

# Collective Inference and the Demand for Financial Information\*

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## Abstract

The tension between two margins of information acquisition is studied in a canonical two-period model of portfolio choice. At the intensive margin, an investor chooses the amount of information. At the extensive margin, investors subscribe to the information source. Only if the number of informed investors at the extensive margin is sufficiently small does acquisition of additional information at the intensive margin become rational in equilibrium. Uninformed investors' collective inference of information from asset price diminishes the asset's expected excess return so that investors value financial information in equilibrium only if sufficient exogenous noise in price burdens collective inference. Under constant relative risk aversion and joint Gaussian payoffs and signals, every signal inflicts a strict negative externality on an uninformed investor whose gain from collective inference does not outweigh the utility loss from diminished excess returns. Information acquisition is informationally inefficient in that informed investors acquire more signals than socially desirable. For information acquisition to occur in equilibrium, information access is priced with a two-part tariff that extracts from informed investors their utility gain over uninformed investors.

**Keywords:** Information acquisition; rational expectations equilibrium; portfolio choice

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Economic information is acquired at two margins. At the intensive margin, an agent chooses the precision or the amount of acquired information. At the extensive margin, more agents become informed and accumulate collective knowledge. In practice, most information is distributed in perfect copies. The business of media such as newspapers is to sell identical copies. A newspaper's extensive margin can be likened to its circulation, whereas the intensive margin is akin to the number of articles per newspaper, or its quality and news coverage. This paper augments a canonical two-period model of portfolio choice by a stage of information acquisition and investigates the consequences of information choice at the two margins for financial market outcomes and allocative efficiency.

There is a fundamental tension between the wider dissemination of information at the extensive margin and better information at the intensive margin. In equilibrium, investors rationally choose to raise the amount of their information at the intensive margin only if the number of uninformed investors at the extensive margin is sufficiently large. The reason is that more information diminishes a risky asset's expected excess return, or equity premium, because information makes investors bid up a risky asset's price. The diminishing excess return strictly reduces expected utility of a risk averse investor in incomplete markets. This negative excess-return effect of information is outweighed by the benefit of a more informed portfolio choice only if the number of other investors, who receive perfect copies of the same information, is sufficiently small. For investors with constant absolute risk aversion to become news subscribers in equilibrium, the majority of investors must remain uninformed about a risky asset with a normally distributed payoff. Even if newspaper articles are free of charge, a symmetric equilibrium with everyone equally informed does not exist. This equilibrium outcome underscores the importance of a distinction between the extensive and intensive margin of information acquisition. The extensive margin is crucial in that there must be at least one group of less informed investors for a news subscriber's information acquisition to be rational in equilibrium.

The necessary presence of uninformed investors is not sufficient for information acquisition to occur in equilibrium. The uninformed investors collectively infer information from observed asset price, and their updating diminishes the excess return. A potential news subscriber anticipates this. So, for an investor to value financial information in equilibrium, it is also necessary that price watchers' collective inference be hampered with sufficient exogenous noise in price. Facing much exogenous noise in equilibrium price, a price watcher rationally chooses to alter his beliefs little in response to informed investors' signal acquisition so that a potential news subscriber expects just a small reduction in the excess return. Intuitively, if a news subscriber can get informed behind the veil of a noisy asset price, collective updating by price watchers has little effect on equilibrium price so that a news subscriber's incentive for information acquisition is strong.

Every signal acquired by news subscribers inflicts a strict negative externality on price watchers, if price watchers have constant relative risk aversion and sig-

nals are jointly normally distributed. For a price watcher, collective inference from observed price never provides a sufficiently strong utility benefit to outweigh the utility loss from the diminished excess return. So, from a benevolent social planner's perspective, information acquisition in market equilibrium is informationally inefficient because news subscribers acquire more signals than the social planner would allocate. The negative externality on uninformed investors underscores that price informativeness, a commonly used statistic in the literature, is a moot welfare criterion because uninformed investors' utility can be strictly higher if price are completely uninformative. If, on the other hand, there is little exogenous noise in asset price so that collective inference would be relatively precise, no investor rationally acquires information because the negative effect of diminishing excess returns now outweighs any utility benefit for both price watchers and potential news subscribers. A social planner considers this no-information equilibrium informationally efficient.

The negative externality of signal acquisition on price watchers implies that information acquisition can only occur in equilibrium if access to information is priced with a two-part tariff. While the marginal cost of an additional newspaper article, or better newspaper quality, can be taken as a fundamental of the economy, the fixed information cost is an equilibrium outcome. News subscribers must incur an endogenous subscription fee for membership in the informed group. In the absence of a fixed information cost, every price watcher would have a strict incentive to become a news subscriber. But a symmetric equilibrium with everyone equally informed does not exist. So, information acquisition is a rational equilibrium outcome in financial markets if and only if information is priced with a two-part tariff. The fixed information cost clears the market for news subscription.

This paper generalizes the single-signal models in Grossman and Stiglitz (1980) and Barlevy and Veronesi (2000) to multiple signals, retaining the common assumptions of Gaussian random variables and CARA utility. These assumptions continue to guarantee closed-form solutions for financial-market equilibrium so that analytic results for information acquisition follow despite a lacking closed-form solution for information-market equilibrium. Beyond Verrecchia (1982), the model in this paper allows investors to acquire an arbitrary number of signals in duplicates but restricts investors to be either news subscribers or uninformed investors, for tractability. The model can be viewed as case of the general but hard-to-tract Admati and Pfleiderer (1987) setting, for which analytic solutions in this paper reveal several previously unnoticed properties. The present analysis uncovers, for instance, that signal acquisition exerts a strictly negative externality on uninformed investors in the Grossman and Stiglitz (1980) and Admati and Pfleiderer (1987) frameworks, rendering measures of the informativeness of price, such as its covariation with payoffs or its precision as a signal, moot statistics for welfare analysis. Tong (2005) emphasizes the intensive margin of signal precision and shows in a (Morris and Shin 2002) beauty-contest model of the financial market that public disclosure may crowd out private information gathering. Beyond the earlier settings, the present paper provides an

analysis of the interaction between collective inference and information acquisition at both the intensive and the extensive margin.

The analysis of information-market equilibrium, prior to financial-market opening, is based on conjugate prior updating in the spirit of Hellwig (1980) and Verrecchia (1982) and the experimentation literature (e.g. Moscarini and Smith 2001). Although the present paper formally adopts a discrete number of investors, the results carry over to the case of a continuum of investors.<sup>1</sup> The present paper extends the experimentation literature by considering Bayesian agents who rationally evaluate the consequences of information acquisition for the economy's general-equilibrium outcome. Instead of allowing investors to vary the precision of signals, the model of this paper gives investors a choice of the number of signals. The main motivation for this modification is that, in general, altering the prior variance of the signal also alters the prior distribution of the asset's payoff (take conjugate prior distributions such as the gamma-Poisson, gamma-gamma, or binomial-beta pairs as examples). This is implausible: most individuals would not alter their beliefs about stock-market performance based on their mere act of buying a newspaper. Although the normal-normal conjugate pair of signals and payoffs would formally permit a variation of signal precision independent of prior beliefs on payoffs, this paper considers the case of multiple signals to represent the intensive margin of better news coverage in more general terms.

Recent related research into demand for financial information includes Barlevy and Veronesi (2000) and Veldkamp (2006).<sup>2</sup> Imposing credit constraints on investors in a Grossman and Stiglitz (1980) style model, Barlevy and Veronesi (2000) show that a larger number of news subscribers at the extensive margin may cause less informative equilibrium price. The reason is that credit-constrained investors cannot freely take portfolio positions that reflect the news so that equilibrium price confounds binding credit constraints with informative asset demand. Barlevy and Veronesi (2000) conclude that the choices to join the subscribers at the extensive margin are strategic complements because additional credit-constrained news subscribers burden collective inference from equilibrium price, thus raising the incentive for news subscription. The analysis in the present paper suggests that more precise information at the intensive margin may reinforce this complementarity at the extensive margin, but an equilibrium derivation under credit constraints is left for future work.

In a Grossman and Stiglitz (1980) model, Veldkamp (2006) specifies an independent information-supply sector that offers a single signal at an endogenous cost of news subscription; the subscription cost is modelled to fall in the number of news

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<sup>1</sup>To close the model for a continuum of investors, assign initial risky-asset holdings to a small but dense measure of investors instead of assigning initial holdings to a single investor.

<sup>2</sup>There is a similar literature on information acquisition and transmission among oligopoly firms Li, McKelvey, and Page (see 1987); Raith (see 1996), which establishes the importance of market conditions for the value of information.

subscribers at the extensive margin. The resulting complementarity in investors' choices to become news subscribers gives rise to multiple equilibria with large or small numbers of informed investors at the extensive margin. The present setting with multiple signals and an intensive margin suggests a potential qualification to that result: if the number of news subscribers increases at the extensive margin because of lower subscription cost, news subscribers optimally choose to reduce the amount of information at the intensive margin.<sup>3</sup>

The present paper emphasizes demand for signals from an independent source of information so as to study incentives and externalities of signal acquisition for investors. Diamond (1985) and Diamond and Verrecchia (1991) study information supply as a decision by firms to disclose private information. In general, the truth-telling problem for security-issuing firms and asset-holding investors remains a little explored area of research (for a review of the disclosure literature see Verrecchia 2001). Admati and Pfleiderer (1988) show that a risk-averse monopolist in information chooses to sell information. Results in this paper suggest that collective inference through asset price introduces a fundamental trade-off between a larger subscriber base and higher-quality information to the subscribers, which opens an array of possibilities for research into information pricing strategies.

The framework has several testable implications. An insight is that collective updating diminishes expected excess return. Easley, Hvidkjaer, and O'Hara (2002) find for a set of NYSE listed stocks between 1983 and 1998 that assets exhibit a lower excess return if public information matters relatively more for their valuation. By extension, collective updating from asset price, a public signal, can be expected to diminish excess returns. Tong (2005) provides empirical evidence from panel data of analysts' forecasts on stocks in thirty countries that disclosure standards promote accuracy but reduce the number of analysts per stock. A testable implication of the trade-off between the extensive and intensive margin in the present framework is, for instance, that—irrespective of disclosure standards—assets whose analysts receive information more frequently or from more sources are studied by fewer analysts.

The paper proceeds in four main steps. Section 1 augments the canonical portfolio choice model with a preceding information acquisition stage. Section 2 presents the closed-form solution for financial-market equilibrium under rational expectations. I derive incentives and externalities of information acquisition in Section 3, while relegating derivation details to the Appendix. In Section 4, I characterize the information-market equilibrium and discuss welfare implications. Section 5 concludes.

