FINAL EXAMINATION —- SUGGESTED ANSWERS

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Ross or Ben B. — who promise to be clueless), until the examination is collected.

The exam is due by 5:00 PM, Tuesday, December 7, 2010. Submit your exam to Prof. Starr's lockbox on the ground floor of the Economics Building (the hallway near Econ 100).

Answer all 5 (five) questions. .

All notation not otherwise defined is taken from Starr's General Equilibrium Theory, *draft second edition. If you need to make additional assumptions to answer a question, that's OK. Do state the additional assumptions clearly.*

1. Consider the general equilibrium of an economy assumed to be pure exchange with the exception of production of a (real valued) public good. A *public good* is here defined as a commodity that all households have equal access to without paying for it. Think of local roadways. Household possible consumption sets (of the N private goods) are $\mathbf{R}^N_+$. The typical household i is endowed with $r^i \in \mathbf{R}^N_+$. r^i is assumed to be strictly positive in all co-ordinates. The households are taxed in kind at the rate $0 < \tau < 1$; i's tax payment is then the N-dimensional vector τr^i . Household income is $M^i(p) = [1-\tau]p{\cdot}r^i$. All households and the public good production authority act as price takers. The resources $\tau \sum_i r^i$ are then sold at market prices and the proceeds, $\mathcal{T} = \tau p \cdot \sum_i r^i$, used to purchase inputs to production of the public good, γ. γ is produced then according to the production function

$$\gamma = \max g(y) \text{ subject to } p \cdot y = \mathcal{T}.$$

We take g to be continuous, strictly concave, strictly increasing in its arguments.

Household utility functions are then characterized as $u^i(x^i; \gamma)$. The households treat γ parametrically. Assume all the usual properties of u^i, including continuity, convexity, nonsatiation. The household budget constraint is $p \cdot x^i \leq M^i(p)$.

   (a) Theorem 18.1 (existence of competitive general equilibrium) is proved in an economy without public goods. Is household i's income above the minimum value in $\mathbf{R}^N_+$ (and why should we care)? Is the excess demand function continuous in p? Is Walras's Law fulfilled? Is there a competitive equilibrium in this economy with taxation and public goods? Explain.

Suggested Answer: One way to think of this problem is to conceive of the public good authority as just another household. Ordinary households have endowments of $(1-\tau)r^i$ and the public good authority has an endowment of $\tau \sum_i r^i$. The authority's utility function is g. Then the question has been reduced to the previous case with the exception of an external effect: $\gamma = g(y)$ entering into all households' utility function. So long as γ and τ don't interfere with C.I - C.VII, the usual results should follow.

$M^i(p) = (1-\tau)p \cdot r^i > 0 =$ minimum value in $\mathbf{R}^N_+$. This is significant inasmuch as this inequality ensures fulfillment of C.VII despite the tax payment. Hence one of the sufficient conditions for existence of general equilibrium in this economy is fulfilled. All of the other sufficient conditions are also fulfilled, continuity of u^i in its arguments, including γ. Given non-satiation, expenditures equal income, including expenditures for production of γ, so Walras's Law is fulfilled. Thus: YES, THERE IS A GENERAL EQUILIBRIUM.

(b) Theorem 19.1 (First Fundamental Theorem of Welfare Economics) is proved in a model without public goods. Consider the allocation of the N private goods, for a given level of γ, τ when there is a competitive equilibrium. Is the allocation of the N private goods Pareto efficient? That is, holding the level of γ, τ fixed, at competitive equilibrium, is there an attainable Pareto preferable allocation of the N private goods? Explain.

Suggested Answer: Denote by y^* the inputs y to $g(y) = \gamma$. y^* is a cost minimizer at p^* for $g(y) = \gamma$. At the competitive equilibrium prices, p^*, we have for each household $i \in H$, $D^i(p^*)$ is a cost-minimizer for $u^i(D^i(p^*), \gamma)$. Then any Pareto improvement for H, for fixed levels of τ, y^*, γ, requires greater expense, and is hence unattainable by the usual argument for 1FTWE. Thus the allocation of the N private goods remaining after inputs to g, $\sum_i r^i - y^*$, is Pareto efficient, subject to given level of τ, y^*, γ.

2. Consider core convergence in a pure exchange economy becoming large through Q-fold replication.

(a) Consider an example where there are two commodities, x and y, and two trader types, 1 and 2.

