Your Name: SUGGESTED ANSWERS

Please answer all questions. Each of the six questions marked with a big number counts equally. Designate your answers clearly.

Short correct answers are sufficient and get full credit. Including irrelevant (though correct) information in an answer will not increase the score.

All notation not otherwise defined is taken from Starr's *General Equilibrium Theory: An Introduction (2nd edition).* Please read questions carefully — technical details can be significant.

If you need additional space to answer a question, write "(over)" at the bottom of the page and continue on the back of the page. If you still need additional space, page 10 provides overflow space. If you need additional overflow space, use an additional sheet of paper, making sure that it has your name on it, designates the question number, and is attached to your exam when submitted.

This examination is open book, open notes (hard copy only). Calculators, cell phones, computers, iPads, etc., advice of classmates, are not allowed. If you need arithmetic advice — e.g. "what is the product of 19×32?" Ross or Isla will be glad to advise.

1 Scale Economies and Existence of General Equilibrium

P.V (Strict convexity of Y^j production technology) is one of the sufficient conditions for existence of general equilibrium. In an alternative treatment P.I (Convexity of Y^j) would be sufficient. **What can go wrong with the with existence of general equilibrium theorem and proof (Theorem 18.1) in the absence of convexity of production technology?**

Circle the letter(s) of the correct answer(s) and explain your reasoning. Draw an illustration if that will help.

A. Non-existence of well defined supply function for firm j. Theorem 15.2 (Boundedness of the attainable set) is not applicable. If scale economies are unbounded, the profit maximizing supply plan for firm j may be unbounded for any $p \in P$ where market-clearing could occur, and $S^j(p)$ may not be well defined.

B. Nothing. There is still always existence of general equilibrium —- assuming the other assumptions are maintained — without convexity of technology sets; it's just harder to prove.

C. $S^j(p)$ may be well defined, but non-convex set-valued; it may appear discontinuous. For example, supply may occur only in integer units or only in multiples of ten. Then there may be no intersection of supply and demand and no market equilibrium.

D. Depends on the time and uncertainty structure of the model. In a single period treatment, P.I and P.V can be eliminated without change in the generally valid results of Theorem 18.1. Only the treatment with futures markets and contingent commodity markets really makes use of the convexity assumptions.

E. None of the above answers is sound.

Explain:

SUGGESTED ANSWER: Both **A** and **C** are correct answers.

P.I is needed to demonstrate boundedness of the attainable set. Without boundedness, if there are unlimited increasing returns (for example, if $y = x^2$) then there may be no well defined limit on profit maximizing supply, so $S^j(p)$ may not be well defined.

Or there may be limited scale economies, as for example in a U-shaped cost function. Then there may be (apparently discontinuous) jumps, gaps, in the supply function resulting in possible non-existence of general equilibrium.

2 Core Convergence in the Replica Economy

Consider the core of a pure exchange economy in a Debreu-Scarf model with replication. Let there be two household types, i = 1, 2, two commodities, x and y. Q denotes the number of replications. Let the type 1 endowment be $r^1 = (48, 2)$ where the first co-ordinate is 1's endowment of x, the second is 1's endowment of y. Let the type 2 endowment be $r^2 = (2, 48)$. Assume Cobb-Douglas utilities, $u^1(x^1, y^1) = x^1y^1, u^2(x^2, y^2) = x^2y^2$. (The superscripts denote the trader types. Nothing in this problem is raised to a power. $2 \times 48 = 96$.)

(2.i) Consider the allocation $(x^1, y^1) = (10, 10), (x^2, y^2) = (40, 40)$. **Show that this allocation is in the core for Q = 1. Show that it is not in the core for Q = 3.**

SUGGESTED ANSWER: $u^1(10, 10) > u^1(48, 2), u^2(40, 40) > u^2(2, 48)$ so the allocation is individually rational. $MRS_{x,y} = \frac{y}{x}$, so the MRS's are equated for the two households, and the allocation is Pareto efficient. Individual rationality and Pareto efficiency are sufficient to give inclusion in the core for $Q = 1$.

To demonstrate that the allocation is not in the core for $Q = 3$ it is sufficient to find a blocking coalition and the blocking allocation. Let the blocking coalition be 3 of type 1 and 1 of type 2. The blocking allocation then can be $(18, 6), (18, 6), (18, 6)$ to the three types 1 and $(90, 30)$ to the single type 3. $u^1(18, 6) = 108 > 100 = u^1(10, 10)$, $u^2(90, 30) = 2700 > 1600 = u^2(40, 40)$.

(2.ii) Find an allocation that is in the core for arbitrarily large Q, $Q \to +\infty$. Demonstrate that it is in the core.