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<sup>3</sup>As a consequence, asset price need not increase in an equilibrium with a large fraction of news subscribers because subscribers may collectively switch from a high-quality newspaper to a tabloid, and rational media frenzies would not occur.

# 1 Information and Portfolio Choice

There are two periods, today and tomorrow. There are two assets: one riskless bond and one risky stock. Assets are traded today, and have a payoff tomorrow. When Wall Street opens today at 10am, investors choose consumption today and their portfolio. The bond will pay the principal plus interest  $R \equiv 1+r$  tomorrow, whereas the stock will yield a risky payoff  $\theta$ . The price of the bond today is standardized to unity, and the stock sells for a price  $P$  to be set by the Walrasian auctioneer in financial-market equilibrium at 10am. All investors hold prior beliefs about the distribution of the payoff  $\theta$ . Newspapers are delivered to all subscribers at 9am today and contain  $N \in \mathbb{N}_0^+$  articles (signals) that inform about the risky asset's payoff at the intensive margin. While assets are assumed to be perfectly divisible, there is a countable number of signals.<sup>4</sup>

Investors can choose not to get the newspaper and only observe security prices at Wall Street. I call an investor who does not subscribe to a newspaper a *price watcher* ( $PW$ ). Price watchers know that the asset price  $P$  conveys market information about tomorrow's payoff since  $P$  is a function of all other investors' asset demands, which in turn reflect informed investors' knowledge. If at least some investors subscribe to the newspaper, price watchers can free-ride on newspaper subscribers' information by merely looking at the price. If the asset price contains exogenous supply or demand noise, however, then asset price never fully reveals a sufficient statistic about informed investors' signals.<sup>5</sup> A price watcher combines his prior knowledge about  $\theta$  with the information that he can extract from price, and then makes his portfolio choice.

Investors can choose to subscribe to a newspaper at 8am today. I call an investor who does so a *news subscriber* ( $NS$ ). News subscribers receive a newspaper copy at 9am. In addition to reading the newspaper, news subscribers could use price  $P$  to extract information but the information content of equilibrium asset price is redundant because price watchers contribute now independent information to equilibrium price. Using newspaper information requires a fixed but not necessarily sunk cost  $F$ . One can think of  $F$  as a fixed cost for reading the newspaper and taking time to interpret the information, or as a component in the newspaper's two-part tariff that will be charged iff the order is for more than zero articles.<sup>6</sup> At the

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<sup>4</sup>For a signal to contain information, its distribution has to depend on  $\theta$ . So, a continuum of signals (or an infinite number of them), will a.s. reveal the exact realization of  $\theta$  to news subscribers. For markets to clear,  $P$  must equal  $\theta/R$  in this case, otherwise news subscribers want to reshuffle their portfolio. But then the price fully reveals  $\theta$  itself and removes all uncertainty—an unrealistic case of little interest.

<sup>5</sup>In the extreme when there is no exogenous noise in price, the stock price at 10am fully reveals a sufficient statistic of the market information on  $\theta$ . This possibility is analyzed in Muendler (2007) in detail, and is a limiting case of the model in this paper when the variance of exogenous noise vanishes.

<sup>6</sup>The fact that  $F$  is fixed but not sunk allows news subscribers, who will be identical in equi-

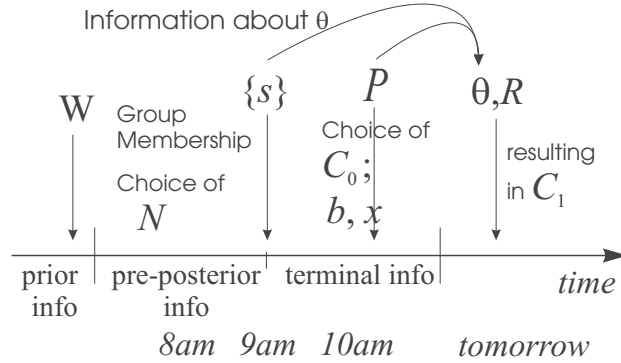


Figure 1: **Timing of Information Revelation and Decisions**

time of subscription at 8am today, news subscribers determine the informativeness of the newspaper, that is the amount of articles (the number of signals) to be put into the newspaper. Each article (signal) sells at a unit cost  $c$ . How many different articles should a news subscriber demand? When looking ahead at 8am today, each news subscriber knows that she will base her portfolio decision, to be taken at 10am today, on the information that she is about to get out of the newspaper articles at 9am. She also knows the statistical distribution of the information in any given article, which is more informative than her own prior beliefs. Taking all this into account, she rationally evaluates what an additional article is worth to her and makes her best choice. Formally, a news subscriber maximizes her pre-posterior expected indirect utility, or *ex ante* utility, with respect to the number of newspaper articles—given her information at 8am, given her expectation of how signals affect her beliefs at 9am, given her expected portfolio choice to be taken at 10am, and given the expected financial-market outcome under news subscribers' expected demands as well as price watchers' expected demands that result from price watchers' collective belief updating using equilibrium price.

The timing of decisions is illustrated in Figure 1. Given her wealth  $W_0^i$  and prior information, a news subscriber chooses at 8am the number  $N$  of newspaper articles (signals) to acquire. A news subscriber receives the realizations  $\{s_1, \dots, s_N\}$  of these  $N$  signals  $\{S_1, \dots, S_N\}$  at 9am (she gets to know the newspaper article contents). Given this information, at 10am she finally chooses consumption today,  $C_0^i$ , and decides how many bonds  $b^i$  and how many risky  $x^i$  assets to hold for consumption tomorrow.

Note the time difference between 8am, when investors choose the number of  
librium, to turn themselves into price watchers by demanding  $N = 0$  signals to be put into the newspaper.

newspaper articles, and 9am, when newspapers are delivered. An investor cannot know what is written in the newspaper articles when she takes her decision on information acquisition. Otherwise, she would not acquire the information (it would be part of her prior beliefs). In other words, there must be a logical intermission between the subscription to a newspaper and the revelation of its articles content, on which the portfolio decision will be based. In the terminology of Raiffa and Schlaifer (1961), the logical intermission makes the difference between *pre-posterior* beliefs, when the number of signals is chosen, and *terminal* beliefs, when the signal realizations are known. In her pre-posterior beliefs, a news subscriber rationally anticipates how the choice of  $N$  signals and their realizations to arrive will affect everyone's portfolio choice and financial-market equilibrium. Her terminal beliefs, in contrast, incorporate the signal realizations themselves. Pre-posterior beliefs are different from prior beliefs because of the anticipation of more precise future information. In utility terms, the logical intermission separates (pre-posterior) *ex ante* expected indirect utility from (terminal) expected indirect utility.<sup>7</sup>

News subscribers are a homogeneous group because they receive identical newspaper copies. They cannot independently decide on different amounts of information. Instead, they must jointly pick the optimal number of articles in the newspaper. For simplicity, think of a representative news subscriber who enters an agreement with the newspaper editor at 8am to include exactly  $N^*$  different articles and to deliver one newspaper copy with those articles to every news subscriber at 9am. If the group representative determines that a strictly positive number of newspaper articles be purchased, each news subscriber pays the cost  $c$  per newspaper article and incurs the fixed cost  $F$  upon reading the newspaper. If the group representative happens to decide that no newspaper be purchased, news subscribers jointly become price watchers and do not pay the fixed cost  $F$ . Because the subscriber representative is identical to every group member, no newspaper subscriber objects to the representative's choice. Among the  $I$  investors, a share  $\lambda \equiv I^{NS}/I \geq 0$  decides to be news subscribers in equilibrium at the extensive margin.

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<sup>7</sup>In the words of Raiffa and Schlaifer (1961):

[There] is a clear distinction between two completely different statistical problems: choice of a terminal act *after* an experiment has already been performed, which we call *terminal analysis*, and choice of the experiment which *is to be* performed, which we call *pre-posterior analysis*. ... [The] utility of any one potential experiment is evaluated by first using terminal analysis to determine the utility of the terminal act which will be optimal given each possible outcome; the pre-posterior expected utility of the experiment is then computed by taking the expectation of these terminal expected utilities with respect to the unconditional prior measure over the experimental outcomes.

In the current framework, terminal acts are portfolio choices and the experiment is the reading of newspaper articles. When referring to utility, I will keep the more common economic term of *ex ante* utility, but use the adjective pre-posterior to describe the according distributions at this stage of beliefs.



In this two-group framework, a rational-expectations equilibrium both at Wall Street and in the newspaper market can be defined as follows.

**Definition 1** Rational Information Choice Equilibrium (RICE). *A rational information choice equilibrium for two investor groups is an allocation of  $x^{i,*}$  risky assets and  $b^{i,*}$  riskless bonds to investors  $i = 1, \dots, I$ , a share  $\lambda^*$  of news subscribers, and an allocation of  $N^*$  signals to the news subscribers. It involves an asset price  $P$ , a signal cost  $c$  and a fixed cost of information receipt  $F$  along with a set of beliefs such that*

1. *Every investor finds membership in either the news subscribers or the price watchers group optimal ex ante while*
  - (a) *the choice of  $N^*$  signals is optimal for every news subscriber given that there are  $\lambda^*I$  news subscribers, and given the costs  $c$  and  $F$ , and*
  - (b) *receiving no signal is optimal for every price watcher given that there are  $\lambda^*I$  news subscribers receiving  $N^*$  signals,*
2. *Asset demands  $x^{i,*}$  and  $b^{i,*}$  are optimal for all investors  $i = 1, \dots, I$  given opportunity cost  $RP$  and their respective information sets,*
3. *The market for the risky asset clears,  $\sum_{i=1}^I x^{i,*} = X$ , where  $X$  is asset supply, and*
4. *Investors' beliefs are consistent with the equilibrium outcome.*

On the first stage of the game, at 8am, investors choose group membership and news subscribers simultaneously select the number of signals. A Bayesian Nash equilibrium results. The equilibrium is Bayesian since investors anticipate their rational updating of beliefs when signal realizations arrive. On the second stage, at 10am, a Walrasian competitive equilibrium results, given the Bayesian Nash equilibrium on the first stage. For simplicity, I consider a passive newspaper editor with no pricing power and focus on the informational properties of the equilibrium price. Note that any choice of  $c$  and  $F$  is compatible with definition 1 so that an extension to endogenous newspaper pricing continues to satisfy the definition of RICE.

RICE is an extension of REE to information acquisition. Condition 1 is the defining requirement for information-market equilibrium. In equilibrium, a news subscriber must not want to object to the group representative's choice of  $N$ . She must want to read the  $N$  newspaper articles that she is required to pay for and not want any additional article. Similarly, a price watcher must not have an incentive to switch group. If  $N^* = 0$  or  $\lambda^* = 0$  or both, everybody is a price watcher in equilibrium.