Type 1 is characterized as having utility function

$$u^1(x, y) = xy, \text{and endowment}$$
$$r^1 = (10, 0).$$

Type 2 is characterized as having utility function

$$\begin{aligned} u^2(x,y) &= xy, \text{and endowment} \\ r^2 &= (0,10). \end{aligned}$$

Show that the following allocation, a^1 to type 1 and a^2 to type 2, is in the core for the original economy with one of each type, and is not in the core for an economy with $Q \geq 2$:
$a^1 = (9,9)$;
$a^2 = (1,1)$.

Suggested Answer: For the original economy without replication, any Pareto efficient indiviudally rational allocation is in the core. Pareto efficiency of a^1, a^2 follows from
$MRS^1_{x,y}(a^1) = MRS^2_{x,y}(a^2)$. Individual rationality follows from $u^1(a^1) = 81 > 0 = u^1(r^1)$, $u^2(a^2) = 1 > u^2(r^2) = 0$.
But for $Q = 2$ there should be a blocking coalition consisting of one household of type 1 and two of type 2. They can achieve $(16,8)$ to type 1, and $(2,1), (2,1)$, to the two type 2's, a blocking allocation.

(b) H represents an economy with a finite number of households of strictly convex, continuous preferences; the typical endowment is r^i and the typical allocation is x^i for $i \in H$ with preferences $\succeq_i$. Let Q be a positive integer. Let Core $(Q \times H)$ denote the set of core allocations of the Q-fold replica of the original economy H. Under the equal treatment property, a typical core allocation will be represented by allocations to type, $\{x^i | i \in H\}$. Recall that blocking coalitions do not need to provide equal treatment in the blocking allocation. Denote the set of households of this economy as $Q \times H = \{i, q | i \in H, q = 1, 2, ..., Q\}$, where "$i, q$" is read as "the qth household of type i."

Demonstrate that Core $((Q+1) \times H) \subseteq$ Core$(Q \times H)$.

Suggested Answer: The informal argument is that any equal treatment allocation blocked in $Q \times H$ will be blocked in $(Q+1) \times H$, because any blocking coalition that can form in $Q \times H$ can also form in $(Q+1) \times H$. We can make that argument a bit more formal. Let $(x^{o1}, x^{o2}, ..., x^{o\#H}) \notin$ Core$(Q \times H)$ be an equal treatment allocation blocked in $(Q \times H)$. Then there is $S \subset (Q \times H)$ forming the blocking coalition. But $S \subset ((Q+1) \times H)$. Then S blocks $(x^{o1}, x^{o2}, ..., x^{o\#H})$ in $((Q+1) \times H)$. This completes the argument.

3. Consider an Edgeworth box economy. Household 1 has endowment $r^1 = (r^1_x, r^1_y) = (5, 5)$, household 2 has endowment $r^2 = (r^2_x, r^2_y) = (10, 10)$. Household 1 has preferences summarized by the utility function, $u^1(x, y) = xy$. Household 2 has preferences summarized by the utility function $u^2(x, y) = inf[xy, 64]$ where inf stands for infimum or minimum. That is, household 2 is satiated with consumption when his utility level gets to 64.

   (a) Demonstrate that this economy has a competitive equilibrium at prices $(\frac{1}{2}, \frac{1}{2})$ with the equilibrium allocation equal to the endowment.

   Suggested Answer: $MRS^1_{xy}(5, 5) = 1 = (\frac{1}{2})/(\frac{1}{2})$. $u^2(10, 10) = 64$. So both households 1 and 2 are optimizing utility subject to budget constraint at $p = (\frac{1}{2}, \frac{1}{2})$ while consuming their endowments, and trivially markets clear.

   (b) Demonstrate that the equilibrium allocation in part (i) is Pareto inefficient.

   Suggested Answer: A Pareto preferable allocation is $(x^1, y^1) = (7, 7), (x^2, y^2) = (8, 8)$. Hence the allocation in (a) is Pareto inefficient.

   (c) Is this a counterexample to the First Fundamental Theorem of Welfare Economics (Theorem 19.1)? If so, explain why. If not, explain why not.

   Suggested Answer: NO. Theorem 19.1 requires non-satiation. Household 2 is subject to satiation because of the ceiling on his utility. Hence the theorem does not correctly apply.

