irrelevant Alternatives, but determining the **median voter** under single-peaked preferences depends on the location in a linear ordering of the alternatives other than those chosen in pairwise voting. Hence the single-peaked preferences result depends on violating the Independence of Irrelevant Alternatives. Thus allowing both results to be sustained as valid.

B. The Arrow Possibility Theorem includes the condition of "non-dictatorship." Under single-peaked preferences the **median voter** always gets his way. He/she becomes the dictator, hence failing to fulfill the conditions of the Arrow Possibility Theorem.

C. The Arrow Possibility Theorem includes the condition of "unrestricted domain." Single-peaked preferences restricts the domain to a simple class of preferences, allowing both results to be sustained as valid.

D. The two competing results are contradictory and cannot both be valid. The textbook has three authors, with cyclic majorities. Two out of three favored each of the two contradictory results, so they were both included.

E. None of the above explanations is correct.

Write your explanation for the answer to Question 1 on this page.

Suggested Answer: C In the case of single-peaked preferences, the class of voting choices available is limited to the following situation: There is consensus on a linear arrangement of propositions; each voter has a favorite position and all movement away from the favorite position represents a decline in preference. That family of limitations constitutes a violation of "Unrestricted Domain" since any disagreement over the linear ordering or multiple local favorite positions is disallowed.

2 Robinson Crusoe

Robinson Crusoe lives on an island with two perishable outputs of his harvesting effort: oysters and guavas. He has 24 hours a day of labor, L, and does not value leisure.

Oyster production takes place with the production function

$$Q = \frac{1}{[24 - L^Q]} - \frac{1}{24}, \qquad 0 \leq L^Q \leq 23.99$$

where L^Q is the volume of labor devoted to oyster harvesting. Guava production takes place using the production function

$$G = L^G$$

where L^G is the volume of labor devoted to guava harvesting. The resource constraint is

$$L^Q + L^G \leq 24.$$

Each day there is a competitive market for labor, guavas, and oysters. Describe the competitive equilibrium. The superscripts Q and G merely designate the application of L; nothing in this problem is raised to a power.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. The production function for Q includes a scale economy contrary to the assumption of convex production technology. There may be no competitive equilibrium.

B. The linear production function for guavas, G, violates the convexity assumption. There may be no competitive equilibrium.

C. If there is a competitive equilibrium, the First Fundamental Theorem of Welfare Economics (1FTWE) applies. The allocation is Pareto efficient.

D. Competitive equilibrium will exist only in the case of a linear utility function.

E. None of the above answers is correct.

Write your explanation for the answer to Question 2 on this page.

Suggested Answer: A & C The production function for Q has increasing marginal product throughout its range, hence a scale economy and a violation of the the convexity assumption. That may lead to a non-convex supply correspondence. Thus the usual existence of equilibrium results do not apply. Nevertheless, an equilibrium may exist and 1FTWE does not require convexity, so an equilibrium allocation is Pareto efficient.

3 2FTWE

Consider a single-household (Robinson Crusoe) economy. Let there be two commodities, x and y with a production frontier defined by

$$\sqrt{x} + \sqrt{y} = 10; \qquad x, y \geq 0.$$

The single household's preferences are defined by the Cobb-Douglas utility function

$$U(x, y) = x \cdot y.$$

The Pareto efficient allocation is $(x, y) = (25, 25)$. Applying the Second Fundamental Theorem of Welfare Economics (2FTWE), there should be competitive prices that will support this allocation as a competitive equilibrium. Find supporting prices and discuss the equilibrium.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. The Second Fundamental Theorem of Welfare Economics (2FTWE) does not apply. In a Robinson Crusoe economy there is no interpersonal allocation issue so simple efficiency, not Pareto efficiency is the appropriate criterion.

B. The production possibility set is non-convex; there is a scale economy. Hence the Second Fundamental Theorem of Welfare Economics (2FTWE) does not apply. 2FTWE requires convexity.

C. Supporting prices are $p = (.5, .5)$ where MRS = MRT = 1.

D. At any price vector, profit maximizing production plans will be at a corner, either $x = 100, y = 0$ or $x = 0, y = 100$. The Pareto efficient allocation cannot be supported by a price system.

E. None of the above answers is correct.

Write your explanation for the answer to Question 3 on this page.

Suggested Answer: B & D There is a scale economy; the production possibility set is non-convex. It includes (100, 0), (0, 100) but not (50,50). 2FTWE requires convexity, so the efficient allocation (25, 25) may not be supported by efficiency prices. Supporting prices for the allocation (25, 25), on the demand side, are (.5, .5) but at those prices, the profit maximizing supply is (100, 0) or (0, 100).