The derivation of RICE in this paper is based on an extension of Hellwig (1980) to investors with a choice of group membership at the extensive margin and multiple

signal acquisition at the intensive margin. Whereas Hellwig’s general model does not have an analytic solution, I aim at obtaining a closed-form financial-market solution to make information-market analysis (which continues to lack a closed form) tractable. There are two assumptions so far. First, the setup of the above-described model imposes that all information is sold in perfect copies (by bundling signals into a single newspaper). This is realistic considering that the large majority of investors obtains information from copies of publicly accessible media in practice. The assumption rules out, however, that an investor can receive additional information from the CEO of the stock-issuing firm. Second, there is no tabloid that would carry a subset of signals. Instead, every investor has a choice between just two groups at the extensive margin. She can either become a news subscriber, or become a price watcher. Both assumptions are incorporated in equilibrium definition 1.

I add the following assumptions.

**Assumption 1** (Common risk aversion) *Investors maximize additively separable utility, are risk averse and, all else equal, share a common degree of risk aversion.*

**Assumption 2** (Common priors) *Investors hold the same prior beliefs about the distributions of the risky asset return, the signals, and the supply of the risky asset.*

**Assumption 3** (Conditional independence) *Signals are conditionally independent given the payoff’s realization. Formally,  $S_j^i | \theta \stackrel{i.i.d.}{\sim} f(s_j^i | \theta)$ .*

**Assumption 4** (Equal precision) *All signals have a constant common precision.*

**Assumption 5** (No borrowing constraint) *Investors can carry out unlimited short sales.*

**Assumption 6** (Rationality) *Investors understand the correlation between signals and the asset price in equilibrium.*

**Assumption 7** (Random asset supply) *Asset supply is random, independent of any other variable, and initially owned by one investor who is prevented from single-handed information provision.*

**Assumption 8** (Price taking) *Investors are price takers in all markets.*

Assumptions 1 through 6 make the framework closely comparable to prior research. In a realistic information market equilibrium, asset price does not fully reveal a sufficient statistic of tomorrow’s payoff. Following much of the existing literature, I add exogenous noise to asset price through random asset supply under

Assumption 7.<sup>8</sup> If it were not for Assumption 7, Assumptions 1 through 6 (and 8) would make asset price fully revealing.

To close the model in the presence of random asset supply under Assumption 7, the initial asset endowment is assigned to a single investor, or equity issuer, who only learns the realization of her risky-asset endowment after the signal acquisition stage. A certain initial risky-asset endowment to every investor would introduce heterogeneity in the model that cannot be handled within a closed-form financial-market equilibrium. As Muendler (2007) shows, initial holdings of the risky asset would provide an additional incentive for the owner to acquire information because information raises expected asset price  $P$  and thus an asset owner's expected wealth.<sup>9</sup> So, heterogeneous initial asset holdings would prevent the formation of a homogeneous newspaper subscriber group.

The equity issuer holds the complete initial endowment of the risky asset but does not have a vote over the number of signals. In economic terms, the equity issuer's identity is publicly known but the issuer cannot commit to a fixed asset supply to the open market *ex ante*. The equity issuer can also not single-handedly determine the newspaper subscription because  $I^{NS} - 1$  other investors will outvote her. The equity issuer's signal is assumed to be the newspaper's first signal so that the equity issuer has no superior information.<sup>10</sup> Facing the certainty to be outvoted by the majority of newspaper subscribers, the initial holder of the risky asset is indifferent between price watching and a newspaper subscription in equilibrium and her presence has no bearing on the subsequent analysis.

As Hellwig (1980) observed, the price taking Assumption 8 stands in a certain contrast to Assumption 6 if there is a finite number of investors. Investors are assumed not to take into account how their asset demand affects price. Yet, they are assumed to perceive how the equilibrium price correlates with their own information through their demand. Hellwig called investors of this kind "schizophrenic." The concern is mitigated in this paper even for a discrete number of investors because equilibrium price only contains redundant information for news subscribers so that they rationally ignore asset price for updating, while price watchers rationally extract information from price but their actions contribute no information. Results in this paper carry over to the case of a continuum of investors.

For tractability, I impose the following functional forms.

**Assumption 9** (CARA) *Investors have CARA utility with  $u(C) = -e^{-\gamma C}$ .*

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<sup>8</sup>An alternative assumption would be to add liquidity traders (Barlevy and Veronesi 2000). Asset supply risk has the advantage that welfare analysis can be based on first principles of von-Neumann-Morgenstern utility.

<sup>9</sup>The special case with a binary choice of  $N \in \{0, 1\}$  in Muendler (2007) carries over to multiple signals. A formal proof for multiple Gaussian signals is given in Muendler (2002).

<sup>10</sup>The newspaper is an objective source of information in the sense that every signal is unbiased by Assumption 3. An investigation into an equity issuer's truth-telling incentives is beyond the scope of this paper.

**Assumption 10** (Normality) *Random variables are Gaussian.*

The assumption of Gaussian payoffs implies that the payoff realization can be negative or positive. Consequently, it entails the more profound assertion that investors are prevented from rejecting a negative payoff through a well-working legal system.

At 10am, investor  $i$  has individual information about the two parameters  $\mu_i$  and  $\tau_i$  of the risky asset's payoff distribution:  $\theta \sim \mathcal{N}(\mu_i, \tau_i^2)$  under investor  $i$ 's beliefs. At 8am, however, all investors still share the same priors about the distribution of  $\theta$  (Assumption 2). So,  $\mu_{i,prior} = \bar{\mu}_\theta$  and  $\tau_{i,prior} = \bar{\tau}_\theta$  at 8am. The normal distribution is a conjugate prior distribution to itself so that, under conditional independence of the signals (Assumption 3) with  $S_j|\theta \sim \mathcal{N}(\theta, \sigma_S^2)$  and under constant signal precision  $1/\sigma_S^2$  (Assumption 4), the normality Assumption 10 implies that investor  $i$  rationally updates her information between 8am and 10am in the following way.

**Fact 1** *Suppose that the prior distribution of  $\theta$  is a normal distribution with given mean  $\bar{\mu}_\theta$  and variance  $\bar{\tau}_\theta^2$ . Suppose also that the signals  $S_1, \dots, S_N$  are independently drawn from a normal distribution with unknown mean  $\theta$  and conditional variance  $\sigma_S^2$ . Then the terminal distribution of  $\theta$ , given the realizations  $s_1, \dots, s_N$  of the signals, is a normal distribution with a mean-variance ratio*

$$\frac{\mu_i}{\tau_i^2} = \frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2} \sum_{j=1}^N s_j$$

and variance

$$\tau_i^2 = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2} N}.$$

Fact 1 is a special case of Fact 2 in the Appendix. The mean-variance ratio  $\mu_i/\tau_i^2$  plays an important role for investors' decisions. The terminal mean  $\mu_i$  can be inferred by multiplying the mean-variance ratio with  $\tau_i^2$ . Since a sum of normal variables is normally distributed, Fact 1 implies that the pre-posterior expectation of the terminal mean is  $\mathbb{E}_{pre}^i[\mu_i] = \bar{\mu}_\theta$ . It is independent of the number of signals as it has to be in general by the law of iterated expectations. While the terminal mean is a random variable, the normal-normal conjugate pair of distributions has the rare property that the terminal variance  $\tau_i^2$  is certain given the chosen number of signals. It is important that the pre-posterior variance changes in the number of signals  $N$ , otherwise information would have no value. Indeed, Fact 1 is encouraging for risk averse individuals: news subscribers can lower the pre-posterior variance of the risky asset  $\mathbb{E}_{pre}^i[\tau_i^2] = \tau_i^2 = [(1/\bar{\tau}_\theta^2) + (1/\sigma_S^2)N]^{-1}$  by purchasing more information.

By CARA utility under Assumption 9, initial wealth will not matter for financial-market equilibrium. Initial wealth does matter for incentives of information acquisition, however. To obtain tractable analytic results for information-market equilibrium, investors within each group need to be identical. For the results in Section 4, I thus make an additional assumption.

**Assumption 11** (Identical wealth) *Initial bond holdings are identical,  $W_0^i = W_0$ , across all investors  $i = 1, \dots, I$ .*

This assumption would not be needed if we allowed for more than only two groups of investors. Then, however, no closed-form financial-market equilibrium would exist and properties of information-market would become intractable.

## 2 Financial Market Equilibrium

On the second stage at 10am, after having received the realizations of her  $N$  signals, every newspaper subscriber decides on asset holdings and consumption today, given the asset price and signal realizations. A price watcher receives no signals and simply relies on the price. In general, investor  $i$  maximizes additively separable utility under a discount factor  $\delta < 1$  and an instantaneous CARA utility function  $u(\cdot)$ . So, a newspaper subscriber maximizes

$$U^i = \mathbb{E} [u(C_0^i) + \delta u(C_1^i) | RP, \{s_1, \dots, s_N\}] \quad (1)$$

with respect to consumption today,  $C_0^i$ , and tomorrow,  $C_1^i$ , and a portfolio choice.  $RP$  is the opportunity cost of holding the risky asset. A price watcher's information set, in contrast, does not include  $\{s_1, \dots, s_N\}$ .

The budget constraint of investor  $i$  today is

$$b^i + Px^i = W_0^i - C_0^i - F^i - cN^i \quad (2-a)$$

so that

$$C_1^i = Rb^i + \theta x^i \quad (2-b)$$

will be available for consumption tomorrow. The investor is endowed with initial bond holdings  $W_0^i$ , and decides about her consumption  $C_0^i$  and  $C_1^i$  in each period, her holdings of the riskless bond  $b^i$ , her holdings of the risky stock  $x^i$ , and how much information  $N^i$  she wants. If the investor reads at least one newspaper article, she incurs the fixed cost  $F$ . To indicate this, I use the shorthand  $F^i \equiv \mathbf{1}(N^i > 0) \cdot F$ . Similarly, define  $N^i \equiv \mathbf{1}(i = NS) \cdot N$  because news subscribers receive identical copies of the  $N$  signals.