   (d) Consult the proof of the First Fundamental Theorem of Welfare Economics, Theorem 19.1 in Starr's *General Equilibrium Theory, 2^{nd} edition* . It uses the assumptions of nonsatiation C.IV and convexity C.VI(C), implying local nonsatiation (near every consumption point there is a strictly preferred consumption point — typically including more of some commodity). The theorem is invalid (that is, the conclusion may not be true) without these assumptions (or a monotonicity assumption). Note that locally satiated preferences will be characterized by thick indifference curves (zones of satiation). Explain how the assumption of nonsatiation is used in the proof of the theorem. Where does the logic of the proof of the theorem break down without C.IV? The proof by contradiction says that if there is an attainable Pareto preferable allocation, it must be more expensive evaluated at competitive equilibrium prices and more profitable. How does this argument fail?

   Suggested Answer: The proof of Theorem 19.1 depends on each household fulfilling his budget constraint as an equality. That will be true under universal non-satiation, C.IV. It may fail without C.IV (as demonstrated in part (a)).

4. Household preferences are assumed to be continuous in C.V. That is, they can be represented by a continuous real valued utility function. In this problem we see what can happen when that assumption fails. Let there be two goods, x and y. An allocation to household i will be represented by (x, y). Consider household preference ordering $\succ_i$ of the following form.

   $\succ_i$ is read "is strictly preferred to;" $\sim_i$ is read " is indifferent to."

$$(x, y) \succ_i (x', y') \text{ if } 3x + y > 3x' + y'; \text{ or if}$$
$$(x, y) \succ_i (x', y') \text{ if } 3x + y = 3x' + y' \text{ and } x > x'.$$
$$(x, y) \sim_i (x', y') \text{ if } (x, y) = (x', y').$$

   That is, a bundle (x, y) is evaluated by the value of the expression $3x+y$ except when two bundles are tied. Then the tie breaker is which one has more x. Consider the following Edgeworth box (two person pure exchange economy), with two households of identical tastes, as above, and different endowments.

	Household 1	Household 2
Preferences	$\succ_\mathbf{i}$, $\sim_\mathbf{i}$	$\succ_\mathbf{i}$, $\sim_\mathbf{i}$
Endowment	$r^1 = (100, 100)$	$r^2 = (50, 50)$

   (a) Does this Edgeworth Box have a competitive equilibrium? If so, describe the price and allocation. If not, explain why.

   Suggested Answer: NO. The preference ordering does not satisfy C.V, so Theorem 18.1 does not apply. At $(p_x/p_y) \geq 3$ there is an excess demand for x. At $(p_x/p_y) < 3$ there is an excess demand for y. There is no equilibrium price.

   (b) Find the core of this economy. Is it nonempty?

   Suggested Answer: Since there is no competitive equilibrium, Theorem 21.1 does not apply and does not provide a nonempty core. However, the only requirements for a core allocation in the Edgeworth Box are Pareto efficiency and individual rationality. The endowment allocation fulfills both and is the core allocation.

   (c) Theorem 21.1 says that a competitive equilibrium allocation will be in the core. Does the core of this economy include its competitive equilibrium? Is this an example of Theorem 21.1? A counterexample?

   Suggested Answer: This setting is neither an example nor a counterexample to Theorem 21.1. There is no competitive equilibrium, so Theorem 21.1 does not apply. Theorem 21.1 does not provide a necessary and sufficient condition for a nonempty core, merely a sufficient condition.

5. Consider a firm planning to start operations in an intertemporal Arrow-Debreu certainty economy with a full set of futures contracts. There are profitable opportunities to produce widgets for supply at $t + 2$; this production requires inputs at t. The firm is inactive prior to t. How does the firm finance its production plan?

   (a) Does the firm borrow needed capital from a bank? Why or why not (in the structure of this model)?

   Suggested Answer: No. This is a non-monetary model. There are no financial intermediaries and no banks. There is no repository of capital awaiting application.

   (b) Does the firm float new stock shares or a bond issue? Why or why not (in the structure of this model)?

   Suggested Answer: No. This is a non-monetary model. There are no proceeds from a stock issue. The ownership of the firm is set exogenously in this model. It is not subject to endogenous decisions.

   (c) Does the firm use other means to acquire needed inputs prior to delivering output? Explain (in the structure of this model).

   Suggested Answer: Yes. The firm sells its output (for future delivery at $t + 2$) on the futures market and uses the proceeds to purchase needed inputs (for future delivery at date t) also on the futures market.