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Liquidity of secondary capital markets: Allocative efficiency and the maturity composition of the capital stock*

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Summary. We investigate the function of liquid financial markets for the allocation of productive capital. We consider an economy where agents endogenously choose among capital production technologies with differing gestation periods. Long-gestation capital investments must be "rolled-over" in secondary capital markets. The use of such investment technologies therefore requires the support of liquid financial markets. We investigate how changes in the liquidity of these markets (i.e., in the costs of transacting) affect (a) the choice of capital production technology, (b) per capita income and the per capita capital stock, (c) the level of financial market activity, (d) the real return on savings and (e) welfare in a steady state equilibrium. Improvements in financial market liquidity raise rates of return on savings, and favor the increased use of long gestation capital investments. However, such improvements may or may not lead to higher levels of real activity or steady state welfare. We describe conditions under which various outcomes occur.

The financial sector of the economy performs two distinct functions: allocating capital and providing liquidity. We will investigate the role of liquidity in enhancing the supply of capital and allowing capital to be channeled to its highest yielding uses. It is widely held that the development and technical efficiency of financial markets are an important – and perhaps essential – component of economic develop-

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ment.[1] Yet most financial market transactions simply rearrange the ownership of existing capital without altering its allocation in production. Why then should financial market development be so important for capital accumulation and growth?

A potential answer to this question was offered by Hicks [20] in the context of the larger question: What made the industrial revolution a revolution? Hicks argued that the industrial revolution was revolutionary because it was associated with the implementation of new technologies that required very large scale investments in highly illiquid capital.[2] Hicks further argued that these technologies would not have been utilized, and the associated capital investments would not have been undertaken, had there not existed a set of financial markets available to provide liquidity to owners of capital:

> What happened in the Industrial Revolution ... is that the range of fixed capital goods that were used in production ... began noticeably to increase.... But fixed capital is sunk; it is embodied in a particular form, from which it can only gradually ... be released. In order that people should be willing ... to sink large amounts of capital,... it is the availability of liquid funds which is crucial. This condition was satisfied in England ... by the first half of the eighteenth century.... The liquid asset was there, as it would not have been even a few years earlier. ([20], p. 143–5)

In other words, according to Hicks, it was necessary that the Financial Revolution[3] occur before the industrial revolution in order to provide the financial market structures necessary to support the adoption of technologies requiring illiquid capital investments.

Hicks' argument provides a specific mechanism by which the existence of an appropriate set of financial markets – and the costs of transacting in them – are linked to the choice of technology in use, and ultimately to the level of capital accumulation. In particular, some technologies require greater investments in illiquid capital than others. Hence, their adoption is possible only if financial markets provide adequate liquidity. The adoption of technologies that are "liquidity intensive" can only occur when financial markets are sufficiently well-developed.

This paper is an attempt to formalize this notion. It presents a model in which agents endogenously choose among different technologies for converting current output into future capital. These technologies differ according to their productivity and their gestation period. Long-gestation technologies will not be adopted, however, in the absence of sufficient liquidity provision by financial markets. These markets permit resale of long-lived capital investments and thus mediate two conflicting forces in the investment process: the most productive capital investments will often be illiquid, while investors (for a variety of reasons) desire access to some liquid assets.

In considering these issues, we provide a model which jointly determines the equilibrium choice of capital production technologies, the equilibrium levels of per

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[1] For arguments to this effect, see [3], [8], [17], [19], [23], and [25]. Recent formalizations of some of their arguments appear in [4], [9], [18], and [28]. Some recent evidence appears in [1], [22], and [32].

[2] Hicks argues that, except for shipping, this was not true of the production technologies in use prior to the industrial revolution.

[3] The Financial Revolution is a term applied in [13] to the rapid development of English financial markets prior to 1750.

capita output and the rate of return to savings, and the equilibrium level of secondary capital market activity. The central focus of the analysis is on how these aspects of an equilibrium depend on the technical efficiency (or liquidity) of financial markets. In order to capture the notion of financial market efficiency (liquidity), we assume that financial market transactions are costly to undertake and implement. The measure of market liquidity used here will be the transaction costs of selling an asset. Such costs in actual markets include commissions and bid/ask spreads, as well as less easily quantified notions such as the time required for sale. Increases in the liquidity (technical efficiency) of financial markets have implications for (i) the equilibrium choice of production technology, (ii) per capita income and the per capita capital stock, (iii) the equilibrium rate of return on savings, (iv) the level of financial market activity, and (v) steady state welfare levels. We seek to investigate these implications. In doing so, we believe that ours is the first analysis in which all of these aspects of economic equilibrium are fully endogenous. Analyzing the equilibrium effects of low versus high transaction cost (liquid versus illiquid financial markets) represents assessing the importance of a successful financial sector to real economic performance.

The basic framework for investigating these issues is Diamond's [11] two period lived, overlapping generations model with production. We add two features to that model.

First, there are several technologies for converting current output into future capital. These technologies vary by gestation period (time to receiving capital in usable form) and productivity (the amount of capital ultimately received per unit invested). Individuals make an endogenous choice of which capital production technology will be employed. Second, financial transactions will be costly to undertake and implement. Since agents are two period lived, the use of technologies with long gestations requires agents to "roll over" capital investments in secondary capital markets (that is, in markets that transfer ownership of existing capital). This provides a link between the cost of transacting in these markets and the equilibrium choice of technology that is central to the determination of the per capita income level and capital stock, the real return to savings, the level of financial market activity, and steady state welfare.

Under the assumptions of proportional transactions costs and linear technologies for producing capital (the latter is obviously the assumption made in [11]), we establish the existence and uniqueness of a non-trivial steady state equilibrium under standard assumptions. We then investigate the consequences of improvements in the transactions technology for all aspects of this equilibrium.

The decisions of investors regarding which capital production technologies to use, and hence which investments to make, will be based on comparison of investment yields net of transactions costs. In equilibrium, the technologies in use will be those that generate the highest internal rate of return net of transactions costs. If transactions costs are high, long-gestation investments will be unattractive because they must be resold several times (that is, they are transactions intensive). Transactions cost reductions favor the use of longer maturity (more transactions intensive) investments. This observation provides the link between financial market technical efficiency, the choice of technology, and real activity which is the focus of this paper.

The results we obtain (for steady state equilibria) are as follows. First, a reduction in transactions costs necessarily raises the equilibrium return on savings. (That increases in financial market efficiency raise returns to savings is a theme of much of the literature on economic development. See, for instance, [15], [23], and [25].) Second, reductions in transactions costs favor the use of longer-lived (more transactions intensive) capital production technologies. They also necessarily raise the volume of financial market activity. Third, a reduction in transactions costs can either raise or lower the per capita capital stock and the level of per capita income. Fourth, if transactions cost reductions lead to a higher steady state capital stock, they necessarily raise steady state welfare – even if there was initially capital over-accumulation. If transactions cost reductions reduce the steady state capital stock, they may either raise or lower steady state welfare.

The possibility that a decline in transactions costs (an improvement in the transactions technology) can reduce the steady state per capita capital stock (and income level) presents an interesting contrast to the standard Diamond model. There, if there is a unique non-trivial steady state equilibrium, any technological improvement that does not impair the marginal productivity of either capital or labor must raise the (steady state) per person capital stock and income level. An improvement in the transactions technology (increased liquidity of secondary capital markets) can operate differently, however, for two reasons. First, as Hicks [20] emphasized, increases in the liquidity of capital resale markets affect the utilization of different technologies. In particular, reductions in transactions costs favor a movement toward the use of longer gestation capital production technologies. By implication, there is more use of secondary capital markets, and some savings is diverted from new capital investment to the purchase of already existing, but not yet mature capital. The proceeds of the sales of immature capital investments are, of course, simply consumed by their owners. Hence, as the use of longer gestation technologies increases, more of the savings of younger generations goes to finance the consumption of older generations, and there is less initiation of new capital investments. In addition, as more transactions are undertaken in these markets, more resources will be consumed in the transactions process. Both effects operate to reduce the steady state capital stock. When the two effects together outweigh the fact that the net of transactions cost productivity of each investment technology has increased, a reduction in transactions costs will act to reduce the steady state capital stock and income level.[4]

As an empirical matter, it does sometimes transpire that increases in the volume of financial market activity are associated with declines in the level of real activity. Indeed, some intentional attempts to stimulate financial market development in developing countries (through reductions in the perceived costs of transacting in

[4] Notice that, for the first part of this story to transpire as described, it is strictly necessary that a reduction in transactions costs leads to the increased use of longer gestation capital investments. If this does not occur, then our results establish that a reduction in transactions costs necessarily raises the steady state capital stock.

these markets) seem to have been counterproductive.[5] Our results offer some suggestions as to why this might be the case.

Before we proceed, it deserves emphasis that our focus on transactions costs – and transactions cost reductions – can be given several interpretations. One, of course, is simply that there is a transactions technology, and that we are investigating the consequences of varying (exogenously) its efficiency. However, other interpretations are also possible. For instance, a reduction in transactions costs might occur because agents find lower cost methods of transferring ownership of assets – for example by selling shares in capital rather than by transferring actual machines between individuals. Alternatively, transaction costs may decline due to an increase in competition among securities market makers or from an exogenous reduction in market makers' costs. Our analysis applies equally to each interpretation.

The remainder of the paper proceeds as follows. Section I describes the environment we consider and the nature of transactions. Section II defines and characterizes a steady state equilibrium and establishes conditions for the existence and uniqueness of a non-trivial steady state equilibrium. It also characterizes the level of activity in secondary capital markets. Section III considers the comparative static effects of changes in transactions costs. Section IV presents a fully worked out example, and Section V concludes.

Finally, all of the discussion in the present paper applies to steady state equilibria. Other work [5] shows how the analysis can be adapted to the context of an endogenous growth model. Most of the results obtained here for steady state output levels apply to equilibrium rates of growth in that model.

I. The Model

A. Environment

We consider a two period lived, overlapping generations economy with production. Time is indexed by $t = 1, 2, \ldots$, and at each date t a new young generation appears with N members. All young agents are identical,[6] being endowed with one unit of labor when young (which is supplied inelastically), and no labor when old. In addition, agents have no endowment of capital or consumption goods.[7]

There is a single consumption good at each date, which can either be eaten or converted into capital. Let $C_{it} \in \mathbb{R}_+$ denote period i consumption of a representative agent born at t. All agents have the common utility function $u(C_{1t}, C_{2t})$, with u being twice continuously differentiable, increasing in each argument, and strictly quasi-concave.

With respect to production, there is a commonly available constant returns to scale technology for converting labor and capital into the consumption good. In particular, a labor input of L_t and a capital input of K_t produces $F(K_t, L_t)$ units of

[5] For a discussion of some examples, see [16], [21] or [30]–[31].

[6] Allowing for heterogeneity creates no problems, but also does not introduce any additional substantive issues.

[7] Except for the initial old. Since our focus is on steady state equilibria, we omit a description of initial conditions.

consumption at t. We again assume that F is twice continuously differentiable, increasing in each argument, and strictly concave. Finally, we let $f(k_t)$ denote the intensive production function, with k_t being the time t capital-labor ratio. f is assumed to satisfy $f(0) = 0$ and the usual Inada conditions.