For CARA utility, the marginal utility ratios for tomorrow's and today's consumption are  $u'(C_1^i)/u'(C_0^i) = e^{-\gamma(C_1^i - C_0^i)}$ . So, an investor's first order conditions for bond and stock holdings become

$$\frac{1}{\delta} = R \mathbb{E}^i [e^{-\gamma(C_1^i - C_0^i)}] = R \cdot H^i \mathbb{E}^i [e^{-\gamma x^i \cdot \theta}] \quad (3-a)$$

$$\frac{P}{\delta} = \mathbb{E}^i [\theta e^{-\gamma(C_1^i - C_0^i)}] = H^i \mathbb{E}^i [\theta \cdot e^{-\gamma x^i \cdot \theta}] \quad (3-b)$$

where  $H^i \equiv \exp(-\gamma[(1+R)b^i + Px^i - W_0^i + F^i + cN^i])$  is certain. The expected values in (3-a) and (3-b) have simple closed-form solutions for a normally distributed payoff. They are reported as Facts 3 and 4 in Appendix A (p. 35). Applying these facts to (3-a) and (3-b), and dividing one by the other, yields demand for the risky asset

$$x^{i,*} = \frac{1}{\gamma \tau_i} \frac{\mu_i - RP}{\tau_i} \quad (4)$$

with  $\mu_i = \mathbb{E}^i[\theta]$ . As is well known, demand for the risky asset is independent of wealth for CARA utility. Throughout this paper, the term  $(\mu_i - RP)/\tau_i$  in (4) will be key. It denotes investor  $i$ 's expected excess return of the risky asset over the opportunity cost of the risky asset, normalized by the updated standard deviation of the payoff. Investor  $i$  goes short in the risky asset whenever  $\mathbb{E}^i[\theta] = \mu_i < RP$ , that is whenever her terminal expectation of the payoff falls short of opportunity costs  $RP$ , and goes long otherwise.

Investors extract information from asset price, which is a function of informed investors' signals. So, we need a generalization of Fact 1 to the case of correlated signals. The according property is reported as Fact 2 in Appendix A (p. 35). Rational investors, who know the correlation in equilibrium, update their beliefs accordingly. They infer a conditional distribution of  $\theta$ —given the signal realizations that they receive, given the equilibrium price that they observe, and correcting for the equilibrium correlation between signal realizations and equilibrium price. Rational investors base their portfolio choice on this inference (Assumption 6), and a rational-expectations equilibrium is a fixed point that results in no excess asset demands and consistent beliefs.

Guess that there is a financial market equilibrium under partly informative prices, in which asset price is a linear function of the signals  $\sum_j s_j$  and asset supply. In particular, suppose equilibrium price takes the form

$$RP = \pi_0 + \pi_S \sum_{j=1}^N S_j - \pi_X X \quad (5)$$

for three coefficients  $\pi_0, \pi_S, \pi_X$  to be determined. Risky asset supply is normally distributed with  $X \sim \mathcal{N}(\bar{x}, \varsigma_x^2)$ .

**News subscribers' terminal beliefs.** When Wall Street opens, a news subscriber knows that no other investor has superior information because price watchers receive no independent signal and other news subscribers receive exact copies of her own  $N$  signals. So the asset price only contains redundant information for her.<sup>11</sup>

Therefore, a news subscriber disregards  $RP$  and applies the common conjugate updating rule of Fact 1 (p. 12). Her terminal beliefs about the payoff are that the payoff is normally distributed with conditional mean

$$\mathbb{E}[\theta | RP; s_1, \dots, s_N; \lambda, N] = \mu_{NS} = m_0^{NS} + m_S^{NS} \sum_{j=1}^N s_j \quad (6)$$

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<sup>11</sup>Appendix C provides the formal proof for redundancy of  $RP$ .

and conditional variance  $\mathbb{V}(\theta | RP; s_1, \dots, s_N; \lambda, N) = \tau_{NS}^2$ , where

$$m_0^{NS} = \frac{\sigma_S^2 \bar{\mu}_\theta}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad (7-a)$$

$$m_S^{NS} = \frac{\bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad (7-b)$$

$$\tau_{NS}^2 = \frac{\sigma_S^2 \bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}. \quad (7-c)$$

**Price watchers' terminal beliefs.** To make his portfolio choice at 10am, a price watcher extracts information from the observation of  $RP$  and infers the expected payoff realization  $\theta$  and its variance applying Fact 2 (Appendix A, p. 35). At 9am, the price watcher knows that there are  $\lambda I$  news subscribers and that they read  $N$  newspaper articles. So, based on a price watcher's pre-posterior beliefs, the joint normal distribution of  $\theta$  and  $RP$  has a vector of means  $\bar{\mu}_{PW} = (\bar{\mu}_\theta; \pi_0 + \pi_S N \bar{\mu}_\theta - \pi_X \bar{x})^T$  and a variance-covariance matrix

$$\bar{\Sigma}^{PW} = \begin{pmatrix} \bar{\tau}_\theta^2 & \pi_S N \bar{\tau}_\theta^2 \\ \pi_S N \bar{\tau}_\theta^2 & \pi_S^2 N (N \bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \zeta_X^2 \end{pmatrix}.$$

Recall that signals are conditionally normally distributed  $S_j | \theta \sim \mathcal{N}(\theta, \sigma_S^2)$  so that  $\mathbb{V}(\sum_j^N S_j) = \mathbb{V}_\theta(\mathbb{E}[\sum_j^N S_j | \theta]) + \mathbb{E}_\theta[\mathbb{V}(\sum_j^N S_j | \theta)] = N^2 \bar{\tau}_\theta^2 + N \sigma_S^2$ .

When Wall Street opens, a price watcher observes  $RP$ , updates his pre-posterior to terminal beliefs applying Fact 2, and arrives at the updated expected value of the payoff

$$\mathbb{E}[\theta | RP; \lambda, N] = \mu_{PW} = m_0^{PW} + m_{RP}^{PW} RP \quad (8)$$

and the updated variance of the payoff  $\mathbb{V}(\theta | RP; \lambda, N) = \tau_{PW}^2$ , where

$$m_0^{PW} = \frac{(\pi_S^2 N \sigma_S^2 + \pi_X^2 \zeta_X^2) \bar{\mu}_\theta - \pi_S N (\pi_0 - \pi_X \bar{x}) \bar{\tau}_\theta^2}{\pi_S^2 N (N \bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \zeta_X^2}, \quad (9-a)$$

$$m_{RP}^{PW} = \frac{\pi_S N \bar{\tau}_\theta^2}{\pi_S^2 N (N \bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \zeta_X^2}, \quad (9-b)$$

$$\tau_{PW}^2 = \frac{(\pi_S^2 N \sigma_S^2 + \pi_X^2 \zeta_X^2) \bar{\tau}_\theta^2}{\pi_S^2 N (N \bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \zeta_X^2}. \quad (9-c)$$

**Financial-market equilibrium.** Investors base their portfolio decisions on their terminal beliefs. Their demand  $x^{i,*}$  is given by (4) for  $i = PW, NS$ . Asset markets at Wall Street must clear. So,

$$(1 - \lambda) \cdot x^{PW,*} + \lambda \cdot x^{NS,*} = \frac{x}{I},$$

where  $x$  is the realization of the uncertain asset supply  $X$ . Hence, the realization of equilibrium price must satisfy

$$\begin{aligned}
RP &= \frac{1}{(1-\lambda)\frac{1-m_{RP}^{PW}}{\tau_{PW}^2} + \lambda\frac{1}{\tau_{NS}^2}} \\
&\quad \left( (1-\lambda)\frac{m_0^{PW}}{\tau_{PW}^2} + \lambda\frac{m_0^{NS}}{\tau_{NS}^2} + \lambda\frac{m_S^{NS}}{\tau_{NS}^2} \sum_{j=1}^N s_j - \gamma\frac{x}{I} \right) \\
&= \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left[ (1-\lambda)\frac{\pi_S(\pi_S N - 1)}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \varsigma_X^2} + \lambda\frac{1}{\sigma_S^2} \right] N} \\
&\quad \left( \frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} - (1-\lambda)\frac{\pi_S N(\pi_0 - \pi_X \bar{x})}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \varsigma_X^2} + \lambda\frac{1}{\sigma_S^2} \sum_{j=1}^N s_j - \gamma\frac{x}{I} \right). \tag{10}
\end{aligned}$$

The second step follows from (7-a) through (7-c) and (9-a) through (9-c). We can now match the coefficients  $\pi_0$ ,  $\pi_S$ , and  $\pi_X$  in equation (5) with the according terms in (10). This yields a non-linear equation system in three equations and the three unknowns  $\pi_0$ ,  $\pi_S$ ,  $\pi_X$ . The equation system has a unique closed-form solution.<sup>12</sup>

**Lemma 1** *Under Assumptions 1 through 10, there exists a unique two-group financial-market equilibrium for a given share  $\lambda$  of news subscribers and a given number of signals  $N$ .*

**Proof.** Appendix D (p. 38) reports the derivation of this equilibrium and the matched coefficients  $\pi_0$ ,  $\pi_S$ ,  $\pi_X$  that satisfy conditions 2 through 4 of definition 1. Conditions 2 through 4 characterize a financial-market equilibrium for a given share  $\lambda$  of news subscribers and a given number of signals  $N$ . Uniqueness is established by assuming price to be a higher-order functional of  $\sum_{j=1}^{NS} S_j$  and  $X$ , and leading that assumption to a contradiction. ■

This financial-market equilibrium is a partial equilibrium, given  $\lambda I$  news subscribers who purchase  $N$  signals. The financial-market equilibrium is unaffected by investors' individual wealth because asset demand is independent of wealth for CARA utility. Given common priors and common risk aversion, whatever is optimal for one newspaper subscriber is also optimal for all other group members. It is thus an admissible simplification to consider a subscriber representative for information acquisition.

**Informativeness of equilibrium price.** The literature mostly considers two measures for the informativeness of price as a signal: its covariance with the fundamental, and its conditional precision. The equilibrium covariance between  $RP$

<sup>12</sup>The equation system would have no closed-form solution if there were a third group of investors (a second newspaper).



and  $\theta$  equals  $\pi_S N \bar{\tau}_\theta^2$  and strictly increases in the share  $\lambda$  of news subscribers and in the number of signals  $N$ . The conditional precision of the price can be defined as the inverse of its variance, conditional on signal realizations. This conditional variance equals  $\pi_X^2 \varsigma_X^2$  and strictly falls (precision increases) in the news subscriber share  $\lambda$  if  $N > (1 - 2\lambda)\sigma_S^2/(2\lambda\bar{\tau}_\theta^2)$ . Similarly, the conditional variance (precision) of price strictly falls (increases) in the number of signals  $N$  if  $N > (1 - \lambda)\sigma_S^2/(2\lambda\bar{\tau}_\theta^2)$ . The intuition for falling precision of price at low information levels  $N$  is that collective updating by price watchers moves expected asset price strongly even though the precision of the sum of the signals is low if there are few  $N$ . However, these measures of informativeness are somewhat moot in the context of information acquisition because, as the next section will show, price watchers suffer a strict negative externality from signal acquisition by news subscribers so that they would strictly prefer an uninformative asset price.

### 3 Incentives and Externalities

This section establishes necessary properties of information-market equilibrium and considers a non-trivial share of news subscribers  $\lambda \in (0, 1)$  as given. Note that news subscribers can set the optimal choice of signals to none so that an equilibrium with no information acquisition remains a special case even if  $\lambda$  is considered given. The existence proof for RICE follows in the next section. The necessary equilibrium properties in this sector establish the incentive for signal acquisition by news subscribers and the externality that information inflicts on price watchers through asset price moves.