SUGGESTED ANSWER: The competitive equilibrium allocation is always in the core (Theorem 21.1). So $(x^1, y^2) = (25, 25) = (x^2, y^2)$ is always in the core, supported by prices $p = (\frac{1}{2}, \frac{1}{2})$.

3 Robinson Crusoe meets the Second Fundamental Theorem of Welfare Economics

Consider a Robinson Crusoe economy with two outputs, x and y and one input, Robinson's labor $L = 168$. The resource constraint is $L^x + L^y = 168$ where L^x and L^y are the labor inputs to production of x and y. Their production technologies are identical and have scale economies: $x = (L^x)^{1.5}; y = (L^y)^{1.5}$. Robinson's tastes are (Leontieff style) summarized by the utility function $u(x, y) = min[x, y]$; that is, x and y are perfect complements. The Pareto efficient allocation is then, $L^x = L^y = 84; x = y = (84)^{1.5}$. The Second Fundamental Theorem of Welfare Economics (2FTWE) says that a Pareto efficient allocation can be supported by competitive equilibrium prices.

How does 2FTWE apply in this setting?

Circle the letter(s) of the correct answer(s) and explain your reasoning.

A. 2FTWE applies successfully here inasmuch as the marginal rate of transformation (MRT) at the Pareto efficient allocation is 1. So the prices, $p_x = p_y = 1$ and the wage rate $w = \frac{2\times(84)^{1.5}}{168}$ supports the allocation as competitive equilibrium.

B. 2FTWE fails. This is a counterexample.

C. 2FTWE is inapplicable here. The utility function fails non-satiation since it is not strictly monotone — additional quantities of the more abundant good, x or y do not increase utility when $x \neq y$.

D. 2FTWE is inapplicable here. The production technology is not convex.

E. None of the responses above is sound.

EXPLAIN:

SUGGESTED ANSWER: The correct answer is **D**. 2FTWE requires convexity, P.I or P.V. The Pareto efficient allocation in this example cannot be supported by prices. Any price system will drive profit maximizing production to a corner solution (all x or all y) but efficiency occurs at the (interior) allocation posited in the statement of the problem.

4 Robinson Crusoe meets the First Fundamental Theorem of Welfare Economics

Consider a Robinson Crusoe economy with two consumption goods, x and y, and one input to production L. Production of x is by simple constant returns,

$$x = L^x,$$

where L^x is the amount of L used as an input to x. Production of y involves a set-up cost, $S > 0$ (a nonconvexity),

$$\begin{aligned} &y = 0 && \text{if } L^y \leq S \\ &y = (L^y - S) && \text{if } L^y > S \\ &S = 20 \\ &L^y \leq 120. \end{aligned}$$

where L^y is the amount of labor used as an input to y. The final inequality reflects limited capacity in production of y. The firm producing y reaches full capacity and cannot expand beyond $L^y = 120, y = 100$. The total labor input supplied is

$$L^x + L^y = 168.$$

Robinson is the sole owner of the firms producing x and y and receives any profits they produce. Robinson's labor is supplied inelastically up to 168, at a wage rate of w. Robinson's utility function is
$u(x, y) = x + 2y$.
The following prices and allocation represent a competitive equilibrium:
$p_x = 1, p_y = 2, w = 1, x = 48, y = 100, L^x = 48, L^y = 120$.

Is the competitive equilibrium allocation Pareto efficient?

Circle the letter(s) of the correct answer(s) and explain your reasoning.

A. Nonconvexity in the production technology means that the First Fundamental Theorem of Welfare Economics (1FTWE) is inapplicable. It is not possible to determine whether the allocation is Pareto efficient.

B. Nonconvexity in the production technology means that the Second Fundamental Theorem of Welfare Economics (2FTWE) is inapplicable. It is not possible to determine whether the allocation is Pareto efficient.

C. 1FTWE applies independent of nonconvexity. The competitive equilibrium allocation is Pareto efficient.

D. 1FTWE does not apply. The allocation is not Pareto efficient.

E. None of the above proposed answers is sound.

Explain your answer on the next page

Explanation for answer to question 4 (Please keep your answer brief; please try not to use the full page.):

SUGGESTED ANSWER: The correct answer is **C**. 1FTWE (Theorem 19.1) does not require convexity. A competitive equilibrium, if it occurs despite non-convexity, is Pareto efficient.

