Problem Set 5

It’s OK to work together on problem sets.


2. Consider an Edgeworth Box for two households. The two goods are denoted x, y. The households have identical preferences:

   \[(x,y) > (x',y') \text{ if } 3x + y > 3x' + y', \quad \text{or}\]
   \[(x,y) > (x',y') \text{ if } 3x + y = 3x' + y' \quad \text{and} \quad x > x'.\]
   \[(x,y) \sim (x',y') \text{ only if } (x,y) = (x',y').\]

They have identical endowments of (10, 10). Demonstrate that there is no competitive equilibrium. Is this example a counterexample to Theorem 7.1 (does it demonstrate that Theorem 7.1 is false?)? Explain.

3. Consider a small economy, with two goods and three households. The two goods are denoted x, y. The households have identical preferences described by the utility function

\[u(x, y) = \sup [x, y]\]

Where sup indicates the supremum or maximum of the two arguments. Demonstrate that these preferences are nonconvex; they do not fulfill Starr’s *General Equilibrium Theory* assumptions C.VI or CVII.

The households have identical endowments of (10, 10). Demonstrate that there is no competitive equilibrium in this economy [Hint: Show that price vector \((1/2 + \varepsilon, 1/2 - \varepsilon)\), \(\varepsilon > 0\), cannot be an equilibrium; similarly for \((1/2 - \varepsilon, 1/2 + \varepsilon)\); and finally \((1/2, 1/2)\). That pretty well takes care of it.]

4. Starr’s *General Equilibrium Theory* problem 7.6, parts (i), (ii). Part (iii) is rewritten below. “competitive equilibria” means “competitive general equilibria.”

(iii) Assuming in addition continuity of \(\tilde{Z}(p)\), Q has a fixed point \(p^* \in P\) so that \(Q(p^*) = p^*\). Does this prove that under these assumptions the economy has a competitive general equilibrium?

5. Let \(f: P \rightarrow P\), \(f\) continuous. Define \(Z(p) = f(p) - \left[ \frac{p \cdot f(p)}{p \cdot p} \right] p\). The term in square brackets is just a scalar multiplying the vector \(p\). Show that \(p \cdot Z(p) = 0\). \(Z\) is a continuous function, \(Z: P \rightarrow \mathbb{R}^N\). Why? Assume there is a competitive
equilibrium price vector \( p^* \) so that \( Z(p^*) = 0 \) (the zero vector; ignore excess supplies of free goods). Is \( p^* \) also a fixed point of \( f \) so that \( f(p^*) = p^* \)? Review Theorem 11.2 in Starr’s *General Equilibrium Theory* to see what you’ve demonstrated.