Thus far the model is identical to that of Diamond [11] in the absence of national debt. The difference we introduce is that there are assumed to be J different technologies available for converting consumption goods into capital. These technologies are indexed by $j = 1, 2, \ldots, J$, and differ as follows: one unit of the consumption good invested in the jth technology at t returns $R_j > 0$ units of capital at $t + j$. Thus capital technologies vary by gestation period (j), and productivity (R_j). The capital produced by any technology is perfectly substitutable as an input in final goods production: that is, K_t is the sum of the capital stocks available at t, however produced.

For technology j, then j describes the *maturity* of capital investments. Our analysis thus has an "Austrian" flavor, in that capital production technologies vary purely by productivity and gestation period, and capital investments are unproductive until they mature. It is possible to reproduce the analysis where all capital becomes productive in one period, but where different types of capital have different productive lifetimes. Such a model is somewhat more complicated than the one we analyze here and leads to no qualitative difference in results. Thus, we focus only on the simpler "Austrian" capital model.

Given this focus, if an investment technology with $j > 1$ is employed, our assumptions on agents' life cycles will force agents to sell capital goods in process on secondary capital markets. In particular, "capital in the pipeline" (CIP) is not productive until it matures, so this capital must be "rolled over" by investors from inception to maturity. Our interest is in examining how the liquidity of the secondary capital markets that accomplish this affects capital accumulation, national income, equilibrium returns, and the equilibrium choice of capital production technology (maturity of investments).

In order to capture the notion of the liquidity of secondary capital markets, we assume that there are transactions costs associated with trade in these markets. The level of transactions costs is inversely related to the liquidity of these markets. For simplicity, we assume a proportional transactions cost structure. In particular, one unit of CIP in technology j, that has been in place h periods (is $j - h$ periods from maturity), has a proportional transactions cost of $\alpha^{j,h}$. More specifically, a fraction $\alpha^{j,h} \in [0, 1)$ of the project is "used up" in the process of selling it h periods after inception.

Finally, we assume that when CIP matures, it is used in the production process and then depreciates completely. This assumption simplifies notation.

B. Trade

We assume that young agents at each date sell their labor to producers in a competitive labor market. One unit of labor earns the real wage rate w. (Since we confine attention to steady state equilibria, we omit time subscripts in what follows.) In addition, producers rent capital in a competitive rental market, paying the rental rate r. In order to keep the model as similar to [11] as possible, we assume that there are no transactions costs in factor markets. Thus the conventional factor pricing relationships obtain, and

$$w = f(k) - kf'(k) \equiv w(k) \tag{1}$$

$$r = f'(k). \tag{2}$$

After earning the wage income w, a young agent makes a savings decision and a set of portfolio choices. We represent the agent's possible portfolio strategies in the following way. Let $S^{j,h}$ denote the amount of type j capital that is h periods old acquired by a representative agent, measured in units of CIP.[8] Then, for example, $S^{j,0}$ is the amount of new investment in type j capital, while $S^{j,j-1}$ is the investment in type j capital that will mature in one period. Similarly, let $P^{j,h}$ denote the price (in units of current consumption) of one unit of CIP in technology j that is $j-h$ periods from maturity. Since agents initiate new projects with consumption goods, and since one unit of foregone consumption initiates one unit of new CIP in technology j, $P^{j,0} = 1\, \forall j$. In addition, mature capital is simply rented in factor markets. As one unit of CIP in technology j produces R_j unit of rentable capital on maturity, $P^{j,j} = rR_j$; i.e., $P^{j,j}$ is just the rental value of mature capital. For $j > 1$ and $0 < h < j$, $P^{j,h}$ will have to be determined.

We assume (without loss of generality) that transactions costs are borne by sellers of CIP. Thus a young agent chooses a vector of consumption levels (c_1, c_2), and a matrix of capital investment choices $(S^{j,h})$ to maximize $u(C_1, C_2)$, subject to

$$C_1 + \sum_{j=1}^{J} \sum_{h=0}^{j-1} P^{j,h} S^{j,h} \leq w \tag{3}$$

$$C_2 \leq \sum_{j=1}^{J} \sum_{h=0}^{j-1} P^{j,h+1} S^{j,h} (1 - \alpha^{j,h+1}), \tag{4}$$

and non-negativity.

It should be evident that, if technology j is in use in a steady state equilibrium, then $S^{j,h} > 0$ must hold for some agent, for all $h = 0, 1, \ldots, j-1$. This obviously requires that the return to holding technology j CIP be equated for all possible times to maturity; i.e.,

$$(1 - \alpha^{j,h+1}) P^{j,h+1}/P^{j,h} = (1 - \alpha^{j,h}) P^{j,h}/P^{j,h-1}, \forall h = 1, \ldots, j-1. \tag{5}$$

Similarly, if technologies j and ℓ are in use at all dates, then

$$(1 - \alpha^{j,h+1}) P^{j,h+1}/P^{j,h} = (1 - \alpha^{\ell,m+1}) P^{\ell,m+1}/P^{\ell,m};$$
$$\forall h = 0, \ldots, j-1, \forall m = 0, \ldots, \ell - 1. \tag{6}$$

Let γ denote this common (gross) rate of return. Then from (5), if technology j is

[8] Recall that one unit of consumption foregone at t and invested in technology j becomes one unit of technology j CIP.

in use,

$$P^{j,h+1}/P^{j,h} = \gamma/(1-\alpha^{j,h+1}); \forall h = 0, \ldots, j-1. \tag{7}$$

When rates of return on all capital investments in use are equated, obviously young agents are individually indifferent regarding portfolio composition. Hence for each young agent only the real value of savings $\tilde{S} \equiv \sum_j \sum_h P^{j,h} S^{j,h}$ is determinate, and

$$\tilde{S} \equiv \operatorname{argmax} u(w_t - \tilde{S}, \gamma\tilde{S}) \equiv s(w, \gamma). \tag{8}$$

We assume throughout that $s_1(w,\gamma) \geq 0$ and $s_2(w,\gamma) \geq 0$, so that savings is non-decreasing both in income and the rate of return.

For future reference, it will be useful to have a notation for the fraction of savings – in real terms – that is held in type j CIP that is h periods old. Denoting this fraction by $\theta^{j,h}$, we have that

$$\theta^{j,h} \equiv P^{j,h} S^{j,h}/\tilde{S}.$$

Apparently, then, $\theta^{j,h} \geq 0$ and

$$\sum_{j=1}^{J} \sum_{h=0}^{j=1} \theta^{j,h} = 1 \tag{9}$$

must hold.

II. Stationary equilibrium

Individuals have two kinds of investment decisions to make in this model: how much to save, and what capital investment technologies to use. (While individuals may be indifferent about the latter choice in equilibrium, in the aggregate the mix of technologies in use will be determinate.) An equilibrium is fully characterized by the level of the capital stock and its maturity composition. We begin with the issue of maturity composition.

Recall that

$$\begin{aligned} P^{j,j} &= rR_j \\ P^{j,0} &= 1, \end{aligned} \tag{10}$$

and write

$$P^{j,j} = P^{j,0} \prod_{h=0}^{j-1} (P^{j,h+1}/P^{j,h}). \tag{11}$$

Then if technology j is in use, substitution of (7) and (10) into (11) yields

$$rR_j = (\gamma)^j \left[\prod_{h=0}^{j-1} (1-\alpha^{j,h+1}) \right]^{-1}, \tag{12}$$

where $(\gamma)^j$ denotes (the stationary value of) γ raised to the jth power. Define

$$\tilde{R}_j \equiv R_j \prod_{h=0}^{j-1} (1-\alpha^{j,h+1}). \tag{13}$$

Therefore, from (12), if technology j is in use,

$$\gamma = (r\tilde{R}_j)^{1/j} = [f'(k)\tilde{R}_j]^{1/j}, \tag{14}$$

where k is the steady state capital–labor ratio.

Evidently, if j^* is an equilibrium choice of project length,

$$[f'(k)\tilde{R}_{j^*}]^{1/j^*} \geq [f'(k)\tilde{R}_j]^{1/j} \forall j \tag{15}$$

must hold. That is, the equilibrium choice of capital investment technology will be the one which maximizes the internal rate of return (given the rental rate on capital) on investments. Let

$$M(k) \equiv \{j | j = 1, \ldots, J; j \text{ maximizes } [f'(k)\tilde{R}_j]^{1/j}\}. \tag{16}$$

As j^* need not be unique, $M(k)$ gives the set of return maximizing project lengths as a function of the capital stock. These are the project lengths in use when the capital stock is k.

The equilibrium capital–labor ratio is determined as follows. For each project length in use $[j^* \in M(k)]$, $\theta^{j^*,0}s(w,\gamma)$ new projects are initiated at each date. Each such new project yields $\tilde{R}_{j^*}$ (net of transactions costs) units of capital j^* periods later. Thus the capital stock at each date is the sum of maturing projects;

$$k = \sum_{j^* \in M(k)} \tilde{R}_{j^*} \theta^{j^*,0} s(w,\gamma), \tag{17}$$

since only technologies in $M(k)$ are in use.

Finally, $\forall j^* \in M(k)$, the market in CIP must clear for $h = 1, \ldots, j^* - 1$. The time t demand for j^* period projects with $j^* - h$ periods to maturity (measured in units of CIP) is $\theta^{j^*,h}s(w,\gamma)/P^{j^*,h}$. The supply of such projects, again (measured in units of CIP) is

$$\theta^{j^*,0}s(w,\gamma)\prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1})/P^{j^*,0} = \theta^{j^*,0}s(w,\gamma)\prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1}),$$

since $1 - \prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1})$ of the initial investment has been consumed by the transactions technology h periods after initiation. Thus the market for CIP clears if

$$\theta^{j^*,h}s(w,\gamma) = P^{j^*,h}\theta^{j^*,0}s(w,\gamma)\prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1}) \tag{18}$$

holds $\forall j^* \in M(k)$, $\forall h = 1, \ldots, j^* - 1$. In addition, (9) becomes

$$\sum_{j^* \in M(k)} \sum_{h=0}^{j^*-1} \theta^{j^*,h} = 1. \tag{19}$$

Equations (17)–(19) and $j^* \in M(k)$ constitute the steady state equilibrium conditions.