4 Production Function

Consider the general equilibrium of an economy with two commodities, x, y and one input to production, labor, L. x and y are produced according to the following constant returns production functions:

$$x = kL^x; \qquad k > 0$$

$$y = kL^y; \qquad k > 0$$

where L^x is the volume of labor applied to production of x, L^y is the volume of labor applied to the production of y and the resource constraint is $L^x + L^y = 24; \qquad L^x, L^y \geq 0$. Under conventional assumptions on preferences and incomes, describe the general equilibrium allocation and prices. The superscripts x and y merely designate the application of L; nothing in this problem is raised to a power.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. Production technologies are *weakly* convex, not *strictly* convex, so sufficient conditions for existence of general equilibrium may not be fulfilled; there may be no general equilibrium.

B. General equilibrium price and output levels for x and y will be equal.

C. General equilibrium prices for x and y will be equal.

D. Scale economies in production technologies may prevent the existence of general equilibrium.

E. None of the above answers is correct.

Write your explanation for the answer to Question 4 on this page.

Suggested Answer: C The production technologies are convex, continuous, so under conventional demand assumptions there will be a market clearing equilibrium price vector and allocation. Since the production technologies are identical and linear, the prevailing equilibrium output prices may be equal, but depending on demand conditions, there is no reason to expect the equilibrium output levels to be equal.

Suggested Answer Alternative E: If demand for one of the goods, for example x, is nil, then price of x in equilibrium may be less than for y, an all production will be of y.

5 Uncertainty

Consider an Arrow-Debreu economy over time and uncertainty with a full set of contingent commodity markets. The market meets and comes to general equilibrium at date 0. Economic activity of production and consumption takes place at dates 1, 2, 3, ..., T. All households are assumed to be expected utility maximizers, subject to budget constraint. Prof. Debreu reminds us that the First Fundamental Theorem of Welfare Economics holds in the general equilibrium of a contingent commodity economy. The equilibrium outcome is Pareto efficient. Describe Pareto efficiency in this setting.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. Pareto efficiency here is the same as in a certainty economy. There is no reallocation of resources that a central planner **endowed with perfect foresight** could arrange that would make a Pareto improvement in realized utilities and outputs.

B. Capital markets are imperfect here, so there may be avoidable intertemporal inefficiencies, since some households may be liquidity constrained.

C. Pareto efficiency is in terms of expected utilities, viewed from date 0. There is no technically achievable reallocation of contingent commodities that would improve some household's expected utility and not reduce another's.

D. The allocation of resources is efficient but with so much activity in contingent commodity markets, distribution of income is skewed to the market-makers. The financial sector in general equilibrium is much larger than in a certainty economy.

E. None of the above answers is correct.

Write your explanation for the answer to Question 5 on this page.

Suggested Answer: C All trading activity is in the market at date 0 aimed for each household at maximizing expected utility of contingent commodity portfolios subject to budget constraint. The efficient allocation that results is Pareto efficiency in terms of expected utilities. There is no attainable reallocation of contingent commodities that improves some household's expected utility without reducing another's. There is no result here concerning the *ex post* realization of utilities from realized consumption and production.

6 Core

Consider the core of the Q-fold replica economy denoted $Q \times H$ where H is a pure exchange economy. Assume all households have continuous convex weakly monotone preferences and strictly positive endowments. Let p° be a competitive equilibrium price vector for H where the allocation $\{x^{\circ i} | i \in H\}$ is the corresponding competitive equilibrium allocation. Denote an allocation in $Q \times H$ as $\{y^{i,q} | i \in H, q = 1, 2, \ldots, Q\}$. Describe the core and competitive equilibrium allocations for $Q = 2, 3, \ldots$.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. $\{y^{i,q} = x^{\circ i} | i \in H, q = 1, 2, \ldots, Q\} \in Core(Q \times H)$ and p° is a competitive equilibrium price vector for $Q \times H$.

B. As Q increases the variety of blocking allocations increases, so $\{y^{i,q} = x^{\circ i} | i \in H, q = 1, 2, \ldots, Q\}$ may be blocked. Nevertheless, the core remains non-empty and includes some competitive equilibrium allocations.

C. As Q increases the variety of blocking allocations increases, so $\{y^{i,q} = x^{\circ i} | i \in H, q = 1, 2, \ldots, Q\}$ may be blocked. The equal treatment property ensures that in a core allocation $y^{i,q}$ is the same for all $q = 1, 2, \ldots, Q$.

D. As Q increases the variety of blocking allocations increases, so $\{y^{i,q} = x^{\circ i} | i \in H, q = 1, 2, \ldots, Q\}$ is blocked and for Q sufficiently large the core may be empty. There may be no core allocation.