At 8am, investor  $i$  chooses group membership and the number of signals she wants to receive. Her optimal signal choice  $N^*$  maximizes (pre-posterior) *ex ante* utility given  $\lambda$  and the fundamentals of the economy. *Ex ante* utility is  $\mathbb{E}_{pre}^i [U^{i,*}] = \mathbb{E}_{pre}^i [u(C_0^{i,*})] + \delta \mathbb{E}_{pre}^i [u(C_1^{i,*})]$ , an instance of the law of iterated expectations.

**Pre-posterior beliefs and the normalized excess return.** To derive *ex ante* utility, begin with terminal expected utility at 10am and work the way back in time. Given optimal asset demand  $x^{i,*}$  for CARA, terminal indirect utility of investor  $i$  in financial-market equilibrium is

$$U^{i,*} = -k^i \cdot e^{\gamma \frac{R}{1+R}(F^i + cN^i)} \mathbb{E}^i \left[ e^{-\gamma x^{i,*}(\theta - RP)} \right]^{\frac{1}{1+R}} \quad (11)$$

with the definitions  $k^i \equiv \frac{1+R}{R}(\delta R)^{\frac{1}{1+R}} \exp \left\{ -\gamma \frac{R}{1+R} W_0^i \right\} > 0$ ,  $F^i \equiv \mathbf{1}(N^i > 0)F$  and  $N^i \equiv \mathbf{1}(i = NS)N$  (see Appendix B, p. 37). For a normal distribution of the payoff, the last factor in (11) becomes

$$\mathbb{E}^i \left[ \exp \left\{ -\gamma x^{i,*}(\theta - RP) \right\} \right] = \exp \left\{ -\frac{1}{2} \left( \frac{\mu_i - RP}{\tau_i} \right)^2 \right\}$$

by Fact 3 and asset demand (4). Note that  $RP$  is known in Walrasian equilibrium at Wall Street and thus certain from a terminal point of view. Using the above expression in (11) and taking pre-posterior expectations, *ex ante* utility of investor  $i = PW, NS$  thus is

$$\mathbb{E}_{pre}^i[U^{i,*}] = -k^i \cdot e^{\gamma \frac{R}{1+R}(F^i + cN^i)} \cdot \mathbb{E}_{pre}^i \left[ \exp \left\{ -\frac{1}{2} \frac{1}{1+R} \left( \frac{\mu_i - RP}{\tau_i} \right)^2 \right\} \right]. \quad (12)$$

News subscribers evaluate (12) to decide the optimal number of newspaper articles  $N^*$ . Price watchers cannot choose the level of information but evaluate (12) to assess the effect of information on their utility.

The key term in (12) is

$$\frac{\mu_i - RP}{\tau_i}.$$

I refer to this key term as the *normalized excess return*. The normalized excess return is an investor's expected excess payoff of the risky asset over the risky asset's opportunity cost in terms of the bond, normalized by the expected standard deviation of the payoff. The normalized excess return reflects two objectives for a risk averse investor. For one, a risk averse investor wants a high excess return over opportunity cost, all else equal. The larger the expected difference  $\mu_i - RP$  in incomplete markets, the better for her consumption tomorrow. On the other hand, she also wants a possibly low variance of her portfolio since she is risk averse. The lower  $\tau_i$ , the better. Additional information affects the normalized excess return in several ways.

Given the closed-form financial-market equilibrium of Lemma 1, the normalized excess return can be expressed in closed form as a function of  $\lambda$ ,  $N$ , and parameters for all investors  $i = PW, NS$ . Parameters are: the interest factor  $R$ ; the prior means and variances  $\bar{\mu}_\theta$ ,  $\bar{\tau}_\theta^2$ ,  $\sigma_S^2$ ;  $\bar{x}$ ,  $\zeta_X^2$ ; the degree of risk aversion  $\gamma$ ; and the number of investors  $I$ .<sup>13</sup> The particular solutions are less important than their properties. So, the explicit terms are relegated to Appendix E (p. 39). As will become clear shortly, what matters for information acquisition are the two pre-posterior moments of the normalized excess return. Price watchers and news subscribers rationally hold different pre-posterior beliefs about these two moments.

The subjective variance of the payoff  $\tau_i^2$  differs for news subscribers and price watchers but it is deterministic for both investor groups by the properties of the normal distribution (see (7-c) and (9-c)). Moreover, the opportunity cost  $RP$  is a sum of normal variables (see (5)) and the terminal mean of the payoff  $\mu_i$  is a sum of normal variables (see (6) and (8)). Since the sum of normal variables is normally distributed, rational investors apply Fact 5 (in Appendix A, p. 36) to (12) and find

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<sup>13</sup>Initial bond holdings  $W_0^i$  and the discount factor  $\delta$  are irrelevant for risky-asset demand under CARA utility.

their (pre-posterior) *ex ante* utility to be

$$\mathbb{E}_{pre}^i[U^{i,*}] = -k^i \cdot \exp \left\{ \gamma \frac{R}{1+R} (F^i + cN^i) \right\} \quad (13)$$

$$\cdot \frac{1}{\sqrt{1 + \frac{1}{1+R} \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right)}} \exp \left\{ -\frac{1}{2} \frac{1}{1+R} \frac{1}{1 + \frac{1}{1+R} \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right)} \left( \mathbb{E}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i} \right] \right)^2 \right\}.$$

Appendix F (p. 40) reports the parametric expressions for the normalized excess return's pre-posterior mean  $\mathbb{E}_{pre}^i[(\mu_i - RP)/\tau_i]$  and variance  $\mathbb{V}_{pre}^i((\mu_i - RP)/\tau_i)$ .

Since  $\mathbb{E}_{pre}^i[U^{i,*}]$  is negative for CARA utility, any change that reduces (13) in absolute value is beneficial. Hence, *ex ante* utility is increasing in the pre-posterior mean of the excess return  $\mathbb{E}_{pre}^i[\mu_i - RP]$ . A higher expected excess return means higher expected consumption tomorrow. The variance of the expected excess return  $\mathbb{V}_{pre}^i(\mu_i - RP)$ , in contrast, has a twofold effect on *ex ante* utility. First, given expected excess returns, a higher variance means that the investor faces a more volatile expected consumption tomorrow and thus suffers a utility loss. This effect is captured by the variance expression in the last factor in (13), which reduces the expected excess return in the exponent. Second, as the variance of the excess return increases, expected asset price in financial-market equilibrium drops. A lower asset price, in turn, means a lower opportunity cost of the risky asset and therefore a higher excess return, thus benefitting the investor in incomplete markets. The square root of the variance expression in the denominator reflects this effect.

**Utility effects of the number of signals.** The number of signals  $N$  affects utility through the moments of the normalized excess return. A news subscriber's optimal signal choice  $N^*(\lambda; I, \gamma, R; \bar{x}, \varsigma_X^2; \sigma_S^2, \bar{\tau}_\theta^2; c)$  maximizes (pre-posterior) *ex ante* utility (13) given  $\lambda$  and the fundamentals of the economy. The prior expectation of the asset payoff  $\bar{\mu}_\theta$  does not enter  $N^*(\cdot)$  because the expected normalized excess return is net of prior payoff expectations.  $N^*(\cdot)$  does not depend on initial wealth  $W^{NS}$  or the discount factor  $\delta$  because risky asset demand (4) is independent of wealth  $W^{NS}$  and the discount factor  $\delta$  under CARA. The fixed information cost  $F$  does not affect  $N^*(\cdot)$  because  $F$  only alters wealth.

The signal choice is over a discrete number of signals. It is nevertheless instructive to take the derivative of *ex ante* utility (13) with respect to  $N$ . This derivative shows the sign of a local change in information. If the sign is unaltered for ranges of  $N$  or all  $N \in \mathbb{N}_0^+$ , the local derivative reflects information-acquisition incentives for news subscribers and externalities on price watchers.<sup>14</sup> Similarly, I use the derivative of

<sup>14</sup>Under monotonicity conditions, the first derivative of  $\mathbb{E}_{pre}^i[U^{i,*}]$  with respect to  $N$ , set to zero, is similar to a necessary first-order condition for an optimal choice of  $N$  in the following sense. Note that  $\mathbb{E}_{pre}^i[U^{i,*}]$  is differentiable in  $N$  by CARA utility. If  $\mathbb{E}_{pre}^i[U^{i,*}]$  changes monotonically in  $N$ , it

$\mathbb{E}_{pre}^{PW} [U^{PW}]$  with respect to  $N$  to investigate the externality that an additional signal inflicts on price watchers. Taking the derivative and multiplying it by the positive factor  $-(1+R)/\mathbb{E}_{pre}^i [U^{i,*}]$  for clarity yields

$$-\frac{1+R}{\mathbb{E}_{pre}^i [U^{i,*}]} \frac{\partial \mathbb{E}_{pre}^i [U^{i,*}]}{\partial N} = -\gamma Rc \mathbf{1}(i=NS) \quad (14)$$

$$+ E_i(\lambda, N) \cdot \varepsilon_{\mathbb{E},N}^i(\lambda, N)$$

$$+ V_i(\lambda, N) \cdot \frac{1}{2} \varepsilon_{\mathbb{V},N}^i(\lambda, N) \cdot \Delta_i(\lambda, N),$$

where  $\varepsilon_{\mathbb{E},N}^i$  is the elasticity of the mean  $\mathbb{E}_{pre}^i[(\mu_i - RP)/\tau_i]$  and  $\varepsilon_{\mathbb{V},N}^i$  is the elasticity of the variance  $\mathbb{V}_{pre}^i((\mu_i - RP)/\tau_i)$  with respect to  $N$ . The definitions of the terms  $E_i(\lambda, N)$ ,  $V_i(\lambda, N)$ , and  $\Delta_i(\lambda, N)$  are

$$E_i(\lambda, N) \equiv \frac{1}{N} \frac{\left( \mathbb{E}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i} \right] \right)^2}{1 + \frac{1}{1+R} \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right)}, \quad (14-a)$$

$$V_i(\lambda, N) \equiv \frac{1}{N} \frac{\frac{1}{1+R} \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right)}{\left[ 1 + \frac{1}{1+R} \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right) \right]^2}, \quad (14-b)$$

$$\Delta_i(\lambda, N) \equiv (1+R) + \mathbb{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i} \right) - \left( \mathbb{E}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i} \right] \right)^2. \quad (14-c)$$

Derivative (14) has the following interpretation. For a news subscriber,  $\gamma Rc$  is the marginal utility loss from the expenditure on an additional signal. A price watcher does not receive a signal and not pay. The second term reflects the marginal utility change from a signal-induced change in the mean normalized excess return  $\mathbb{E}_{pre}^i[(\mu_i - RP)/\tau_i]$  for investor  $i = NS, PW$ . Similarly, the third term captures the utility change from a signal-induced change in the variance  $\mathbb{V}_{pre}^i((\mu_i - RP)/\tau_i)$ . Note that  $E_i(\lambda, N)$  and  $V_i(\lambda, N)$  are strictly positive for both news subscribers and price watchers but that the factor  $\Delta_i(\lambda, N)$  can be positive or negative for either investor. It reflects the ambiguous effect that an increase in the variance has on *ex ante* utility: more signals reduce the pre-posterior variance, thus raising expected consumption through improved portfolio choice, but a reduced pre-posterior variance also depresses pre-posterior expectations of the excess return in incomplete markets because more informed investors bid up expected relative asset price  $RP$ .