A. Characterization of equilibrium

Using equation (10) and

$$P^{j,h} = P^{j,0}\prod_{\ell=0}^{h-1}(P^{j,\ell+1}/P^{j,\ell})$$

in (18) gives

$$\theta^{j^*,h} = \theta^{j^*,0} \prod_{\ell=0}^{h-1} [P^{j^*,\ell+1}(1-\alpha^{j^*,\ell+1})/P^{j^*,\ell}] \equiv \theta^{j^*,0}(\gamma)^h,$$

$$\forall j^* \in M(k), \forall h = 0, \ldots, j^*-1. \quad (20)$$

Therefore (19) can be written as

$$\sum_{j^* \in M(k)} \sum_{h=0}^{j^*-1} \theta^{j^*,h} = \sum_{j^* \in M(k)} \theta^{j^*,0} \sum_{h=0}^{j^*-1} (\gamma)^h = 1. \quad (21)$$

Furthermore, since for all γ

$$\sum_{h=0}^{j^*-1} (\gamma)^h = (1-\gamma^{j^*})/(1-\gamma) = [1-\tilde{R}_{j^*}f'(k)]/\{1-[\tilde{R}_{j^*}f'(k)]^{1/j^*}\}$$

holds, (21) is equivalent to

$$\sum_{j^* \in M(k)} \theta^{j^*,0}[1-\tilde{R}_{j^*}f'(k)]/\{1-[\tilde{R}_{j^*}f'(k)]^{1/j^*}\} = 1. \quad (21')$$

Equation (17) defines a self-reproducing capital stock. In order to characterize a solution to (17), it is useful to proceed as follows. Suppose that we exogenously impose that (only) technology j is in use. Then (21′) reduces to $\theta^{j,0} = \{1-[\tilde{R}_j f'(k)]^{1/j}\}/[1-\tilde{R}_j f'(k)]$. Substituting this expression, along with (1) and (14), into (17) would then yield

$$k = \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}][1-(\tilde{R}_j f'(k))^{1/j}]/[1-\tilde{R}_j f'(k)] \equiv G_j(k). \quad (22)$$

The function $G_j(k)$ describes how much capital is obtained (given the stationarity of allocations), after j periods, starting from a capital stock of k, if technology j is in use. Or, in other words, $G_j(k)$ describes how much capital is provided by the combination of savings decisions and financial market decisions if (only) technology j is in use and the current capital stock is k.

Evidently, (17) can now be written as

$$k = \sum_{j^* \in M(k)} \theta^{j^*,0}([1-\tilde{R}_{j^*}f'(k)]/\{1-[\tilde{R}_{j^*}f'(k)]^{1/j^*}\})G_{j^*}(k). \quad (23)$$

In view of (21′) and $\theta^{j^*,0}[1-\tilde{R}_{j^*}f'(k)]/\{1-[\tilde{R}_{j^*}f'(k)]^{1/j^*}\} \geq 0$, (23) asserts that the equilibrium per capita capital stock k equals a convex combination of the values $G_{j^*}(k)$. In order to express (23) more compactly, we define the correspondence $G(k)$ by

$$G(k) \equiv CH\{G_{j^*}(k) | j^* \in M(k)\}, \quad (24)$$

where CH denotes convex hull. Then (23) can be written as

$$k \in G(k). \quad (25)$$

Equation (25) is now the sole equilibrium condition.

It will evidently be useful to know more about the correspondence $G(k)$. First, since each $G_j(k)$ is a continuous function, $G(k)$ is point-valued if $M(k)$ consists of a single element. Second, from (24) it is apparent that G is upper hemi-continuous and convex-valued. Third, it is straight-forward to show that k can be bounded by some finite value k_{max}. Thus, by Kakutani's fixed point theorem an equilibrium exists satisfying $k^* \in G(k^*)$. However, since $0 \in G(0)$, we would like to provide conditions under which a non-trivial stationary equilibrium capital–labor ratio ($k^* > 0$) exists. We now turn our attention to this problem, which requires a sharper characterization of the correspondence G.

In order to provide such a characterization, we begin by stating four lemmas. The proof of each appears in the appendix.

Lemma 1. (a) *Suppose that $j, \ell \in M(\hat{k})$ for some $\hat{k}$, and that $j > \ell$. Then there exist values $\varepsilon_1, \varepsilon_2 > 0$ such that $\{\ell\} = M(k) \forall k \in (\hat{k} - \varepsilon_1, \hat{k})$ and $\{j\} = M(k) \forall k \in (\hat{k}, \hat{k} + \varepsilon_2)$.* (b) *If $j > \ell$ and $j \in M(k')$ for some $k' > 0$, then $\ell \notin M(k)$ for any $k > k'$.*

Lemma 1 asserts that $M(k)$ has more than one element only at (finitely many) isolated points, and hence that $G(k)$ fails to be singleton-valued only at those same points. When $G(k)$ is set-valued, of course, this occurs because there is more than one choice of capital production technology that maximizes the internal rate of return on investments. By lemma one, there are no more than $J-1$ values of k at which the choice of capital production technology fails to be unique. In addition, (loosely speaking) the lemma implies that the equilibrium maturity of capital investments is nondecreasing in k. This result obtains because, as the rental rate r declines, it becomes less painful to delay the receipt of the proceeds from an investment. Hence as r falls, the gestation period that maximizes the internal rate of return rises. It follows that the equilibrium choice of project length is non-decreasing in k.

Lemma 2. *For k sufficiently near zero, $\{1\} = M(k)$. For k sufficiently large, $\{J\} = M(k)$.*

Lemma 2 implies that when k is small (when r is large), agents will wish to receive the return on capital investments at the earliest possible date. As $k \to \infty$ ($r \to 0$), agents are happy to delay the return on capital investments as long as possible.

Lemma 3. *Let $j, \ell \in M(\hat{k})$ for some $\hat{k}$ and let $j > \ell$. Then*

$$G_j(\hat{k}) < G_\ell(\hat{k}) \tag{26}$$

Lemma 3 establishes that, if technologies j and ℓ both maximize the internal rate of return to investments when the capital stock is $\hat{k}$ [and the rental rate is $f'(\hat{k})$], and if $j > \ell$, then $\tilde{R}_\ell \theta^{\ell,0} > \tilde{R}_j \theta^{j,0}$. Hence, from equation (17), less total capital is received when technology j is in use than is received when technology ℓ is in use. Or, in other words, the use of the longer-gestation technology results in less capital formation than the use of the shorter-gestation technology. This is true (potentially) for two reasons. First, the use of longer-gestation technologies (for a given k) results in more saving being absorbed by purchases of existing CIP, and hence in less initiation of new capital investment. Second, the use of longer gestation technologies (again for a given k) may result in an increase in the total quantity of transactions costs borne. Both effects operate to reduce capital formation.[9]

[9] As technologies with gestation periods in excess of one are employed, it is necessary for agents to trade claims to CIP. The purchase of existing CIP from old agents absorbs some of the savings generated by young agents. This, in turn, acts to reduce new capital formation, other things equal.

Parenthetically, the implication that the purchase of existing assets absorbs some savings and potentially reduces capital formation is present in a variety of models. Three examples include [14], [26], and [27].

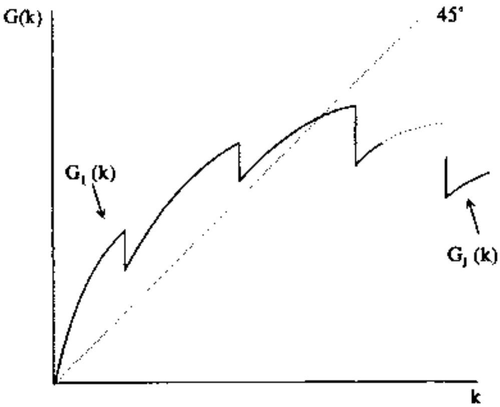

Figure 1.

Lemma 4. *For each* $j = 1, \ldots, J$,

$$\lim_{k \to \infty} G_j(k)/k = 0.$$

Lemma 4 asserts that each function $G_j(k)$ eventually lies below the 45° line in Figure 1, and hence that $G_J(k)$ does.[10]

Suppose we now assume that[11]

$$\lim_{k \downarrow 0} G_1(k)/k > 1. \tag{a.1}$$

Assumption (a.1) implies that, as $k \downarrow 0$, $G_1(k)$ [and hence $G(k)$] lies above the 45° line, and hence (a.1) and Lemmas 1–4 imply that the correspondence $G(k)$ has the configuration depicted in Figure 1.

Evidently, then (a.1) is sufficient to imply the existence of a non-trivial steady state equilibrium per capita capital stock. It would further appear, say from Figure 2, that there is considerable scope for the existence of multiple stationary equilibria. In fact, this is less the case than might be suspected. We now state

Proposition 1. *Suppose that*

$$s(w, \gamma)/w \text{ is non-increasing in } w, \tag{a.2}$$

$$f \text{ displays an elasticity of substitution, denoted } \sigma, \text{ satisfying } \sigma \geq 1. \tag{a.3}$$

(a.1)–(a.3) *imply the existence of a unique, non-trivial stationary equilibrium.*

---

[10] Lemmas 1–4 seem to exhaust the general statements that can be made about the properties of the correspondence G. In particular, it is not generally possible to state "how many segments" will comprise G, or which technologies may be "missing from" G. For example, if $\tilde{R}_{j+2}/R_{j+1} < \tilde{R}_{j+1}/\tilde{R}_j$ holds for all $j = 1, \ldots, J-2$, then for each j there exists a value k_j such that $j \in M(k_j)$. On the other hand, if $\tilde{R}_{j+2}/\tilde{R}_{j+1} > \tilde{R}_{j+1}/\tilde{R}_j$ holds for all $j = 1, \ldots, J-2$, then it is possible to show that $1 \in M(k)$ iff $f'(k) \in [(\tilde{R}_J)^{1/(J-1)}/(\tilde{R}_1)^{J/(J-1)}, \infty)$, while $J \in M(k)$ otherwise. No other technologies can maximize the internal rate of return on investment. These comments indicate that G can have as few as 2 or as many as J "segments."

[11] Observe that (a.1) is the standard assumption made to guarantee the existence of a non-trivial stationary equilibrium in [11]. See, for example, the discussion in [2].

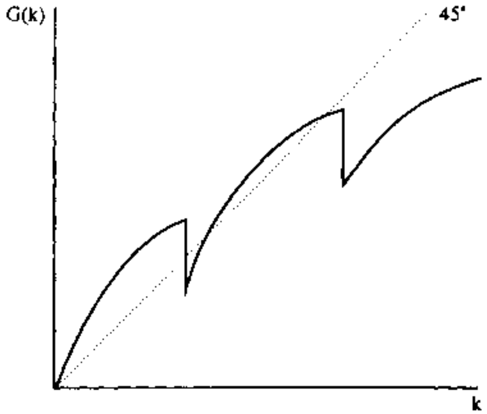

Figure 2.

The proof appears in the appendix. (a.2) holds if the income elasticity of savings does not exceed unity.

The determination of an equilibrium capital–labor ratio is depicted in Figure 1. By Lemma 3, for k near zero, the only maturity in use is $j = 1$ and $G(k)$ lies above the 45° line [by (a.1)]. As k increases, there will be values $\hat{k}$ that are switch points between maturities (say ℓ, j). At these, $G(\hat{k})$ is set-valued as capital investments are allocated between technologies ℓ and j. As k increases beyond $\hat{k}$, the equilibrium choice of investment maturity lengthens and $G(\cdot)$ shifts downward since, in particular, we have $G_j(\hat{k}) < G_\ell(\hat{k})$. Thus G is a sequence of continuous segments, linked by vertical declines associated with the increasing maturity of capital investments. Finally, for k sufficiently large, $G(k)$ lies below the 45° line (by Lemma 4). A steady state equilibrium then occurs where $G(k)$ intersects the 45° line.

The multiplicity of (non-trivial) steady state equilibria depicted in Figure 2 is ruled out by Assumptions (a.2) and (a.3). In particular, those assumptions imply that none of the functions G_j intersect the 45° line from below. Thus, by Lemma 3, the correspondence G cannot cross the 45° line from beneath, and there must exist a unique (non-trivial) steady state equilibrium.

