Answers to Exam No. 1 on Topics from Chapters 2 through 4

1. 
   \( H_0 \) is that the coefficients for HSGPA, VSAT, and MSAT are all zero. \( H_1 \) is that at least one of the coefficients is nonzero. The test statistic is given in Equation (4.4) as
   \[
   F_c = \frac{(0.22/3)}{(0.78/423)} = 39.8
   \]
   Under the null hypothesis this has an \( F \)-distribution with d.f. 3 and 423. The critical \( F^*(0.01) = 3.8 \) which is well below \( F_c \). Therefore we reject the null hypothesis and conclude that at least one of the regression coefficients is nonzero.

2. A single regression coefficient is tested with a \( t \)-test. The critical \( t \) is \( t^*(0.01) = 2.33 \) (note that the alternative is one sided). The \( t \)-statistics for the coefficients of constant, HSGPA, VSAT, and MSAT are obtained by dividing the corresponding regression coefficients by their standard errors. These values are 1.92, 6.52, 2.63, and 3.46. Except for the constant term, all the rest are above 2.33. Therefore we conclude that all the coefficients are significant at the one percent level with the exception of the constant term which is not significant.

3. \[
\Delta \text{COLGPA} = 0.0007375 \Delta \text{VSAT} + 0.001015 \Delta \text{MSAT} = 0.07375 + 0.1015 = 0.17525
\]
   The expected average increase in COLGPA is therefore 0.175.

4. Let the general unrestricted model (U) be
   \[
   \text{COLGPA} = \beta_1 + \beta_2 \text{HSGPA} + \beta_3 \text{VSAT} + \beta_4 \text{MSAT} + u
   \]
   The marginal effect of VSAT is \( \beta_3 \) and the marginal effect of MSAT is \( \beta_4 \). The test is therefore \( \beta_3 = \beta_4 \). The alternative is that these two coefficients are unequal.

   **Method 1 (the Wald test):** Assume this condition and obtain the restricted model (R) as
   \[
   \text{COLGPA} = \beta_1 + \beta_2 \text{HSGPA} + \beta_3 (\text{VSAT} + \text{MSAT}) + v
   \]
   Generate the new variable \( Z = \text{VSAT} + \text{MSAT} \). Next regress COLGPA against a constant, HSGPA, and \( Z \), and save the error sum of squares. The Wald \( F \)-statistic is given by Equation (4.3). Reject \( H_0: \beta_3 = \beta_4 \) if \( F_c > F^* \), where \( F^* \) is obtained from \( F(1,423) \) such that the area to the right is equal to the level of significance.

   **Method 2 (an indirect \( t \)-test):** Let \( \beta = \beta_3 - \beta_4 \). Solving for \( \beta_4 \), we get \( \beta_4 = \beta_3 - \beta \). Substitute this in Model U. The modified model is
COLGPA = \( \beta_1 + \beta_2 \text{HSGPA} + \beta_3 \text{VSAT} + (\beta_1 - \beta) \text{MSAT} + \nu \)

Combining the \( \beta \) terms together, we get

\[
\text{COLGPA} = \beta_1 + \beta_2 \text{HSGPA} + \beta_3 Z - \beta \text{MSAT} + \nu
\]

where \( Z \) has been defined before. The test is conducted by regressing COLGPA against a constant, HSGPA, \( Z \), and MSAT, and using the regular \( t \)-test on the coefficient of MSAT.

**Method 3 (direct \( t \)-test):** The variance of the estimated difference \( \hat{\beta}_3 - \hat{\beta}_4 \) is given by

\[
\text{Var} (\hat{\beta}_3) + \text{Var} (\hat{\beta}_4) - 2 \text{Cov} (\hat{\beta}_3, \hat{\beta}_4)
\]

The computed \( t \)-statistic is therefore

\[
t_c = \frac{\hat{\beta}_3 - \hat{\beta}_4}{\text{Var}(\hat{\beta}_3) + \text{Var}(\hat{\beta}_4) - 2 \text{Cov}(\hat{\beta}_3, \hat{\beta}_4)}
\]

For a two-tailed test, \( H_0 \) is rejected if the numerical value of \( t_c \) exceeds \( t_{n-k}^* \) (level/2).

5. The major of a student is an important determinant of the GPA because some disciplines are easier to get good grades in and others are more difficult. Also, if a student went to a private school, he or she might have a better training and hence might do better in college. Thus, whether the student went to public or other types of school is important. If a student has to work for a living, then the grades are likely to suffer. Therefore the number of hours of employment might significantly affect the GPA. If a student spends a great deal of time commuting, then the grades might be lower as compared to another person who lives on campus. Not including these variables causes the "omitted variable bias" with biased and inconsistent estimates and forecasts. Furthermore, tests of hypotheses are invalid.