Table 1 displays the signs of elasticities for moments of the excess return when the number of signals increases. The pre-posterior mean of the raw excess return falls

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gives rise to a differentiable function  $N^*(\cdot)$  of economic fundamentals and, for  $N^* \in \mathbb{N}_0^+$ , there is a monotonic and continuous step function  $\tilde{N}^*(\cdot)$  around  $N^*(\cdot)$ . Then  $\tilde{N}^*(\cdot)$  captures optimal signal choice in the discrete case.

Table 1: ELASTICITIES OF EXCESS RETURN AND PAYOFF MOMENTS

	$i = NS$	compare	$i = PW$
$\varepsilon_{(\mathbb{E}_{pre}^i[\theta - RP]), N}^a$	$< 0$	$=$	$< 0$
$\varepsilon_{\tau^2, N}^i{}^b$	$< 0$	$\varepsilon_{\tau^2, N}^{NS} < \varepsilon_{\tau^2, N}^{PW}$ iff $\lambda IN < \gamma \sigma_S^2 \varsigma_X / \bar{\tau}_\theta$	$< 0$
$\varepsilon_{\mathbb{E}, N}^i{}^b$	<i>ambiguous</i> ( $> 0$ only if $\varsigma_X^2$ large) <sup>c</sup>	$\varepsilon_{\mathbb{E}, N}^{NS} > \varepsilon_{\mathbb{E}, N}^{PW}$ iff $\lambda IN < \gamma \sigma_S^2 \varsigma_X / \bar{\tau}_\theta$	$< 0$
$\frac{1}{2} \varepsilon_{\mathbb{V}, N}^i{}^b$	<i>ambiguous</i> ( $> 0$ only if $\varsigma_X^2$ large) <sup>c</sup>	$>$	$< 0$

<sup>a</sup>This follows from (G.7) in Appendix G, p. 42.

<sup>b</sup>Elasticities are reported in (G.1), (G.4), and (G.8) through (G.15) in Appendix G, p. 41.

<sup>c</sup>Given  $I$ ,  $\gamma$ ,  $\sigma_S^2$  and  $\bar{\tau}_\theta^2$ .

both for news subscribers and price watchers ( $\varepsilon_{(\mathbb{E}_{pre}^i[\theta - RP]), N}$ ). Note that the drop in the expected raw excess return solely occurs through an increase in equilibrium price because, by the law of iterated expectations, pre-posterior and terminal expectations of the asset payoff are the same:  $\mathbb{E}_{pre}^i[\theta] = \mathbb{E}^i[\theta]$ . More signals make investors bid up (pre-posterior) expected asset price for two reasons: news subscribers obtain more precise independent information so that they demand more of the asset for any given price, and price watchers use observed asset price as a signal in rational-expectations equilibrium and also demand more of the asset for any given price. Both news subscribers and price watchers rationally anticipate the asset-price effect of collective updating and fully agree on the magnitude of the asset-price move. Consequently, their pre-posterior expectations of a signal's impact on the raw excess return are identical, as indicated by the equality sign in the comparison column. Lemma 2 restates this insight more formally.

**Lemma 2** *Under Assumptions 1 through 10 and for a given share  $\lambda \in (0, 1)$  of news subscribers, an increase in the number of signals  $N$  strictly reduces the pre-posterior expectation of the risky asset's raw excess return  $\mathbb{E}_{pre}^i[\theta - RP]$  for every investor  $i$ . News subscribers and price watchers expect an equivalent drop in  $\mathbb{E}_{pre}^i[\theta - RP]$ .*

**Proof.** See (G.7) in Appendix G, p. 42. ■

As Table 1 also shows, the pre-posterior variance of the asset's raw payoff  $\tau_i^2$  falls when more information becomes available ( $\varepsilon_{\tau^2, N}^i$ ). The magnitude of the drop, however, differs between news subscribers and price watchers. The reason is that news

subscribers receive direct information on the asset payoff whereas price watchers update information from observed asset price. If the total amount of information in the market,  $\lambda IN$ , is small relative to the supply-noise in price  $\zeta_X^2$ , then price watchers respond less strongly than news subscribers in their pre-posterior variance beliefs. News subscribers hold more precise payoff information. If, on the other hand,  $\lambda IN$  is large relative to price noise  $\zeta_X^2$ , then price watchers rationally over-interpret observed price moves as due to news subscribers' information, and not to price noise, so that they reset their pre-posterior variance beliefs more elastically than news subscribers.

The pre-posterior mean of the normalized excess return reflects the reverse responses ( $\varepsilon_{\mathbb{E},N}^i$ ). The *normalized* excess return is the raw excess return divided by the standard deviation of the asset's raw payoff. Since both news subscribers and price watchers rationally update the raw excess return in exactly the same way, only their difference in updating the standard deviation of the asset's raw payoff matters for the pre-posterior mean of the normalized excess return. So, if  $\lambda IN$  is small relative to the supply-noise in price  $\zeta_X^2$ , then news subscribers reduce their pre-posterior payoff variance beliefs more elastically than price watchers and thus reduce their pre-posterior expectation of the normalized excess return less elastically. In fact, if  $\zeta_X^2$  is sufficiently large, then news subscribers do not reduce but raise their pre-posterior expectation of the normalized excess return (see Appendix G for a proof, p. 41). The reason is that, if  $\zeta_X^2$  is large, price watchers do not alter their payoff beliefs much in response to news subscribers' signal acquisition so that pre-posterior expected asset price increases little and news subscribers do not suffer a strong reduction in the expected excess return from signal acquisition. Intuitively, if news subscribers can get informed behind the veil of a noisy asset price, collective updating by price watchers has little effect on equilibrium price. Then news subscribers have a strong incentive for information acquisition.

The pre-posterior variance of the normalized excess return drops for price watchers ( $\varepsilon_{\mathbb{V},N}^i$ ). News subscribers, however, do not reduce the pre-posterior variance of the normalized excess return as elastically as price watchers. The reason is that, whenever a price watcher observes a deviation of asset price from its prior mean, he rationally infers that news subscribers' information moved asset price with a pre-posterior probability. Price watchers' collective inference exaggerates exogenous price moves that are due to mere supply noise. In cases of negative supply shocks, for instance, a price watcher rationally assigns a probability that the higher-than-prior-expected asset price is due to news subscribers' favorable return information and not necessarily to noise so that price watchers' collective inference bids asset price up even further. As a consequence of the presence of price watchers, a news subscriber's pre-posterior variance of the normalized excess return does not necessarily drop as much with additional signals as does a price watcher's pre-posterior variance belief. Intuitively, if the prior payoff variance is small, then collective updating by price watchers has a relatively strong effect on normalized excess returns

Table 2: UTILITY RESPONSES TO SIGNAL ACQUISITION

	$i = NS$	compare	$i = PW$
$E_i \cdot \varepsilon_{\mathbb{E},N}^i$ <sup>a</sup>	<i>ambiguous</i> ( $> 0$ only if $\zeta_X^2$ large) <sup>b</sup>	.	$< 0$
$V_i \cdot \frac{1}{2}\varepsilon_{\mathbb{V},N}^i$ <sup>a</sup>	<i>ambiguous</i> ( $> 0$ only if $\zeta_X^2$ large) <sup>b</sup>	.	$< 0$
$\Delta_i^{a,c}$	$> 0$ iff $\bar{x} < \bar{x}_{cutoff}^{\Delta,NS}$	$\bar{x}_{cutoff}^{\Delta,NS} < \bar{x}_{cutoff}^{\Delta,PW}$	$> 0$ iff $\bar{x} < \bar{x}_{cutoff}^{\Delta,PW}$

<sup>a</sup>For derivations see Appendix H, p. 45.

<sup>b</sup>Given  $I$ ,  $\gamma$ ,  $\sigma_S^2$  and  $\bar{\tau}_\theta^2$ .

<sup>c</sup>Definitions of the threshold values  $\bar{x}_c^{\Delta,i}$  are given in (H.9) and (H.14), p. 46.

through equilibrium price.

Table 2 reports how the number of signals  $N$  affects *ex ante* utility through the moments of the normalized excess return. Multiplication of  $E_{NS}$  with  $\varepsilon_{\mathbb{E},N}^{NS}$  and of  $V_{NS}$  with  $\frac{1}{2}\varepsilon_{\mathbb{V},N}^{NS}$  does not resolve the sign ambiguity for news subscribers, but the effect of an additional signal on a price watcher can be signed.

**Utility effect of signals on a price watcher.** For a price watcher, the utility effect of an additional signal is negative at least if  $\Delta_{PW} > 0$ . If the risky-asset market is thin with expected asset supply  $\bar{x} < \bar{x}_{cutoff}^{\Delta,PW}$ , then  $\Delta_{PW} > 0$  and a signal to news subscribers necessarily inflicts a strict negative externality on price watchers. The reason is that the negative utility effect of a diminished excess return strictly outweighs a price watcher's utility benefit from more informed portfolio choice and less risky consumption. The relative magnitudes of  $E_{PW} > 0$  and  $V_{PW} > 0$  matter, however, so that *ex ante* utility of price watchers can be negatively affected by signal acquisition even for lower levels of  $\bar{x}$ . In fact, price watchers suffer a strictly negative externality at any level of  $\bar{x}$ .