B. Secondary market transactions

The value of per capita purchases in secondary capital markets, measured in units of current consumption, is just per capita savings by young agents less per capita investment in new projects. Let $\rho(k^*)$ denote the real value of these purchases in equilibrium. Then

$$\rho(k^*) = s(w,\gamma) - \sum_{j^*} \theta^{j^*,0} s(w,\gamma) = s(w,\gamma)\left[1 - \sum_{j^*} \theta^{j^*,0}\right] \geq 0 \tag{27}$$

where the last inequality is strict if $\{1\} \neq M(k^*)$. From (21),

$$\sum_{j^* \in M(k)} \theta^{j^*,0}[1-(\gamma)^{j^*}]/(1-\gamma) = 1,$$

which, upon rearranging terms, yields

$$\sum_{j^*\in M(k)} \theta^{j^*,0} = 1 - \gamma + \sum_{j^*\in M(k)} \theta^{j^*,0}(\gamma)^{j^*} \tag{28}$$

Multiplying both sides of (17) by $f'(k^*)$ and using (14), one obtains

$$\sum_{j^*\in M(k)} \theta^{j^*,0}(\gamma)^{j^*} = kf'(k)/s(w,\gamma).$$

Using the latter two expressions in (27) implies that

$$\rho(k^*) = \gamma s(w,\gamma) - rk^* = [\tilde{R}_{j^*}f'(k^*)]^{1/j^*}s\{w(k^*),[\tilde{R}_{j^*}f'(k^*)]^{1/j^*}\} - k^*f'(k^*). \tag{29}$$

Equation (29) describes how the volume of secondary market transactions depends on the equilibrium capital stock (k^*), as well as its maturity composition (j^*). We will be interested in examining how the liquidity of secondary capital markets and the volume of secondary market activity are related.

III. The comparative statics of changes in transactions costs

We now wish to investigate how reductions in transactions costs (which can be regarded as increasing the liquidity of secondary capital markets)[12] affect (i) the steady state equilibrium levels of the capital stock, per capita income, and the return on savings, (ii) steady state welfare, and (iii) the level of secondary capital market activity. Unambiguous results along these dimensions require that there be a unique non-trivial steady state equilibrium: therefore for the remainder of the section we impose the Assumptions (a.2) and (a.3). In addition, it will be useful to have a single parameter which controls the transactions cost structure. We therefore assume that

$$\tilde{R}_j = \tilde{R}_j(\beta); \quad j = 1, \ldots, J. \tag{a.4}$$

β is a scalar parameter; for concreteness we assume that increases in β represent reductions in transactions costs. Thus $\tilde{R}'_j(\beta) \geq 0\ \forall j$.

Definitive results on the consequences of a change in the transactions cost parameter require some assumptions on the functions $\tilde{R}_j(\beta)$. First, since there are no transactions costs associated with one period length projects we assume that, $\forall\beta$,

$$\tilde{R}'_1(\beta) = 0. \tag{a.5}$$

Second, since longer-lived projects involve more transactions than shorter-lived projects, we assume that a change in β results in a larger proportional reduction in total transactions costs for projects of longer maturities than for projects of shorter

[12] Recall that reductions in transactions costs can occur either because the "transactions technology" becomes more efficient or because society discovers new methods of transferring ownership of capital.

maturities.[13] Our specific technical assumption is that, $\forall \ell, j$,

$$\tilde{R}_j'(\beta)/j\tilde{R}_j(\beta) > \tilde{R}_\ell'(\beta)/\ell\tilde{R}_\ell(\beta) \tag{a.6}$$

whenever $j > \ell$. This assumption is satisfied by some obvious transactions cost structures. For instance, suppose that $\alpha^{j,0} = \alpha^{j,j} = 0$ holds $\forall j$, and that $\alpha^{j,h} = \alpha \in [0,1]\ \forall j,\ \forall h = 1, \ldots, j-1$. Then $\beta = 1 - \alpha$ governs the transactions cost structure, and

$$\tilde{R}_j(\beta) = R_j\beta^{j-1}; \quad j = 1, \ldots, J. \tag{30}$$

These functions satisfy (a.6). In addition, if $J = 2$ and $\tilde{R}_2' > 0$, (a.5) implies (a.6). This observation is relevant to the example in Section IV, where (a.6) necessarily holds.

A. Capital stock, income, and returns

There are two ingredients in investigating how a change in β affects the steady state equilibrium capital stock: (a) the effect of a change in β on the correspondence $G(k)$ if $G(k)$ is point-valued [i.e., if $M(k)$ is singleton-valued], and (b) the effect if $G(k)$ is not point-valued. We begin with case (a).

If $G(k)$ is point-valued, then there exists a $j \in M(k)$ such that $\{G_j(k)\} = G(k)$. The following lemma then establishes how $G(k)$ is affected by a change in β:

Lemma 5. *An increase in $\tilde{R}_j$ increases $G_j(k)$, $\forall j, \forall k$.*

Lemma 5 is proved in the appendix. It asserts the unsurprising result that – if there are no changes in the (unique) equilibrium choice of capital investment technology – then a reduction in transactions costs results in more net capital formation. More formally, then, if $\{j\} = M(k)$ and $\tilde{R}_j'(\beta) > 0$, an increase in β shifts $G(k)$ upwards in Figure 3.

It now remains to describe the effect of a change in β on the correspondence G when $G(k)$ is *not* singleton-valued. If $G(k)$ consists of two or more elements, then there exist values ℓ and j with $\{\ell, j\} \subset M(k)$. In this event define the capital–labor ratio $\hat{k}_{\ell,j}$ by

$$\{\ell, j\} \in M(\hat{k}_{\ell,j}) \tag{31}$$

and $0 < \hat{k}_{\ell,j} < \infty$. Thus $\hat{k}_{\ell,j}$ is that capital–labor ratio which yields ℓ and j as equilibrium project lengths. Lemma 1 implies that for any pair (ℓ, j), there exists at most one value $\hat{k}_{\ell,j}$ satisfying (31): if one exists it is defined by

$$[\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j} = [\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}. \tag{32}$$

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[13] The idea that transactions costs are most significant for assets of long maturity has a genuine empirical basis. For example, the Wall Street Journal of July 23, 1993, reported a bid/ask spread on three-month treasury bills of the previous day equal to 0.005 percent of price. For a thirty-year treasury bond, the bid/ask spread was 0.062 percent of price, while for a thirty-year treasury strip (a pure discount instrument, equivalent to a long-term bill), the bid/ask spread was 0.7 percent of price. Thus, these bid/ask spreads vary by a factor of 100 with maturity alone. Moreover, this observation abstracts from the obvious fact that a typical long-term instrument will be rolled-over many more times than a typical short-term instrument during its lifetime.

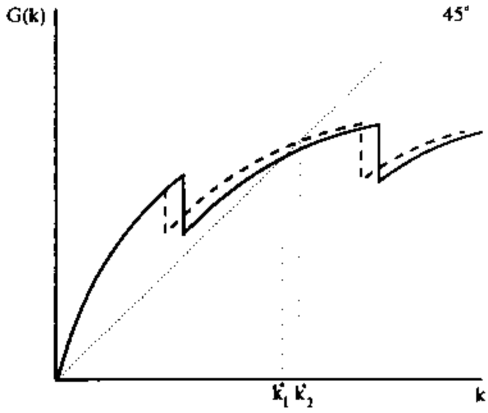

Figure 3. A reduction in transactions costs.

The following lemma then states how $G(k)$ is affected by a change in β if $G(k)$ is *not* singleton-valued:

Lemma 6. *Suppose that* $\{\ell, j\} \subset M(\hat{k}_{\ell,j})$ *and that* $j > \ell$. *Then* $d\hat{k}_{\ell,j}/d\beta < 0$.

Lemma 6 is proved in the appendix. It asserts that a reduction in transactions costs reduces $\hat{k}_{\ell,j}$, for all ℓ, j such that $\hat{k}_{\ell,j}$ exists.

Why does this reduction in $\hat{k}_{\ell,j}$ occur? Recall that $\hat{k}_{\ell,j}$ is the capital–labor ratio that results in technologies ℓ and j having identical internal rates of return (*and* internal rates of return in excess of those generated by other capital production technologies). Assumption (a.6) implies that a reduction in transactions costs, ceteris paribus, raises the internal rate of return on long-gestation relative to short-gestation capital investments. In order to restore the equality of internal rates of return on investments in technologies ℓ and j, then the rental rate on capital must rise (in order to increase the "impatience" of investors to realize investment returns), or in other words $\hat{k}_{\ell,j}$ must fall as β is increased.[14]

Lemmas 5 and 6 now allow us to infer how a change in transactions costs affects the entire correspondence G. In Figure 3 the solid (dashed) locus represents a high (low) transactions cost economy. A change in transactions costs does not affect the correspondence $G(k)$ when $\{1\} = M(k)$, since by assumption $\tilde{R}_1'(\beta) = 0$. If $\{j\} = M(k)$ for some $j = 2, \ldots, J$, then $\tilde{R}_j'(\beta) > 0$, and a reduction in transactions costs shifts $G(k)$ upwards. Finally, if $\{\ell, j\} = M(\hat{k}_{\ell,j})$ for some ℓ, j and $j > \ell$, then $G(\hat{k}_{\ell,j})$ is a vertical segment. Lemma 6 implies that this vertical segment shifts to the left with an increase in β, as shown in the figure.

As is apparent from Figures 3, 4, and 5, a reduction in transactions costs (an increase in β) has an ambiguous effect on the steady state per capita capital stock (and

[14] It bears emphasis that $\hat{k}_{\ell,j}$ is determined purely by properties of the production and transactions technologies. It does not depend on the composition of savings.

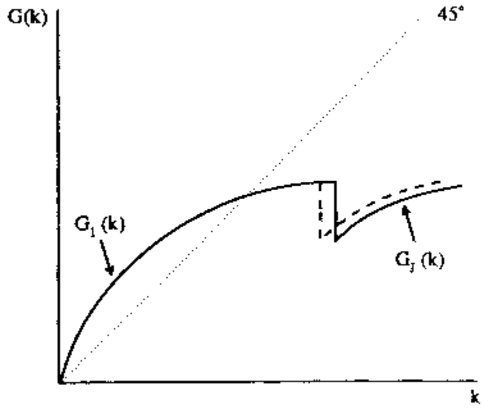

Figure 4. Reductions in transactions costs.

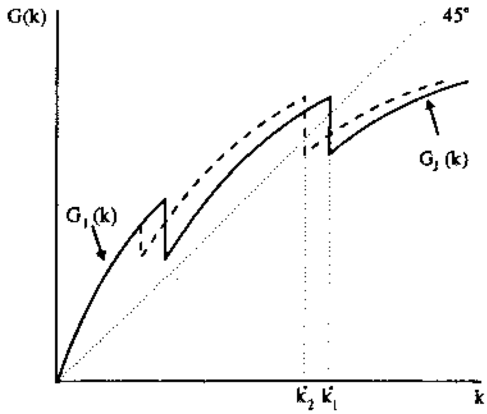

Figure 5. Reductions in transactions costs.

hence on per capita income). There are three general possibilities for a "small"change in β.[15]

Case 1. $\{j^*\} = M(k)$ both before and after the change in transactions costs, and $j^* > 1$. This situation is depicted in Figure 3. Evidently here a reduction in transactions costs leads to a higher steady state capital stock; this corresponds to the event that an increase in financial market activity is associated with capital deepening. Obviously this situation obtains whenever there is no change in the equilibrium choice of capital production technology (and whenever this choice is unique). Thus, if there is no change in the composition of technologies in use as a result of an increase in financial market efficiency, the consequence must be a higher steady state capital stock and output level.

