Consider the following double-log model of new car sales.

$$LQ_t = \alpha + \beta LY_t + \gamma LP_t + \delta Lr_t + \varepsilon LU_t + u_t$$

where L refers to the logarithm, Q is the number of new cars sold (in thousands) per million population, Y is per capita disposable personal income (in thousands of dollars), r is the prime interest rate in percents, and U is the unemployment rate in percents. Quarterly data are available for 16 years (64 observations).

1. (10 points)
You believe that the Greek letter parameters are not constant but vary across the four seasons. Derive a general model that can test that belief. Clearly define any new variables you create.

2. (5 points)
Because the data are quarterly, it is likely that an AR(4) model is appropriate for the error structure. To test this, write down the auxiliary regression model for $u_t$ and state the null hypothesis for no serial correlation.

3. (5 points)
Such a model was estimated with observations from 5 through 64 and the unadjusted $R^2$ was obtained as 0.557. Carry out an appropriate test (at the 1 percent level) and state your conclusions as to whether AR(4) is present or not.

4. (10 points)
Regardless of your conclusion in (3), describe step by step how you would use the Generalized Cochrane-Orcutt procedure to obtain the parameter estimates for the model you derived in (1) with AR(4) error terms.

A model was estimated along the lines of that in (1) with AR(3) error structure (because the fourth-order term was insignificant). Variables were then omitted based on the insignificance of the coefficients. The final estimated model is given below.

$$LQ_t = 6.5123 - 1.0481 \text{SUMMER} - 2.3187 \text{LP} + 0.2961 (\text{SUMMER} \times \text{LP}) + 2.7734 \text{LY} + 0.2214 (\text{SPRING} \times \text{LY}) - 0.1772 (\text{SPRING} \times \text{Lr}) - 0.1197 (\text{SUMMER} \times \text{Lr}) - 0.1140 (\text{FALL} \times \text{Lr}) + 0.1237 (\text{FALL} \times \text{LU})$$

where SUMMER = 1 for the Summer quarter and 0 elsewhere, SPRING = 1 for Spring quarter only, and FALL = 1 for the Fall quarter only, with Winter as the control.
5. (8 points)
Write down the estimated model for each of the four seasons without using any dummy variables.

6. (12 points)
First state what signs you would expect for each of the variables in the original model and why. Next state whether the signs in the models in (5) agree with your intuition or are there any counterintuitive signs? Finally, state which elasticities are elastic and which are inelastic.