Take-Home Midterm EVEN-EVEN-EVEN

This exam is take-home, open-book, open-notes. You may consult any published source (cite your references). Other people are closed. The exam you turn in should be your own personal work. Do not discuss with classmates, friends, professors (except with Prof. Starr or Ms. Fried — who promise to be clueless), until the examination is collected.

1

Consider the general competitive equilibrium of a production economy with redistributive taxation of income from endowment. Half of each household's income from endowment (based on actual endowment, not net sales) is taxed away. The proceeds of the tax are then distributed equally to all households. We then have,

$$M^i(p) = p \cdot (.5r^i) + \sum_{j \in F} \alpha^{ij} p \cdot y^j + T,$$

where T is the transfer of tax revenues to the household,

$$T = (1/\#H) \sum_{h \in H} p \cdot (.5r^h).$$

Assume that Walras's Law holds as an equality.

1. Define a competitive equilibrium in this economy.
2. Does a competitive equilibrium generally exist in this economy? Explain.

2

Consider the following Edgeworth Box example. Demonstrate that competitive equilibrium prices and allocations do not exist.

Do both parts 1 and 2.

Superscripts are used to denote the name of the households *and unfortunately, raising the consumption to a squared value — we'll try to keep them straight.* Households are characterized by a utility function and an endowment vector. The possible consumption set is the nonnegative quadrant, R^2_+. There are two commodities, x and y.

Household A is characterized as $\mathrm{u}^A(x,y) = [x]^2 + [y]^2$(where the terms in brackets are raised to the power 2), with endowment $r^A = (5,5)$. Household A's optimizing consumption subject to budget constraint will typically be a corner solution, so marginal equivalences will not be fulfilled as an equality.

Household B is characterized as $\mathrm{u}^B(x, y) = xy$(where neither term is raised to a power; it's just x^B times y^B), with endowment $\mathrm{r}^B = (5, 5)$.Denote A's demand as (x^A, y^A), B's as (x^B, y^B).

1. We claim there is no competitive equilibrium in this Edgeworth Box. Demonstrate this argument in the following way — clearly explain why each step is sound:

   $p_x > p_y$ implies there is an excess demand for y;

   $p_x < p_y$ implies there is an excess demand for x;

   $p_x = p_y$ implies there is an either an excess demand for x and an excess supply of y, or the opposite.

2. Explain which of the assumptions of Theorem 14.1 is not fulfilled.

3

Consider an Edgeworth Box for two households. The two goods are denoted x, y. The expression $\succ$ denotes strict preference; the expression $\sim$ denotes indifference, equivalence in preference. The households have identical preferences:

$$\begin{aligned}
(x, y) &\succ (x', y') \text{ if } x + 4y > x' + 4y', \text{ or} \\
(x, y) &\succ (x', y') \text{ if } x + 4y = x' + 4y' \text{ and } y > y'. \\
(x, y) &\sim (x', y') \text{ only if } (x, y) = (x', y').
\end{aligned}$$

They have identical endowments of (100, 100). Demonstrate that there is no competitive equilibrium. Is this example a counterexample to Theorem 18.1 (does it demonstrate that Theorem 18.1 is false?) ? If so, explain why Theorem 18.1 is false. If not, state which of Theorem 18.1's assumptions is not fulfilled and demonstrate that it is not fulfilled. (Hint: Consider demand behavior in the neighborhood of the price vector $(p_x, p_y) = (.2, .8)$.)