[15] Changes in β that are not "local" in nature are considered in Section IV.

Case 2. $\{\ell, j\} \subset M(k)$ both before and after the change in β. This case is depicted in Figure 4. Here a reduction in transactions costs actually causes a reduction in the steady state capital stock. We establish below that this situation occurs when a reduction in transactions costs simply results in a portfolio shift from less to more transactions (and transactions cost) intensive investments, without channeling additional resources into capital accumulation. In other words, when multiple capital productions technologies are in use (in equilibrium), a reduction in transactions costs will cause the longer gestation technology to be used more intensively. The result is that more savings are absorbed by purchases of existing CIP, and less new capital investment is initiated. As a result the steady state capital stock declines.

Case 3. $\{1\} = M(k)$, both before and after the change in transactions costs. This event is depicted in Figure 5; evidently a change in β has no effect on the steady state capital stock. This situation arises when secondary capital markets are undeveloped, so that no transactions costs are incurred in any event.

The possibility, illustrated in Case 2, that an increase in financial market activity need not lead to capital deepening is more than simply a theoretical curiosity. It is argued in [23] and [25], for instance, that an increase in financial market efficiency should generally lead to increased investment and capital formation. However, in practice, attempts to stimulate the growth of financial markets in developing countries have met with mixed success. It is even asserted in [7], [12], and [29]–[31] that such attempts have often been detrimental to capital accumulation. This is, in fact, what occurs in Figure 4, where improvements in the liquidity of secondary capital markets simply increase activity in those markets (see below) without resulting in capital deepening. Thus, our analysis suggests why financial deepening need not lead to higher levels of economic developments.

It remains to consider the consequences for the equilibrium return to savings of a change in β. As before, if the change in β is not large enough to affect $M(k)$ in equilibrium, there are three possibilities. We consider each in turn.

Case 1. $\{j^*\} = M(k), j^* > 1$. In this case the equilibrium (gross) rate of return on savings is given by $[\tilde{R}_{j^*}(\beta) f'(k)]^{1/j^*}$. We now state

Lemma 7. *Suppose that* $\{j^*\} = M(k)$. *Then an increase in* β *(weakly) increases* $[\tilde{R}_{j^*}(\beta) f'(k)]^{1/j^*}$.

Lemma 7 is proved in the appendix. Thus, in this case, an increase in β does not reduce the steady state equilibrium rate of return. This return will rise if $\sigma > 1$ holds, or if the income elasticity of savings is strictly less than one.

Case 2. $\{\ell, j\} \subset M(k)$. In this event an increase in β reduces k (which equals $\hat{k}_{\ell,j}$). The equilibrium return on saving is given by $[\tilde{R}_\ell(\beta) f'(\hat{k}_{\ell,j})]^{1/\ell} = [\tilde{R}_j(\beta) f'(\hat{k}_{\ell,j})]^{1/j}$. Since at least one of the values $\tilde{R}_\ell(\beta)$ and $\tilde{R}_j(\beta)$ must rise, the return on savings does as well.

Case 3. $\{1\} = M(k)$. In this situation a change in β affects neither $\tilde{R}_1$ nor k. Thus the equilibrium rate of return, which equals $\tilde{R}_1 f'(k)$, is unaffected by β.

To summarize:

Proposition 2. *The steady state equilibrium rate of return is not reduced by a reduction in transactions costs; it is increased if $M(k)$ is not singleton-valued, or if $\{1\} \neq M(k)$ and either* (a.2) *or* (a.3) *holds as a strict inequality.*

The result that an increase in financial market efficiency raises the return on savings is widely asserted in the development literature; for instance in [17], [23], or [25].

B. Welfare

In this section we consider how a small change in β [small enough to leave $M(k)$ unaffected] affects the steady state equilibrium level of agents' utilities. As before there are three possibilities.

Case 1. $\{j^*\} = M(k), j^* > 1$. Here Lemma 7 implies that $\gamma = [\tilde{R}_{j^*}(\beta)f'(k)]^{1/j^*}$ rises (weakly) with a reduction in transactions costs. Furthermore, as in Figure 3, the steady state capital stock rises, and so therefore does the real wage rate $w(k)$. Thus, by the envelope theorem, the steady state welfare level $u\{w(k) - s[w(k), \gamma],\ \gamma s[w(k), \gamma]\}$ necessarily increases when transactions costs are reduced.

Case 2. $\{\ell, j\} \subset M(k), j > \ell$. In this case, as before, the steady state welfare level is given by $U = u\{w(\hat{k}_{\ell,j}) - s[w(\hat{k}_{\ell,j}), \gamma],\ \gamma s[w(\hat{k}_{\ell,j}), \gamma]\}$ since $\hat{k}_{\ell,j}$ is the steady state capital–labor ratio. Then, by the envelope theorem,

$$dU/d\beta = u_1(-)\{w'(\hat{k}_{\ell,j})d\hat{k}_{\ell,j}/d\beta + [s(-)/\gamma]d\gamma/d\beta\}. \tag{33}$$

Moreover, $\hat{k}_{\ell,j}$ satisfies (31), so that

$$f'(\hat{k}_{\ell,j}) \equiv [\tilde{R}_j(\beta)]^{\ell/(j-\ell)}/[\tilde{R}_\ell(\beta)]^{j/(j-\ell)}. \tag{34}$$

Differentiating (34) and using $w'(k) = -kf''(k)$ yields

$$w'(\hat{k}_{\ell,j})d\hat{k}_{\ell,j}/d\beta = -kf'(k)\{[\tilde{R}'_j(\beta)/j\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\ell\tilde{R}_\ell(\beta)]\}[j\ell/(j-\ell)]. \tag{35}$$

In addition, γ is given by

$$\gamma = [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}. \tag{36}$$

Substituting (34) into (36) gives

$$\gamma = [\tilde{R}_j(\beta)/\tilde{R}_\ell(\beta)]^{1/(j-\ell)}. \tag{37}$$

From (37) it follows that

$$d\gamma/d\beta = \gamma\{[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\}/(j-\ell). \tag{38}$$

Equations (33), (35), and (38) imply that $dU/d\beta \leq 0$ holds iff

$$\begin{aligned} &kf'(k)\{\ell[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - j[\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\} \\ &\geq s[w(k), \gamma]\{[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\} \end{aligned} \tag{39}$$

Equation (39) can easily be satisfied. For instance

Lemma 8. *Suppose that*

$$\tilde{R}'_j(\beta)/(j-1)\tilde{R}_j(\beta) \geq \tilde{R}'_\ell(\beta)/(\ell-1)\tilde{R}_\ell(\beta) \tag{40}$$

whenever $j > \ell$, and that

$$kf'(k)/w(k) \geq s[w(k), \gamma]/w(k) \tag{41}$$

always holds. Then $dU/d\beta \leq 0$.

The lemma follows from straight-forward algebraic manipulation.

Equation (40) will be satisfied for some obvious transactions cost structures [such as that displayed in equation (30)]. Equation (41) will hold whenever the ratio of capital's share to labor's share is at least as great as the savings rate of young agents. An example satisfying these conditions is given in Section IV. Thus it can easily happen that reductions in transactions costs result in reductions in steady state welfare. When this transpires, of course, steady state welfare declines because of the reduction in the real wage rate associated with the fall in the steady state capital stock.

Case 3. $\{1\} = M(k)$. In this case a reduction in transactions costs affects neither k nor γ, and consequently has no welfare effects.

In summary, just as a reduction in transactions costs can have ambiguous consequences for the steady state equilibrium capital stock, it can have ambiguous consequences for steady state welfare. It therefore seems appropriate to develop a criterion for determining when an increase in β will reduce the capital stock (and possibly welfare). We now consider this issue.

C. Secondary market transactions

The level of secondary market activity, in real terms, is described by equations (27) and (29). Evidently, if $M(k^*) = \{1\}$, $\rho(k^*) = 0$. Thus for the remainder of this section we consider only the case where $\{1\} \neq M(k^*)$.

Our objective is to describe how the real volume of secondary market transactions is affected by changes in transactions costs. As before, we will consider only small enough changes in β to leave $M(k^*)$ unaffected. Again the analysis involves two possibilities.

Case 1. $\{j^*\} = M(k^*), j^* > 1$. In this case we can use the equilibrium condition

$$k^* = G_{j^*}(k^*) = \tilde{R}_{j^*}s[w(k^*), (\tilde{R}_{j^*}f'(k^*))^{1/j^*}][1 - (\tilde{R}_{j^*}f'(k^*))^{1/j^*}]/[1 - \tilde{R}_{j^*}f'(k^*)] \tag{42}$$

as follows. Multiply both sides of (42) by $f'(k^*)$, use the relation $\gamma = [\tilde{R}_{j^*}f'(k^*)]^{1/j^*}$, and substitute the result into (29) to obtain

$$\rho(k^*) = s(w, \gamma)[\gamma - (\gamma)^{j^*}]/[1 - (\gamma)^{j^*}] \equiv s(w, \gamma)Q_{j^*}(\gamma) \tag{43}$$

Now we have established that an increase in β increases both w and γ. Thus $s(w, \gamma)$ rises with a reduction in transactions costs. It is also straightforward to show that $Q'_{j^*} \geq 0$. Therefore we have

Lemma 9. *Let $\{j^*\} = M(k^*)$, with $j^* > 1$. Then a reduction in transactions costs increases the level of secondary market activity.*

We can say more than this, however. The ratio $\rho(k^*)/s(w, \gamma)$ gives the fraction of savings devoted to purchases in secondary capital markets. We have

Lemma 10. *Suppose that $M(k^*)$ is a singleton. Then the ratio $\rho(k^*)/s(w,\gamma)$ (weakly) increases with a reduction in transactions costs.*

Proof. From (43),

$$\rho(k^*)/s(w,\gamma) = Q_{j^*}(\gamma). \tag{44}$$

Moreover, $Q_j' > 0$, and $d\gamma/d\beta \geq 0$. □

Thus in an economy with a unique equilibrium project length, improvements in the liquidity of financial markets will result in an increase in the ratio of secondary capital market activity (which can be thought of as financial transactions) to total savings. In addition, these increases in liquidity (financial deepening) will lead to capital deepening. In short, when β increases the ratio of financial transactions to total assets will rise, as will the per capita capital stock and income level. Goldsmith [17] has noted the strongly positive observed cross-country correlation between these objects.

Case 2. $\{\ell, j\} \subset M(k^*)$, with $j > \ell$. In this case $k^* = \hat{k}_{\ell,j}$, and from (29),

$$\rho = \rho(\hat{k}_{\ell,j}) = [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}s\{w(\hat{k}_{\ell,j}), [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}\} - \hat{k}_{\ell,j}f'(\hat{k}_{\ell,j}). \tag{45}$$

$d\rho/d\beta$ is not easily signed; however it is possible to show that as β is increased (transactions costs are reduced), the fraction of savings consumed by secondary capital market purchases increases. More specifically,

Lemma 11. *The ratio $\rho(\hat{k}_{\ell,j})/s(-)$ increases with an increase in β.*

Lemma 11 is proved in the appendix. It asserts that, in the economy depicted in Figure 4, a reduction in transactions costs results in a reduction in the fraction of savings devoted to new capital formation (as previously).