5 Structure of Production Technology and Existence of General Economic Equilibrium

Let the notation $[v]$ for a real number v denote the smallest integer greater than or equal to $(\geq)v$. Thus for example, $[0] = 0, [0.5] = 1$, and so forth. Consider a two-commodity Arrow-Debreu economy. There is a nonempty subset of firms $F' \subset F$ so that $j \in F'$ have production technologies of the form $Y^j = \{(y,x)|y \geq 0, x \leq 0, y \leq (\sqrt{|x|})$, when $[|x|]$ is 0 or even; $y < (\sqrt{|x|})$, when $[|x|]$ is odd$\}$. That is, the weak versus strict inequality in the production technology switches on or off as inputs move between even and odd input levels. The rest of the firms in F fulfill the usual assumptions P.II - P.V, and we assume C.I-C.V, C.VI(SC), C.VII. **Can we demonstrate existence of general equilibrium in this economy, applying Theorem 18.1 (of *General Equilibrium Theory: An Introduction*, 2nd edition)?**

Circle the letter(s) of the correct answer(s) and explain your reasoning.

A. Yes. The technologies of F' are a bit strange, but all of the assumptions of Theorem 18.1 are fulfilled.

B. No. The technologies in F' are inconsistent with P.II, "$0 \in Y^j$". Theorem 18.1 is inapplicable.

C. No. The technologies in F' are inconsistent with P.III, "Y^j is closed for all $j \in F$". Theorem 18.1 is inapplicable.

D. No. The technologies in F' are inconsistent with P.I (convexity) and P.V (strict convexity). Theorem 18.1 is inapplicable. (Please ignore the (irrelevant) convex, but not strictly convex structure of Y^j along the x-axis where $y = 0$.)

E. None of the above responses is sound.

Explain:

SUGGESTED ANSWER: The correct answer is **C**. Theorem 18.1 cannot be correctly applied, because for $j \in F'$, Y^j is not closed. There may be prices where $S^j(p)$ is not well defined — where there may be no clear profit maximizing supply for firm $j \in F'$.

6 Futures and contingent commodity markets over time

Consider an Arrow-Debreu pure exchange economy with uncertainty and a full set of contingent commodity markets. The market takes place at a market date before any consumption is realized. There are two households, i = α, β. There are two periods, 0 and 1. Period 0 is certain; 1 has two states 1A and 1B. There is a single commodity deliverable in each date/event. The households differ in their probability judgments, their subjective probabilities denoted θ. For α, $\theta^{\alpha,1A} = 1, \theta^{\alpha,1B} = 0$. For β, $\theta^{\beta,1A} = 0, \theta^{\beta,1B} = 1$. The households have very similar maximands, optimizing an expected utility: $u(x^0, x^{1A}, x^{1B}) = v(x^0) + \theta^{1A}v(x^{1A}) + \theta^{1B}v(x^{1B})$, where the single period utility function $v(x) \equiv \sqrt{x}$ for $x \geq 0$.

The households have identical endowments $r^i = (r^{i,0}, r^{i,1A}, r^{i,1B}) = (1,1,1)$. Equilibrium prices (off the simplex to keep the algebra simple) then are $p = (p_0, p_{1A}, p_{1B}) = (1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. The equilibrium allocation on the contingent commodity market then is $x^\alpha = (x^{\alpha,0}, x^{\alpha,1A}, x^{\alpha,1B}) = (1, 2, 0)$, $x^\beta = (x^{\beta,0}, x^{\beta,1A}, x^{\beta,1B}) = (1, 0, 2)$.

6.i

In period 1, event 1A takes place. That leaves the allocation $x^\alpha = (x^{\alpha,0}, x^{\alpha,1}) = (1, 2)$, $x^\beta = (x^{\beta,0}, x^{\beta,1}) = (1, 0)$. The allocation is evidently Pareto inefficient, since there could be a mutually beneficial transfer of good 0 from β to α in exchange for good 1. **This appears to be a violation of the First Fundamental Theorem of Welfare Economics. Is it? Explain.**

SUGGESTED ANSWER: No. 1FTWE applies to the *ex ante* allocation of expected utility. Expected utility is allocated Pareto efficiently. In the contingent commodity context, 1FTWE does not apply to realized allocation. The proposed reallocation is intertemporal — requiring knowledge at date 0 of the outcome at date 1; an impossibility.

6.ii

Back at the market date, household β may decide to be more adventurous. Since β's $\theta^{\beta,1A} = 0$, he decides on $x^{\beta,1A} = -1$. In financial market terminology, β is selling the $1A$ good short. This allows him to increase $x^{\beta,0}$, and $x^{\beta,1B}$ consistent with budget constraint. Unfortunately for β, in period 1, event 1A takes place. In the US economy under this setting, β would declare bankruptcy in period 1, and default on his obligations. **How does the Arrow-Debreu model deal with this issue? Explain. Please keep your answer brief.**

SUGGESTED ANSWER: The Arrow-Debreu model does not allow the proposed short sale. β can only sell his endowment; he is not allowed to sell something he does not have or more than he has. Thus this instance of default and bankruptcy is prohibited/prevented by definition in the Arrow-Debreu model.