**Theorem 1** *Under Assumptions 1 through 10 and for a given share  $\lambda \in (0, 1)$  of news subscribers, a signal to news subscribers inflicts a strict negative externality on price watchers in a two-group RICE.*

**Proof.** Note that  $\mathbb{E}_{pre}^{PW}[(\mu_{PW} - RP)/\tau_{PW}]^2 = a \cdot \bar{x}^2$  for some  $a > 0$  (see (9-c) and (F.3) in Appendix F). The pre-posterior variance  $\mathbb{V}_{pre}^{PW}((\mu_{PW} - RP)/\tau_{PW})$  and the elasticities  $\varepsilon_{\mathbb{E},N}^{PW}$  and  $\varepsilon_{\mathbb{V},N}^{PW}$  do not include  $\bar{x}$  (they include only higher moments of  $X$ ). For simplicity, define  $b \equiv \mathbb{V}_{pre}^{PW}(\cdot)/(1+R) > 0$ ,  $d \equiv \varepsilon_{\mathbb{E},N}^{PW}/N < 0$  and

$g \equiv \frac{1}{2}\varepsilon_{v,N}^{PW}/N < 0$ , where the signs of  $d$  and  $g$  follow from Table 1. In financial-market equilibrium (definition 1),  $d + b(d - g) < 0$  (for a parametric solution see Appendix I, p. 46). So, a price watcher's *ex ante* utility strictly increases with signals iff

$$-\frac{1+R}{\mathbb{E}_{pre}^{PW}[U^{PW*}]} \frac{\partial \mathbb{E}_{pre}^{PW}[U^{PW*}]}{\partial N} = \frac{a}{1+b} \frac{d+b(d-g)}{1+b} \bar{x}^2 + \frac{(1+R)bg}{1+b} > 0$$

$$\Leftrightarrow \bar{x}^2 < -\frac{(1+R)(1+b)bg}{a[d+b(d-g)]} < 0.$$

But  $\bar{x} > 0$  by Assumption 7. So, a price watcher's *ex ante* utility strictly decreases with every signal. ■

News subscribers maximize their utility but disregard how they affect *ex ante* utility of a price watcher. News subscribers, who update their priors and thus raise expected asset price  $RP$  with every signal they acquire, strictly diminish the expected excess return regardless of whether a price watcher uses equilibrium price to update his beliefs or not. The expected increase in asset price means foregone excess returns. In incomplete markets, for constant absolute risk aversion and under Gaussian returns and signals, a price watcher's expected loss from diminished excess returns strictly outweighs his gain from consumption smoothing through a more informed portfolio choice.

The result is stark. One might imagine that, when the risky-asset market is thick (a large  $\bar{x}$ ) and the noise in price matters little (a small  $\zeta_X^2$ ), price watchers could extract much information from price and would benefit from the variance-lowering effect of better information. Not so in the current framework. The utility-reducing effect of a shrinking excess return dominates (for any values of  $\bar{x}$  and  $\zeta_X^2$ ) so that every signal to news subscribers inflicts a strict negative externality on price watchers. If a price watcher had an initial holding of the risky asset, the pre-posterior expectation of a rising asset price would amount to a positive wealth effect that could potentially counteract the negative externality for large enough initial asset holdings. The (strict) negative externality remains, however, if a price watcher has small (no) initial holdings of the risky asset.

**Utility effect of signals for a news subscriber.** Though a representative news subscriber disregards the strict utility loss to price watchers, a representative news subscriber does take into account that price watchers extract information from equilibrium price and that their updated demands in turn affect asset price. This collective inference has important consequences for the news subscribers' incentive to acquire information. The marginal utility effect of an additional signal (14) reflects the incentive for optimal signal acquisition. In principle, when (14) is set to zero, it gives rise to an implicit function of the number of signals in the share of news



subscribers  $\lambda$  and the fundamentals of the economy  $N^*(\lambda; I, \gamma, R; \bar{x}, \varsigma_X^2; \sigma_S^2, \bar{\tau}_\theta^2; c)$ . Despite the simplifications of the model, there is no closed-form solution for this function  $N^*(\lambda; \cdot)$ .

Table 3 reports noteworthy limits for a news subscriber's components of (14). When the asset price becomes perfectly informative as  $\varsigma_X \rightarrow 0$ , news subscribers perceive the negative impact on the excess return more strongly than the positive impact on the variance of the excess return and acquire no single signal. In other words, without the veil of noise in asset price, there is no news subscriber. This is the extreme case of a fully revealing asset price.<sup>15</sup>

When asset price completely ceases to be informative, as  $\varsigma_X \rightarrow \infty$ , price watchers cannot extract information. So news subscribers do not face a diminishing excess return effect from price watchers, only from information acquisition by other news subscribers. Then, a news subscriber has a strictly positive marginal incentive to add one more newspaper article iff the present number of newspaper articles satisfies  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  (which also implies that  $\lambda < 1/2$ ). Theorem 2 below establishes this condition more formally.

When investors become risk neutral and  $\gamma \rightarrow 0$ , no one minds receiving signals for free, but no one would pay either: the marginal utility of an additional signal is zero. A risk neutral investor  $i$  does not care about consumption risk and chooses asset holdings so that  $\mathbb{E}_{prd}^i[\theta - RP] = 0$ , and are no excess returns for a risk neutral investor. As absolute risk aversion grows to dominate any other fundamental of the model,  $\gamma \rightarrow \infty$ , a news subscriber's marginal utility gain from an additional signal becomes

$$E_{NS} \varepsilon_{E,N}^{NS} + V_{NS} \frac{1}{2} \varepsilon_{V,N}^{NS} \Delta_{NS} = \frac{(1+R)\bar{\tau}_\theta^2 [(1-2\lambda)\sigma_S^2 - \lambda N \bar{\tau}_\theta^2] (N \bar{\tau}_\theta^2 \bar{x}^2 + \sigma_S^2 \varsigma_X^2)}{2(\sigma_S^2 + N \bar{\tau}_\theta^2)^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)}. \quad (15)$$

Even in this limit, a news subscriber has a strictly positive marginal incentive to add one more newspaper article only if the present number of newspaper articles satisfies  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$ .

When signal precision  $1/\sigma_S^2 \rightarrow 0$  vanishes, signals become useless. Then investors would accept signals for free because they have no effect on actions or asset price. When signals become infinitely precise,  $1/\sigma_S^2 \rightarrow \infty$ , they show nature's draw of  $\theta$  today. The risky asset turns into a second bond and  $RP = \theta$  in financial-market equilibrium. Additional signals become useless also in this limit and investors accept them for free but would not pay. Similarly, removing the prior risk from payoffs,  $\bar{\tau}_\theta^2 \rightarrow 0$ , turns the risky asset into a second bond so that  $RP = \theta$  in financial-market equilibrium. Again, additional signals become useless and investors accept them for free but would not pay.

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<sup>15</sup>If there were a continuum of investors, Grossman and Stiglitz (1980) show that no equilibrium would exist (because the continuum assumption cannot be reconciled with the existence of a sufficient statistic in price; Muendler 2007). In the present model with a countably large number of investors, the two-group RICE exists for perfectly informative asset price but involves no information acquisition (Muendler 2002).

Table 3: INCENTIVES FOR NEWS SUBSCRIBERS IN THE LIMIT

	$\epsilon_{\mathbb{E},N}^{NS}$	$E_{NS} \cdot \epsilon_{\mathbb{E},N}^{NS}$	$\frac{1}{2}\epsilon_{\mathbb{V},N}^i$	$V_{NS} \cdot \frac{1}{2}\epsilon_{\mathbb{V},N}^{NS} \cdot \Delta_{NS}$
$\lim_{\varsigma_X \rightarrow 0^a}$	$-\frac{\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	$-\frac{\gamma^2\sigma_S^2\tau_\theta^4}{2I^2(\sigma_S^2+N\tau_\theta^2)^2}\bar{x}^2$	$-\frac{\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	0
$\lim_{\varsigma_X \rightarrow \infty^a}$	$\frac{\sigma_S^2}{\sigma_S^2+\lambda N\tau_\theta^2} - \frac{\sigma_S^2+\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	0	$\frac{\sigma_S^2}{\sigma_S^2+\lambda N\tau_\theta^2} - \frac{\sigma_S^2+\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	$\frac{(1+R)\sigma_S^2\tau_\theta^2[(1-2\lambda)\sigma_S^2-\lambda N\tau_\theta^2]}{2(\sigma_S^2+N\tau_\theta^2)^2(\sigma_S^2+\lambda N\tau_\theta^2)}$
$\lim_{\gamma \rightarrow 0^a}$	$-\frac{\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	0	$-\frac{\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	0
$\lim_{\gamma \rightarrow \infty^a}$	$\frac{\sigma_S^2}{\sigma_S^2+\lambda N\tau_\theta^2} - \frac{\sigma_S^2+\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	$\frac{(1+R)\tau_\theta^2[(1-2\lambda)\sigma_S^2-\lambda N\tau_\theta^2]}{2(\sigma_S^2+N\tau_\theta^2)(\sigma_S^2+\lambda N\tau_\theta^2)}\bar{x}^2$	$\frac{\sigma_S^2}{\sigma_S^2+\lambda N\tau_\theta^2} - \frac{\sigma_S^2+\frac{1}{2}N\tau_\theta^2}{\sigma_S^2+N\tau_\theta^2}$	$\frac{(1+R)\sigma_S^2\tau_\theta^2[(1-2\lambda)\sigma_S^2-\lambda N\tau_\theta^2]}{2(\sigma_S^2+N\tau_\theta^2)^2(\sigma_S^2+\lambda N\tau_\theta^2)}\varsigma_X^2 - \bar{x}^2$
$\lim_{\sigma_S \rightarrow 0^a}$	0	0	0	0
$\lim_{\sigma_S \rightarrow \infty^a}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
$\lim_{\tau_\theta \rightarrow 0^a}$	0	0	$< 0$	0

<sup>a</sup>Limits for elasticities of news subscribers' moments follow from (G.8) and (G.11) in Appendix G, p. 42, and for the utility components from (H.6), (H.7) and (H.8) in Appendix H, p. 45.

A news subscriber expects a strictly positive marginal utility gain from an additional signal under conditions that have already become clear in the limiting cases above. The conditions generalize.

**Theorem 2** *Under Assumptions 1 through 10 and for a given share  $\lambda \in (0, \frac{1}{2})$  of news subscribers, a signal has a strictly positive marginal utility value for a news subscriber in RICE iff*

- the number of signals satisfies

$$N < \frac{(1-2\lambda)\sigma_S^2}{\lambda\bar{\tau}_\theta^2},$$

and

- either

$$\zeta_X^2 \in (\bar{\zeta}^2, \bar{\zeta}^2) \quad \text{and} \quad \bar{x}^2 < \bar{z}^2$$

or

$$\zeta_X^2 \geq \bar{\zeta}^2,$$

where  $\bar{\zeta}^2$  and  $\bar{z}^2$  are the unique real and positive roots of  $\varepsilon_{\mathbb{V},N}^{NS}$  and  $[\varepsilon_{\mathbb{E},N}^{NS} + \frac{1}{1+R}\mathbb{V}_{pre}^{NW} \cdot (\varepsilon_{\mathbb{E},N}^{NS} - \frac{1}{2}\varepsilon_{\mathbb{V},N}^{NS})]$ , and  $\bar{z}^2 \equiv \|[ (1 + \frac{1}{1+R}\mathbb{V}_{pre}^{NW})\mathbb{V}_{pre}^{NW} \frac{1}{2}\varepsilon_{\mathbb{V},N}^{NS} ] / \{ [\varepsilon_{\mathbb{E},N}^{NS} + \frac{1}{1+R}\mathbb{V}_{pre}^{NW} \cdot (\varepsilon_{\mathbb{E},N}^{NS} - \frac{1}{2}\varepsilon_{\mathbb{V},N}^{NS})] [\partial(\mathbb{E}_{pre}^{NW})^2 / \partial \bar{x}^2] \}\|$ .