The empirical implications of the analysis are now very different, however. An increase in β (enhanced liquidity of secondary markets) raises the ratio of financial transactions to total assets, as before, but also results in a *reduced* capital stock and income level. For many countries this is contrary to observation [17], suggesting that Case 1 is the empirically most common situation. This result also suggests an empirical test for when Case 1 (or 2) obtains: Case 1(2) results in a positive (negative) correlation between $\rho/s(w,\gamma)$ and per capita income across regimes that differ with respect to the liquidity of secondary capital markets.

IV. An example

We now present an example which can be solved explicitly for the steady state equilibrium choice of capital production technology, the steady state equilibrium per capita capital stock and income level, and the steady state equilibrium rate of return to savings. In addition, we demonstrate exactly how the steady state equilibrium is affected by changes in the liquidity of secondary capital markets. In doing so we also provide a characterization of how relatively "large" changes in transactions costs can affect the equilibrium choice of a capital production technology. This choice, of course, has important implications for all other aspects of the steady state equilibrium.

Our example has the following features. First, $J = 2$, so there are only two technologies for converting current output into future capital. Second, $f(k) = Ak^{\theta}$, with $\theta \in (0, 1)$, so that production is Cobb–Douglas. Finally, agents also have Cobb–Douglas utility functions, so that $u(C_{1t}, C_{2t}) = (1-\lambda)\ln C_{1t} + \lambda \ln C_{2t}$; $\lambda \in (0, 1]$. Then young agents have the savings function $s(w, y) = \lambda w$.

For this economy the function $G_1(k)$ has the form

$$G_1(k) = R_1 \lambda w(k), \tag{46}$$

while the function $G_2(k)$ is given by [see equation (22)]

$$G_2(k) = \lambda \tilde{R}_2 w(k)\{1 - [\tilde{R}_2 f'(k)]^{1/2}\}/[1 - \tilde{R}_2 f'(k)] = \lambda \tilde{R}_2 w(k)/\{1 + [\tilde{R}_2 f'(k)]^{1/2}\}. \tag{47}$$

Of course for this economy, $w(k) = (1-\theta)Ak^{\theta}$. Finally, technology 2 will be in use iff it generates at least as high an internal rate of return as technology 1. Thus $\{2\} \subset M(k)$ iff

$$[\tilde{R}_2 f'(k)]^{1/2} \geq R_1 f'(k). \tag{48}$$

(48) is equivalent to $k \geq \hat{k}$, where

$$\hat{k} = [\theta A (R_1)^2/\tilde{R}_2]^{1/(1-\theta)}. \tag{49}$$

It is now possible to display the correspondence $G(k)$ for the example. It is given by

$$G(k) = \begin{cases} \{G_1(k)\}; & k < \hat{k} \\ [G_2(k), G_1(k)]; & k = \hat{k} \\ \{G_2(k)\}; & k > \hat{k} \end{cases}$$

The various possible configurations of the G correspondence are depicted in Figure 6. A steady state equilibrium, of course, occurs where $G(k)$ crosses the 45° line. Since the example satisfies all of our assumptions, a non-trivial steady state equilibrium exists and is unique.

Panel A depicts the correspondence G when transactions costs are "relatively high" ($\tilde{R}_2$ is relatively low). In this case, the locus G_2 is relatively depressed, as transactions costs use up a comparatively high fraction of the output of the long-gestation capital production technology. In addition, $\hat{k}$ will be relatively large, so that only if $f'(k)$ is quite small will agents be willing to use the long-gestation technology in equilibrium. The result is that – in equilibrium – only the short-gestation capital production technology is in use, and a steady state equilibrium occurs at the capital stock $k_1^* = \bar{k}_1$. [$\bar{k}_1$ denotes the intersection of the locus G_1 with the 45° line: that is, $\bar{k}_1 = G_1(\bar{k}_1) > 0$.]

As transactions costs are reduced ($\tilde{R}_2$ rises), two things occur. First, the locus G_2 shifts up as the long-gestation technology becomes more productive, net of transactions costs. Second, $\hat{k}$ shifts to the left as agents become willing to employ the long-gestation technology at lower current capital stocks. Therefore, since the locus G_1 is unaffected by changes in transactions costs, a large enough increase will result in $\hat{k}$ falling below $\bar{k}_1$. In this situation both technologies can be in use, and the steady state equilibrium capital stock is $k_2^* = \hat{k} < \bar{k}_1 = k_1^*$. This is the case depicted in panel

B of Figure 6, and it illustrates that a reduction in transactions costs (between panels A and B) can cause a decline in the steady state per capita capital stock and income level.

As transactions costs fall ($\tilde{R}_2$ rises) even further, G_2 will continue to shift up and $\hat{k}$ will continue to shift to the left. For large enough values of $\tilde{R}_2, G_2$ will intersect the 45° line at a value $\bar{k}_2$ satisfying $\bar{k}_2 > \hat{k}$. Then in equilibrium only the long-gestation technology will be employed, and the steady state equilibrium capital stock will be $k_3^* = \bar{k}_2 > \hat{k}$. This situation is depicted in panel C of Figure 6. Between panels B and C the steady state per capita capital stock and output level can either rise or fall.

As transactions costs fall ($\tilde{R}_2$ increases) even further (panel D), G_2 can intersect the 45° line at a value $\bar{k}_2 > \bar{k}_1$. (Since reductions in transactions costs do not affect the locus $G_1, \bar{k}_1$ remains constant across panels in Figure 6.) Since increases in $\tilde{R}_2$ cause $\hat{k}$ to fall even further, in panel D only the long-gestation technology will be employed. Moreoever, since $\bar{k}_2 > \bar{k}_1$, it will now unambiguously be the case that reductions in

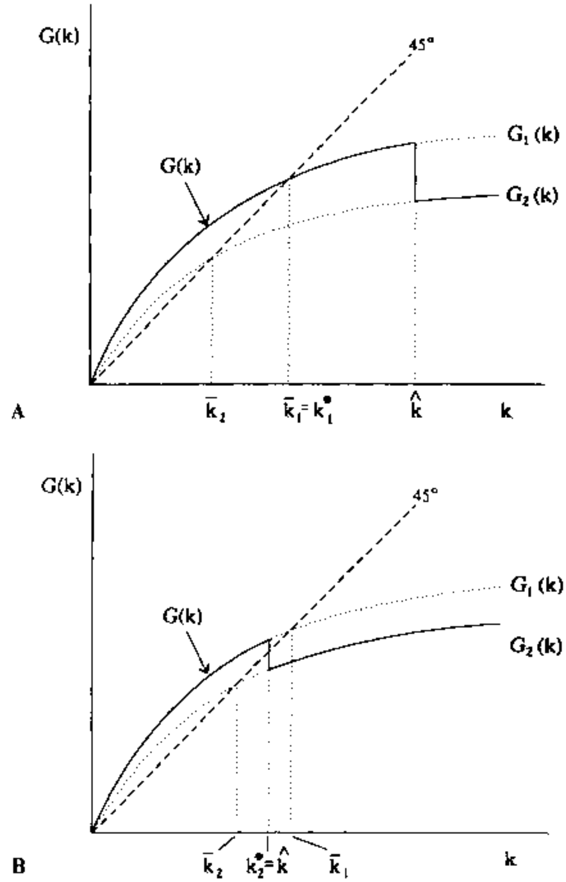

Figure 6. A High transactions costs. **B** Reductions in transactions costs.

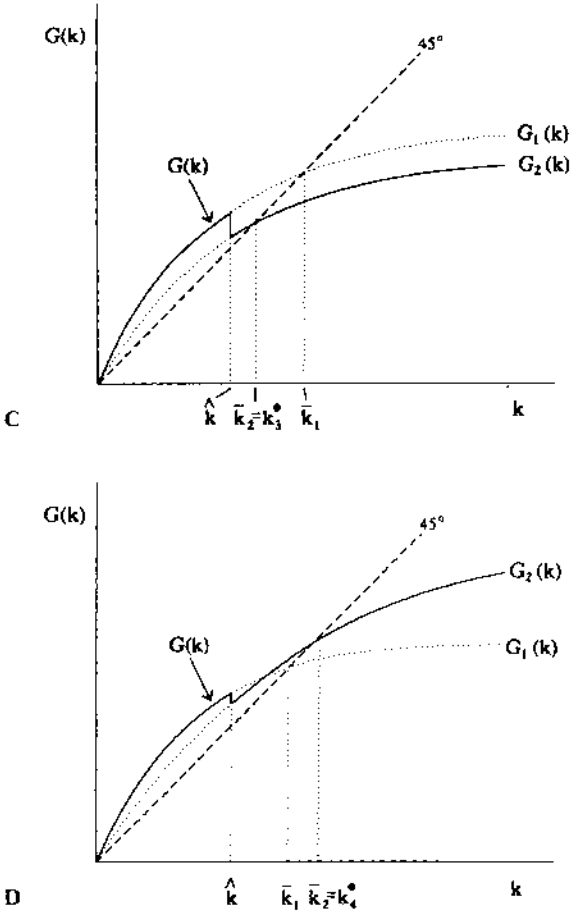

Figure 6. C Lower transactions costs. D Low transactions costs.

transactions costs raise the steady state equilibrium per capita capital stock and output level.

As this discussion makes apparent, to know which case obtains in Figure 6 it is necessary to determine the values $\bar{k}_j = G_j(\bar{k}_j) > 0; j = 1, 2$. These values are unique, and are given by the expressions

$$\bar{k}_1 f'(\bar{k}_1)/w(\bar{k}_1) = \lambda R_1 f'(\bar{k}_1), \tag{50}$$

and

$$\bar{k}_2 f'(\bar{k}_2)/w(\bar{k}_2) = \lambda \tilde{R}_2 f'(\bar{k}_2)/\{1 + [\tilde{R}_2 f'(\bar{k}_2)]^{1/2}\}. \tag{51}$$

Defining $\psi \equiv \lambda(1-\theta)/\theta$, evidently $R_1 f'(\bar{k}_1) = 1/\psi$. From (51) it is straightforward to deduce that $\bar{k}_2$ is given implicitly by the expression

$$[\tilde{R}_2 f'(\bar{k}_2)]^{1/2} = [1 + (1 + 4\psi)^{1/2}]/2\psi. \tag{52}$$

Recalling that $\hat{k}$ is defined by $[\tilde{R}_2 f'(\hat{k})]^{1/2} = R_1 f'(\hat{k})$, it is evident that

$R_1 f'(\hat{k}) = \tilde{R}_2/R_1$. Therefore, $\bar{k}_1 \le \hat{k}$ holds iff

$$1/\psi = R_1 f'(\bar{k}_1) \ge R_1 f'(\hat{k}) = \tilde{R}_2/R_1. \tag{53}$$

When $\bar{k}_1 \le \hat{k}$, the G correspondence has the configuration depicted in panel A of Figure 6.