E. None of the above answers is correct.

Write your explanation for the answer to Question 6 on this page.

Suggested Answer: A This is the Debreu-Scarf model. As Q increases the original economy is merely duplicated. The original competitive equilibrium prices and allocation, p° and $x^{i\circ}$ for a type i household, remain competitive equilibria, with an equal treatment property for the replicated economy. Thus $\{y^{i,q} = x^{\circ i} | i \in H, q = 1, 2, \ldots, Q\}$ is a competitive equilibrium and according to the Debreu-Scarf model, the competitive equilibrium is in the core.

7 External Effects

Two firms are located on a river. The upstream firm produces x using the production function $x = \alpha L^x$ where $\alpha > 0$ and L^x is the quantity of labor used in the production of x. The downstream firm suffers from the effluent of the upstream firm. The downstream firm produces y using the production function $y = \sup[\beta\sqrt{L^y} - x, 0]$ where $\beta > 0$ and L^y is the quantitity of labor used in production of y. The downstream firm treats x parametrically. There is a single household in the economy, Robinson Crusoe, who provides the labor $L^x + L^y = 24$ and does not value leisure. Robinson's utility function is Cobb-Douglas, $U(x, y) = x \cdot y$. Assume there are competitive equilibrium prices p_x, p_y. Describe the Pareto efficiency or inefficiency of the equilibrium allocation. The superscripts x and y refer to the allocation of L; nothing in this problem is raised to a power.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. There is only a single household so the Pareto criterion does not apply.

B. The presence of an external diseconomy means that the First Fundamental Theorem of Welfare Economics does not apply. The equilibrium allocation is likely to be Pareto inefficient.

C. Apply the First Fundamental Theorem of Welfare Economics. The competitive equilibrium allocation is Pareto efficient, maximizing $U(x, y)$ subject to the available technology, including the external effect.

D. The external effect is likely to send the equilibrium allocation of y to the corner solution, $y = 0$. That is inefficient.

E. None of the above answers is correct.

Write your explanation for the answer to Question 7 on this page.

Suggested Answer: B 1FTWE is based on all commodities, inputs and outputs, being traded to competitive equilibrium on competitive markets. In the case of external diseconomies, in this example, significant inputs to production (clean water) are not allocated by the market. Hence 1FTWE does not apply and the allocation may be, indeed is likely to be, inefficient. There may be a utility- or output-improving technically possible reallocation that is not undertaken because the water resources may be misallocated.

8 Fixed Point Theorem

Consider the case of an excess demand function (not a correspondence, to keep the analysis simple). Let the economy's excess demand function be

$$Z : P \to R^N$$

where P is the unit simplex in R^N. Walras's Law, can be stated as

$$p \cdot Z(p) = 0.$$

Then Theorem 5.2 of Starr's *General Equilibrium Theory* (2nd edition) says

Let Z(p) be continuous and fulfill Walras's Law. Then there is $p^* \in P$ so that p^* is an equilibrium.

This result is proved using the Brouwer Fixed Point Theorem.

The Uzawa Equivalence Theorem, (Theorem 18.2) can be stated:

Assume for every continuous $Z : P \to R^N$ fulfilling Walras's Law, there is $p^* \in P$ so that p^* is an equilibrium. Then the Brouwer Fixed Point Theorem follows as a result. Every continuous $f : P \to P$ has $p^* \in P$ so that $f(p^*) = p^*$.

Explain the significance of the Uzawa Equivalence Theorem.

Circle the letter(s) of the correct answer(s). Write an explanation of your reasoning on the next page. Draw an illustration if that will help.

A. The theorem is merely a guide to proving Theorem 5.2. It says that's the way to prove the result.

B. It says that the Brouwer Fixed Point Theorem is mathematically equivalent to Theorem 5.2. Each is the same as the other.

C. Using the Brouwer Fixed Point Theorem is pretty much the only way to prove Theorem 5.2. If you have an alternative proof, you've probably proved the Brouwer Fixed Point Theorem in the process of your alternative proof.

D. The First and Second Fundamental Theorems of Welfare Economics are essentially the same as each other.

E. None of the above answers is correct.

Write your explanation for the answer to Question 8 on this page.

Suggested Answer: B, C Each of the two mathematical results logically implies the other. They are mathematically equivalent. Thus the Brouwer theorem is pretty much the only way to prove the existence of general equilibrium result. That means when you prove existence of general equilibrium — without using the Brouwer Theorem directly — you've either used something equivalent (maybe the Kakutani theorem or — for N=2 intermediate value theorem) or your proof embodies a proof of the Brouwer theorem.