Exercises 13.1, 13.2 — Suggested Answers

13.1 An economy is generally said to be "competitive" if no agent in the economy has a significant effect in determining equilibrium prices. They cannot be price-setters. Is it an assumption or a conclusion in Chapters 11 through 13 that agents are competitive in this sense? If it is an assumption, where is it made? If a conclusion, where does it appear and what hypotheses is it based on?

Suggested Answer: Price taking behavior for firms and households is an assumption, not a conclusion, of the model. It does not show up as an axiom, but rather in the form of the functional form of supply and demand. Representing demand as $\tilde{D}^i(p)$ says that household i responds to prices p. i doesn't set prices, he adjusts to them. Representing supply as $\tilde{S}^j(p)$ says that firm j doesn't set prices, it adjusts to them.

13.2

Prove Theorem 13.1: Assume P.II, P.III, P.VI. $\tilde{\pi}^j(p)$ is a well-defined continuous function of p for all $p \in \mathbf{R}^N_+, p \neq 0$. $\tilde{\pi}^j(p)$ is homogeneous of degree 1.

Suggested Answer: $\tilde{\pi}^j(p) \equiv \sup\{p \cdot y | y \in \mathcal{Y}^j\}$. But under P.III, P.VI, $\mathcal{Y}^j$ is a compact set. By Theorem 7.6 and Corollary 7.2, $\tilde{\pi}^j(p)$ is then well-defined (the sup exists), and there is $y^\circ \in \mathcal{Y}^j$ so that $p \cdot y^\circ = \tilde{\pi}^j(p)$.

Proving that $\tilde{\pi}^j(p)$ is continuous in p without using P.V (strict convexity) is a bit trickier, but it can be done. Let $p^\nu \to p^\circ$, $p^\nu \cdot y^\nu = \tilde{\pi}^j(p^\nu), y^\nu \in \mathcal{Y}^j$. Then since $\mathcal{Y}^j$ is a compact set, y^ν has a convergent subsequence $y^\nu \to y^\circ$ and $y^\circ \in \mathcal{Y}^j$. But the dot product is a continuous function of its arguments, so $p^\nu \cdot y^\nu \to p^\circ \cdot y^\circ = \tilde{\pi}^j(p^\circ)$. That completes the demonstration.