Similarly $\bar{k}_2 \le \hat{k}$ holds iff

$$\tilde{R}_2/R_1 = [\tilde{R}_2 f'(\hat{k})]^{1/2} \le [\tilde{R}_2 f'(\bar{k}_2)]^{1/2} = [1 + (1 + 4\psi)^{1/2}]/2\psi. \tag{54}$$

When $\bar{k}_2 \le \hat{k}$, the G correspondence is as depicted in panels B–D of Figure 6.

Finally, $\bar{k}_1 \ge \bar{k}_2$ holds iff $1/\psi = R_1 f'(\bar{k}_1) \le R_1 f'(\bar{k}_2) = (R_1/\tilde{R}_2)[\tilde{R}_2 f'(\bar{k}_2)]$. This condition is equivalent to

$$\tilde{R}_2/R_1 \le 1 + [1 + (1 + 4\psi)^{1/2}]/2\psi. \tag{55}$$

When $\bar{k}_2 > \bar{k}_1 > \hat{k}$, the G correspondence is as depicted in panel D of Figure 6.

We now consider four Cases.

Case 1. Suppose (53) holds. Then evidently (54) and (55) hold as well, and we have $\hat{k} > \bar{k}_1 > \bar{k}_2$. This is the situation depicted in panel A of Figure 6. Evidently the correspondence $G(k)$ intersects the 45° line at the steady state equilibrium capital–labor ratio $k_1^* = \bar{k}_1$. The steady state equilibrium choice of technology is $j^* = 1$.

Case 2. Suppose that (53) fails but (54) holds. [The latter supposition implies that (55) also holds.] Then $\bar{k}_1 > \hat{k} > \bar{k}_2$. This Case is depicted in panel B of Figure 6. The steady state equilibrium capital–labor ratio is $k_2^* = \hat{k}$, and the steady state equilibrium has both technologies in use. This situation will be relevant for "intermediate" levels of transactions costs satisfying

$$1/\psi \le \tilde{R}_2/R_1 \le [1 + (1 + 4\psi)^{1/2}]/2\psi. \tag{56}$$

Since $\bar{k}_1 > \hat{k}$, Cases 1 and 2 illustrate that a reduction in transactions costs (an increase in $\tilde{R}_2$) can easily cause a decline in the steady state equilibrium capital stock and in production.

Case 3. Suppose that (54) *fails* [which implies that (53) fails as well], while (55) *holds*. Then $\bar{k}_1 \ge \bar{k}_2 > \hat{k}$. The steady-state equilibrium per person capital stock is $k_3^* = \bar{k}_2$, as depicted in panel C of Figure 6, and $j^* = 2$.

This Case obtains for even lower levels of transactions costs (higher levels of $\tilde{R}_2$) than Case 2. However, transactions costs are still not low enough so that $k_3^* > \bar{k}_1$ holds; or in other words, per capita income is not as high as it would be if only technology 1 were in existence.

Case 4. Suppose that (55) fails. This implies that (53) and (54) fail as well, and hence $\bar{k}_2 > \bar{k}_1 > \hat{k}$. This situation is represented in panel D of Figure 6. The steady state equilibrium has $j^* = 2$, and $k_4^* = \bar{k}_2 > \bar{k}_1$.

This situation obtains for high values of $\tilde{R}_2$ (low transactions costs). Liquidity intensive capital production technologies will be in use, and they are productive enough so that $k_4^* > \bar{k}_1$. Moreover, a reduction in transactions costs that (a) moves an economy from Case 3 to Case 4, or (b) occurs in a Case 4 economy, necessarily raises the steady state equilibrium per capita capital stock and income level. Finally, since

$$[\tilde{R}_2 f'(\bar{k}_2)]^{1/2} = [1 + (1 + 4\psi)^{1/2}]/2\psi > 1/\psi = R_1 f'(\bar{k}_1),$$

a Case 3 or 4 economy necessarily displays a higher equilibrium rate of return on savings than a Case 1 economy. Also, a reduction in transactions costs that moves an economy from Case 1 to Case 2, or from Case 2 to Case 3 necessarily raises the rate of return to savings. Reductions in transactions costs that occur in a Case 3 or 4 economy have no effect on this return, however.

Finally, we saw that in a Case 2 economy, a small reduction in transactions costs would *reduce* steady state equilibrium welfare levels if equations (40) and (41) held. For the example, (40) holds if $\tilde{R}_2'(\beta) > 0$, while (41) is simply $\psi \leq 1$. If these conditions and (56) hold, the economy will be in Case 2, and reductions in transactions costs that leave the economy in Case 2 will reduce the utility of agents in a steady state equilibrium. It is easy to produce parameter values such that $\psi \leq 1$ and (56) is satisfied. Thus, this situation can easily be observed.

V. Conclusions

We have presented a model in which secondary capital markets perform the allocative function suggested by Hicks [20]. By providing liquidity, they allow the adoption of technologies that require illiquid capital investments. We have shown how the technical efficiency of these markets affects (i) the equilibrium choice of technology, (ii) capital accumulation and per capita income, (iii) the real return to savings, (iv) the level of financial market activity, and (v) welfare.

It is often argued that the comparative inefficiency of their financial markets accounts for the low level of real activity observed in less developed economies. Yet not all attempts to stimulate financial market activity in such economies have had positive consequences for growth. Our analysis shows that the second observation does not necessarily contradict the view that sufficient increases in financial market efficiency will lead to increases in production and capital accumulation. This point is illustrated by the example of Section IV where, over some range of parameters, transactions cost reductions do have a negative impact on output levels and the capital stock. Even so, however, sufficiently large reductions in transactions costs (ones that move the economy into panel D of Figure 6) will result in higher levels of both the capital stock and national income. The analysis also indicates that improvements in financial market efficiency will necessarily raise the equilibrium return on savings and the equilibrium level of secondary capital market activity.

Admittedly, our results have been obtained under a very particular set of assumptions. Perhaps the most suspect of these is that the capital produced via different investment technologies is perfectly substitutable as an input in the final goods production process. It is this feature of the analysis that is responsible for the result that there are only a finite number of current values for the capital–labor ratio that allow more than one technology to maximize the internal rate of return on capital investments. Moreover, it is exactly that fact which is responsible for the "saw-toothed" configuration of the correspondence $G(k)$ in Figures 1–6. In particular, that configuration results as discrete transitions occur between situations where shorter and longer-maturity capital production technologies maximize the internal rate of return to an investor.

It may appear that these discrete transitions are necessary to the result that reductions in transactions costs can lower the steady state values of the per capita capital stock and welfare. However, that is in fact not the case, as is demonstrated by [6]. That paper modifies the present one by allowing the capital produced by different technologies to be imperfectly substitutable as inputs in production, and in addition, it allows all capital production technologies to be in use simultaneously at all values of k. The latter feature has the consequence that there are no discrete changes in the set of technologies in use under any circumstances, so that small changes in transactions costs cause "smooth" changes in all endogenous variables. Nevertheless, as shown in [6], reductions in transactions costs can still cause declines in the capital stock, production, and steady state welfare if different "types" of capital are sufficiently substitutable as inputs in the final goods production process. Thus, none of our results are particularly dependent on the possibility of discrete changes in the set of technologies employed.

Of course many other extensions of the model are possible. One would be to examine the dynamical properties of non-stationary equilibria. This would permit us to analyze the dynamic interactions between financial market activity and real activity. A second would be to consider the consequences of taxing or subsidizing financial market activity. The model of the previous sections can address these questions by allowing transactions costs to have a tax/subsidy component. A third extension would be to consider financial intermediary institutions. For example, if there are long (finitely) lived and short lived agents in the model, and if the long lived agents can issue liabilities with low transactions costs, then they could intermediate long-gestation capital investments on behalf of the shorter lived agents. Such intermediation would clearly involve a maturity transformation performed by real world intermediaries; indeed the function of intermediaries in the model would be to issue short-term liabilities and hold long-term assets.

A particularly interesting extension would be to consider an open economy version of the model of this paper. It is widely accepted that there are significant cross-country differences in the costs of undertaking financial market transactions ([17], [23], [25], [32]). A multi-country model in the spirit of the current paper could give all economies access to the same production technologies, and even allow agents in all countries to be identical. Even under these circumstances, cross-country differences in transactions costs could still prevent the long-run convergence of capital stocks and income levels, and could allow for interesting patterns of international funds flows. Such a model might also permit a case to be made for increased financial integration between economies, as well as for opening high transactions cost economies to international trade in financial services.

On any business day at the New York Stock Exchange, billions of dollars worth of securities are traded. With few exceptions these are not new public offerings – they do not raise new capital, but rather rearrange the ownership of existing capital. This liquidity trade pursues the convenience of individual wealth holders. Our study argues that it also significantly affects the aggregate allocation of capital by allowing the simultaneous fulfillment of two apparently opposing needs. Some capital must be long-lived and illiquid and yet represent a liquid holding to its owners. The ability of financial markets to fulfill these contradictory requirements allows the economy successfully to allocate investment to uses with payouts far in the future even though wealth-holders may have short time horizons. The ability to extend the maturity of investment in the economy while accommodating investors' demand for liquidity is an essential function of financial institutions and financial markets.

Appendix

A.1. Proof of Lemma 1

The proof of part (a) proceeds in two parts. We first prove: (i) if $j, \ell \in M(\hat{k})$ and $m \notin M(\hat{k})$, then there exists an interval $(\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$ such that $m \notin M(k)$, $\forall k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$. We then establish: (ii) $\{\ell\} = M(k)\, \forall k \in (\hat{k}-\varepsilon_1, \hat{k})$, and $\{j\} = M(k)\, \forall k \in (\hat{k}, \hat{k}+\varepsilon_2)$.

Proof of (i). Since $j, \ell \in M(\hat{k})$ and $m \notin M(\hat{k})$, we have from equation (15) that

$$[\tilde{R}_j f'(\hat{k})]^{1/j} > [\tilde{R}_m f'(\hat{k})]^{1/m} \tag{A.1}$$

$$[\tilde{R}_\ell f'(\hat{k})]^{1/\ell} > [\tilde{R}_m f'(\hat{k})]^{1/m}. \tag{A.2}$$

It then follows from continuity that we can choose values $\varepsilon_1, \varepsilon_2 > 0$ such that, for all $k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$,

$$[\tilde{R}_j f'(k)]^{1/j} > [\tilde{R}_m f'(k)]^{1/m} \tag{A.3}$$

$$[\tilde{R}_\ell f'(k)]^{1/\ell} > [\tilde{R}_m f'(k)]^{1/m}. \tag{A.4}$$

This establishes (i).

