Answers to Exam on Topics from Chapters 2 through 11

I.1 Let $t$ refer to the time trend variable that takes the value 1 for the first observations, 2 for the second observation, and so on until $n$ for the last observation. The models are

(A) Straight Line: $Y_t = \beta_1 + \beta_2 t + u_t$

(B) Quadratic: $Y_t = \beta_1 + \beta_2 t + \beta_3 t^2 + u_t$

(C) Cubic: $Y_t = \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 t^3 + u_t$

(D) Linear-log: $Y_t = \beta_1 + \beta_2 \ln(t) + u_t$

(E) Reciprocal: $Y_t = \beta_1 + \beta_2 \left(\frac{1}{t}\right) + u_t$

(F) Log-linear: $\ln(Y_t) = \beta_1 + \beta_2 t + u_t$

(G) Double-log: $\ln\left(\frac{Y_t}{1-Y_t}\right) = \beta_1 + \beta_2 t + u_t \quad 0 < Y_t < 1$

(H) Logistic: $\ln\left(\frac{Y_t}{1-Y_t}\right) = \beta_1 + \beta_2 t + u_t \quad 0 < Y_t < 1$

I.2 First estimate each of the above models using the first 90 percent of observations and generate forecasts (call them $\hat{Y}_t$) for the last 10 percent. Compute the forecast error sum of squares ESS = $\Sigma(Y_t - \hat{Y}_t)^2$. Use ESS to compute the model selection statistics described in Chapter 4. The model with the lowest selection statistics would be judged the “best” according to these criteria. Also regress $Y_t$ against a constant and $\hat{Y}_t$ for each model forecasts. Ideally we would want the intercept term to be zero and the slope term to be 1. Use these and the previous criteria to select one or more models.

I.3 Regress $Y_t$ against a constant and the $\hat{Y}_t$ values from each of the three models. Use this regression to generate forecasts. They are the “best” in terms minimizing the error sum of squares and having a mean error of zero.

II.

The model is $C_t = \beta_1 + \beta_2 C_{t-1} + \beta_3 Y_t + \beta_4 Y_{t-1} + u_t$

with $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \rho_4 u_{t-4} + \epsilon_t$

A one-step ahead forecast for $t = n+1$ is (denoting forecasts of $Y_t$ made independently as $\hat{Y}_t$),

$\hat{C}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 C_n + \hat{\beta}_3 \hat{Y}_{n+1} + \hat{\beta}_4 Y_n + \hat{u}_{n+1}$

where $\hat{u}_{n+1} = \hat{\rho}_1 \hat{u}_n + \hat{\rho}_2 \hat{u}_{n-1} + \hat{\rho}_3 \hat{u}_{n-2} + \hat{\rho}_4 \hat{u}_{n-3}$

For higher steps, repeat this procedure with $\hat{C}_t$ instead of $C_t$ and $\hat{Y}_t$ for $Y_t$. 