Proof of (ii). Define the function $\delta_{j,\ell}: \mathbb{R}_+ \to \mathbb{R}$ by

$$\delta_{j,\ell}(k) \equiv [\tilde{R}_j f'(k)]^{1/j} - [\tilde{R}_\ell f'(k)]^{1/\ell}. \tag{A.5}$$

Then, since $j, \ell \in M(\hat{k})$, equation (15) implies that $\delta_{j,\ell}(\hat{k}) = 0$. Moreover,

$$\delta'_{j,\ell}(k) = \{(1/j)[\tilde{R}_j f'(k)]^{1/j} - (1/\ell)[\tilde{R}_\ell f'(k)]^{1/\ell}\} f''(k)/f'(k). \tag{A.6}$$

Since $\delta_{j,\ell}(\hat{k}) = 0$ and $j > \ell$, it follows that $\delta'_{j,\ell}(\hat{k}) > 0$. Thus it is possible to choose values $\varepsilon_1, \varepsilon_2 > 0$ such that (A.3) and (A.4) hold $\forall m \notin M(\hat{k})$, $\forall k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$, and such that

$$[\tilde{R}_\ell f'(k)]^{1/\ell} > [\tilde{R}_j f'(k)]^{1/j} \tag{A.7}$$

$\forall k \in (\hat{k}-\varepsilon_1, \hat{k})$, while $\forall k \in (\hat{k}, \hat{k}+\varepsilon_2)$,

$$[\tilde{R}_j f'(k)]^{1/j} > [\tilde{R}_\ell f'(k)]^{1/\ell} \tag{A.8}$$

Proof of (b). Immediate from $\delta'_{j,\ell}(\hat{k}) > 0$. □

A.2. Proof of Lemma 2

$\{1\} = M(k)$ iff $\delta_{1,\ell}(k) > 0\, \forall \ell \neq 1$. From (A.5) this condition obtains iff

$$f'(k) > (\tilde{R}_\ell)^{1/(\ell-1)}/(\tilde{R}_1)^{\ell/(\ell-1)} \tag{A.9}$$

$\forall \ell = 2, \ldots, J$. But (A.9) holds for all k sufficiently close to zero, by the Inada condition.

Similarly $\{J\} = M(k)$ iff $\delta_{J,\ell}(k) > 0\, \forall \ell \neq J$. From (A.5), this condition is satisfied iff

$$f'(k) < (\tilde{R}_J)^{\ell/(J-\ell)}/(\tilde{R}_\ell)^{J/(J-\ell)} \tag{A.9'}$$

$\forall \ell = 1, \ldots, J-1$. But again, satisfaction of (A.9') for large enough k is guaranteed by the Inada condition. □

A.3. Proof of Lemma 3

Since $j, \ell \in M(\hat{k})$, $\delta_{j,\ell}(\hat{k}) = 0$. In addition, it is easy to show that

$$G_j(k) \equiv \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}] \Big/ \sum_{h=0}^{j-1} [\tilde{R}_j f'(k)]^{(h/j)}, \tag{A.10}$$

Therefore $G_j(\hat{k}) < G_\ell(\hat{k})$ holds iff

$$\tilde{R}_j s[w(\hat{k}), (\tilde{R}_j f'(\hat{k}))^{1/j}] \Big/ \sum_{h=0}^{j-1} [\tilde{R}_j f'(\hat{k})]^{h/j} < \tilde{R}_\ell s[w(\hat{k}), (\tilde{R}_\ell f'(\hat{k}))^{1/\ell}] \Big/ \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}. \tag{A.11}$$

But $\delta_{j,\ell}(\hat{k}) = 0$ implies that $(\tilde{R}_j f'(\hat{k}))^{1/j} = (\tilde{R}_\ell f'(\hat{k}))^{1/\ell}$, so that (A.11) reduces to

$$\tilde{R}_\ell \sum_{h=0}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} > \tilde{R}_j \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}. \tag{A.12}$$

Moreover,

$$\tilde{R}_j = [\tilde{R}_\ell f'(\hat{k})]^{j/\ell} / f'(\hat{k}), \tag{A.13}$$

holds. Substituting (A.13) into (A.12), we obtain

$$\sum_{h=0}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} > [\tilde{R}_\ell f'(\hat{k})]^{(j-\ell)/\ell} \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} = \sum_{h=j-\ell}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}, \tag{A.14}$$

which obviously holds, establishing the desired relation.[16] □

A.4. Proof of Lemma 4

From equation (22) it is apparent that

$$G_j(k)/k \leq \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}]/k \leq \tilde{R}_j w(k)/k \tag{A.15}$$

holds $\forall k, \forall j = 1, \ldots, J$. But then, $\forall j$,

$$\lim_{k \to \infty} G_j(k)/k \leq \lim_{k \to \infty} \tilde{R}_j w(k)/k. \tag{A.16}$$

Furthermore, by L'Hospital's rule, $\forall j \ \lim_{k \to \infty} \tilde{R}_j w(k)/k = 0$, establishing the result. □

A.5. Proof of Proposition 1

The existence of a non-trivial stationary equilibrium follows immediately from (a.1) and Lemmas 1–4 (see Figure 1). We now establish uniqueness of the non-trivial steady state equilibrium.

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[16] In equation (A.12),

$$\sum_{h=0}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} = 1/\theta^{j,0}$$

if *only* technology j is in use, while

$$\sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} = 1/\theta^{\ell,0}$$

if *only* technology ℓ is in use. Thus (A.12) is equivalent to

$$\tilde{R}_\ell \theta^{\ell,0} > \tilde{R}_j \theta^{j,0},$$

as asserted in the text. Thus, the use of longer-gestation technologies shifts savings away from the initiation of new capital investment, and toward the holding of already existing CIP. Moreover, as more long-maturity investments are held, more resources are potentially used in the transactions process. Both effects tend to depress capital formation, as the lemma asserts.

To do so it will be useful to begin with two preliminary results.

Result 1. Define the function $h: \mathbb{R}_+ \to \mathbb{R}_+$ by

$$h(x) \equiv x(1 - x^{1/j})/(1 - x). \tag{A.17}$$

Then $h'(x) > 0\ \forall x \in \mathbb{R}_+$.

Proof. Differentiating (A.17) yields

$$h'(x) = \eta(x)/(1 - x)^2, \tag{A.18}$$

where

$$\eta(x) \equiv 1 + (1/j)x^{(j+1)/j} - [(j + 1)/j]x^{1/j}. \tag{A.19}$$

Evidently, $\eta(1) = 0$, and application of L'Hospital's Rule establishes that $h'(1) > 0$. Moreover,

$$\eta'(x) = [(j + 1)/j^2](x - 1)x^{(1-j)/j}. \tag{A.20}$$

Thus

$$\begin{aligned} \eta'(x) &\leq 0, \quad x \leq 1 \\ \eta'(x) &\geq 0; \quad x \geq 1 \end{aligned} \tag{A.21}$$

holds. It follows that $\eta(x) \geq 0\ \forall x \in \mathbb{R}_+$, and hence that $h'(x) > 0\ \forall x \in \mathbb{R}_+$. □

Result 2. For all $j = 1, \ldots, J$, the equation

$$k = G_j(k) \tag{A.22}$$

has at most one non-trivial solution.

Proof. Using the definition of $G_j(k)$ in (22), rewrite (A.22) as

$$kf'(k)/w(k) = s[w(k), (\tilde{R}_j f'(k))^{1/j}]h[\tilde{R}_j f'(k)]/w(k), \tag{A.23}$$

where h is defined in (A.17). Assumption (a.2) (along with $s_2 \geq 0$) implies that $s[w(k), (\tilde{R}_j f'(k))^{1/j}]/w(k)$ is a non-increasing function of k. Similarly, Result 1 implies that $h[\tilde{R}_j f'(k)]$ is a non-increasing function of k. Finally, the assumption that the elasticity of substitution σ satisfies

$$\sigma \equiv -f'(k)[f(k) - kf'(k)]/kf(k)f''(k) \geq 1 \tag{A.24}$$

$\forall k$ implies that $kf'(k)/w(k)$ is a non-decreasing function of k. Thus (A.23) [and hence (A.22)] has at most one solution. □

We are now prepared to prove the remainder of Proposition 1. By result 2 each $G_j(k)$ crosses the 45° line in Figure 1 at most once. By Lemma 4, each $G_j(k)$ lies below the 45° line for large enough values of k. Thus each $G_j(k)$ crosses the 45° line from above (if at all). Therefore, if $k \in G(k)$ and $M(k)$ is a singleton, $G(k)$ also crosses the 45° line from above. If $k \in G(k)$ and $M(k)$ is not a singleton, then k corresponds to a vertical portion of G, and Lemma 3 implies that $G(k)$ crosses the 45° line from above. Thus $\forall k > 0$ satisfying $k \in G(k)$, $G(k)$ intersects the 45° line from above, and there can be at most one non-trivial solution to the equilibrium condition $k \in G(k)$. □

A.6. Proof of Lemma 5

We wish to consider (with reference to Figure 1) the vertical shift in the term

$$\begin{aligned} G_j(k) &= \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}][1 - (\tilde{R}_j f'(k))^{1/j}]/[1 - \tilde{R}_j f'(k)] \\ &= s[w(k), (\tilde{R}_j f'(k))^{1/j}]h[\tilde{R}_j f'(k)]/f'(k) \end{aligned} \tag{A.25}$$

associated with an increase in $\tilde{R}_j$, where the function h is as defined in (A.17). This vertical shift is given by the term

$$s(-)h'[\tilde{R}_j f'(k)] + (1/j)[\tilde{R}_j f'(k)]^{(1-j)/j}h[\tilde{R}_j f'(k)]s_2(-).$$

Result 1 and $s_2 \geq 0$ imply that this expression is positive. □

A.7. Proof of Lemma 6

Differentiating (32) gives

$$\begin{aligned}&\{(1/j)[\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j} - (1/\ell)[\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}\}(f''/f')d\hat{k}_{\ell,j}/d\beta \\ &\quad = [\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}[\tilde{R}'_\ell(\beta)/\ell\tilde{R}_\ell(\beta)] - [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}[\tilde{R}'_j(\beta)/j\tilde{R}_j(\beta)].\end{aligned} \quad (A.26)$$

Thus (32), (a.6), and $j > \ell$ imply that $d\hat{k}_{\ell,j}/d\beta < 0$. □

A.8. Proof of Lemma 7

Using $\{j^*\} = M(k)$ and (A.23), write the equilibrium condition as

$$kf'(k)/w(k) = s[w(k), (\tilde{R}_{j^*}f'(k))^{1/j^*}]h[\tilde{R}_{j^*}f'(k)]/w(k). \quad (A.27)$$

Using (a.2) and (a.3), it is easy to show that (A.27) gives $\tilde{R}_{j^*}f'(k)$ as a (weakly) increasing function of k, say

$$\tilde{R}_{j^*}f'(k) = \psi(k). \quad (A.28)$$

Since an increase in β increases k, in this case, it follows that $\tilde{R}_{j^*}f'(k)$ must (weakly) increase in order to satisfy (A.28). □

A.9. Proof of Lemma 11

From (45),

$$\rho(\hat{k}_{\ell,j})/s(w,\gamma) = \gamma - [\hat{k}_{\ell,j}f'(\hat{k}_{\ell,j})/w(\hat{k}_{\ell,j})][w(\hat{k}_{\ell,j})/s(w,\gamma)]. \quad (A.29)$$

From equation (38), $d\gamma/d\beta > 0$, while from (35), $d\hat{k}_{\ell,j}/d\beta < 0$. Moreover, these facts and Assumption (a.2) imply that $w/s(w,\gamma)$ (weakly) declines when β increases, and Assumption (a.3) implies that $\hat{k}_{\ell,j}f'(\hat{k}_{\ell,j})/w(\hat{k}_{\ell,j})$ (weakly) declines when β increases. Thus an increase in β increases $\rho/s(w,\gamma)$. □

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