*Under Assumptions 1 through 10 and for a given share  $\lambda \in [\frac{1}{2}, 1)$ , a signal has a strictly negative marginal utility value to a potential news subscriber in RICE.*

**Proof.** Set  $c = 0$ . Define  $A = \mathbb{E}_{pre}^{NS}[(\mu_{NS} - RP)/\tau_{NS}]^2/\bar{x}^2 > 0$  ( $A$  does not depend on  $\bar{x}^2$  by (7-c) and (F.1) in Appendix F),  $B \equiv \mathbb{V}_{pre}^{NS}(\cdot)/(1+R) > 0$ ,  $D \equiv \varepsilon_{\mathbb{E},N}^{NS}/N$  and  $G \equiv \frac{1}{2}\varepsilon_{\mathbb{V},N}^{NS}/N$ . In financial-market equilibrium (definition 1), a news subscriber's *ex ante* utility strictly increases with signals iff

$$-\frac{1+R}{\mathbb{E}_{pre}^{NS}[U^{NS*}]} \frac{\partial \mathbb{E}_{pre}^{NS}[U^{NS*}]}{\partial N} = \frac{A}{1+B} \frac{D+B(D-G)}{1+B} \bar{x}^2 + \frac{(1+R)BG}{1+B} > 0. \quad (16)$$

Note that  $D < G$  (see (G.13) in Appendix G, p. 44).  $D$  is a second-order polynomial in  $\zeta_X^2$  and has a single positive root at  $\bar{\zeta}^2$  only if  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  (see (G.9) in Appendix G, p. 42).  $G$  is a third-order polynomial in  $\zeta_X^2$  and has (two imaginary roots and) a unique real root at  $\bar{\zeta}^2$ , which is positive iff  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  (see (G.12) in Appendix G, p. 44). Since  $D < G$ ,  $0 < \bar{\zeta}^2 < \zeta^2$ . Note that, as  $\zeta_X^2 \rightarrow \infty$ , condition (16) is strictly satisfied with the limit (15).

Suppose  $\zeta_X^2 \leq \bar{\zeta}^2$ . Then  $D < 0$ ,  $G \leq 0$ , and thus  $G/D < 1$  by  $D < G$ . So,  $D + B(D - G) < 0$  because, for  $D < 0$ ,  $D + B(D - G) \geq 0$  can only be satisfied if  $(1+B)/B \leq G/D$  while  $G/D < 1$ , a contradiction for  $B > 0$ . Therefore, the first

term on the left-hand side of condition (16) is strictly negative, the second term is weakly negative, and condition (16) is violated.

Suppose  $\zeta^2 < \zeta_X^2 \leq \bar{\zeta}^2$ . Consider strict inequality first,  $\zeta_X^2 < \bar{\zeta}^2$ . Then  $D < 0$ ,  $G > 0$ , and thus  $G/D < 0$ . So,  $D + B(D - G) < 0$  because, for  $D < 0$ ,  $D + B(D - G) \geq 0$  can only be satisfied if  $(1+B)/B \leq G/D$  while  $G/D < 0$ , a contradiction for  $B > 0$ . Therefore, condition (16) is satisfied iff  $\bar{x}^2 < -(1+R)(1+B)BG/\{A[D+B(D-G)]\} \equiv \bar{z}^2$ , where  $\bar{z}^2$  is strictly positive under  $G > 0$  and  $D + B(D - G) < 0$ . Now consider equality,  $\zeta_X^2 = \bar{\zeta}^2$ . Then  $D = 0$  and  $G > 0$ . Therefore, condition (16) is satisfied iff  $\bar{x}^2 < (1+R)(1+B)BG/\{ABG\}$ , a special case of  $\bar{z}^2$  for  $D = 0$ , and strictly positive under  $G > 0$ .

Suppose  $\zeta^2 < \zeta_X^2 < \bar{\zeta}^2$ , where  $\bar{\zeta}^2$  is the positive real root of the third-order polynomial  $D + B(D - G)$ . For  $\zeta^2 < \zeta_X^2$ ,  $D > 0$ ,  $G > 0$ , and thus  $G/D > 1$  by  $D < G$ . Also,  $D + B(D - G) < D$  by  $D < G$ , so the real root of  $D + B(D - G)$  must satisfy  $\bar{\zeta}^2 > \zeta^2$ . It is unique because  $B > 0$  and  $G$  has a unique real root. For  $\zeta_X^2 \leq \bar{\zeta}^2$ ,  $D + B(D - G) < 0$  and  $G/D > (1+B)/B > 1$ . Therefore, condition (16) is satisfied iff  $\bar{x}^2 < -(1+R)(1+B)BG/\{A[D+B(D-G)]\} \equiv \bar{z}^2$  as above, where  $\bar{z}^2$  is strictly positive under  $G > 0$  and  $D + B(D - G) < 0$ .

Suppose  $\zeta_X^2 \geq \bar{\zeta}^2$ . Consider equality first,  $\zeta_X^2 = \bar{\zeta}^2$  so that  $D + B(D - G) = 0$ . Then condition (16) is strictly satisfied for any  $\bar{x}^2$  because  $G > 0$  at  $\zeta_X^2 = \bar{\zeta}^2 > \zeta^2$ . Now consider strict inequality. For  $\zeta_X^2 > \bar{\zeta}^2$ ,  $D + B(D - G) > 0$  and  $(1+B)/B > G/D > 1$ . Therefore, the first term and second term on the left-hand side of condition (16) are strictly positive, and condition (16) is satisfied for any  $\bar{x}^2$ .

Suppose  $\lambda \geq 1/2$ . Then  $G < 0$ ,  $D < 0$  by  $D < G$ , and  $G/D < 1$  by  $D < G$ . So,  $D + B(D - G) < 0$  because, for  $D < 0$ ,  $D + B(D - G) \geq 0$  can only be satisfied if  $(1+B)/B \leq G/D$  while  $G/D < 1$ , a contradiction for  $B > 0$ . Therefore, the first term and the second term on the left-hand side of condition (16) are strictly negative, and condition (16) is violated.  $\blacksquare$

Theorem 2 proves it impossible that all investors become news subscribers. There must be at least one price watcher in a two-group RICE. In fact, strictly more than half of the investors must be price watchers under the assumptions of the model. Even if newspaper articles are free of charge, a symmetric equilibrium with equally well informed investors who receive copies of the same newspaper articles does not exist. Moreover, the higher the share of news subscribers  $\lambda$ , the lower the maximal number of newspaper articles they will rationally acquire:  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  strictly falls in  $\lambda$ . In other words, the more widely available information is across investors, the less precise information must be.

For low levels of exogenous noise in the asset price, no investor becomes a news subscriber because price is informative and collective updating by price watchers would strongly diminish the expected excess return. At intermediate levels of exogenous noise in the asset price, investors start to have a strict incentive to acquire information if expected asset supply is sufficiently small. The intuition is that small

expected asset supply implies low expected asset holdings in financial-market equilibrium. So, the diminished excess return weighs less heavily because the expected portfolio holdings of the risky asset are small. As exogenous noise in the price system becomes high, investors have a strict incentive to acquire information. As the limit (15) shows too, for a large level of exogenous noise in the asset price, news subscribers acquire information up to  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  irrespective of the size of the asset market.

## 4 Information Market Equilibrium

The preceding section has established properties of information equilibrium for a given share  $\lambda$  of news subscribers. There is no closed-form expression for signal demand as a function of  $\lambda$  and fundamentals  $N^*(\lambda; I, \gamma, R; \bar{x}, \varsigma_X^2; \sigma_S^2, \bar{\tau}_\theta^2; c)$ . But Theorem 2 has shown that for  $\lambda < \frac{1}{2}$  and sufficiently large exogenous price noise  $\varsigma_X^2$ , news subscribers have a strict incentive to acquire up to  $N < (1-2\lambda)\sigma_S^2/(\lambda\bar{\tau}_\theta^2)$  newspaper articles. Given the optimal choice  $N^*(\lambda; \cdot)$  in RICE, it remains to find the equilibrium share of news subscribers  $\lambda^*(I, \gamma, R; \bar{x}, \varsigma_X^2; \sigma_S^2, \bar{\tau}_\theta^2; c; W, \delta, \bar{\mu}_\theta, F)$ .

What is the incentive for an investor to start a news subscriber club with one member—herself? If signals are perfectly divisible, a potential news subscriber evaluates<sup>16</sup>

$$\begin{aligned} \lim_{\lambda \rightarrow 0} -\frac{1}{\mathbb{E}_{pre}^{NS}[U^{NS}]} \frac{\partial \mathbb{E}_{pre}^{NS}[U^{NS}]}{\partial N} \Big|_{N=0} - F = \\ -\gamma R c - F \\ + \frac{\bar{\tau}_\theta^2}{2\sigma_S^2} \frac{I^4(1+R) + \gamma^4 \sigma_S^4 \varsigma_X^4 + I^2 \gamma^2 \bar{\tau}_\theta^4 [R\bar{x}^2 + (2+R)\varsigma_X^2]}{(I^2(1+R) + \gamma^2 \bar{\tau}_\theta^2 \varsigma_X^2)^2}. \end{aligned} \quad (17)$$

to judge whether she should become a news subscriber and acquire the first signal. For sufficiently small  $c$  and  $F$ , the investor has a strictly positive incentive to become the first news subscriber. The first derivative of (17) with respect to  $\varsigma_X^2$  is

$$\frac{I^2 R \gamma^2 \bar{\tau}_\theta^4}{2\sigma_S^2} \cdot \frac{I^2(1+R) + \gamma^2 \bar{\tau}_\theta^2 (\varsigma_X^2 - 2\bar{x}^2)}{(I^2(1+R) + \gamma^2 \bar{\tau}_\theta^2 \varsigma_X^2)^3} > 0 \quad \Leftrightarrow \quad \varsigma_X^2 > 2\bar{x}^2 - \frac{I^2(1+R)}{\gamma^2 \bar{\tau}_\theta^2}.$$

So, a sufficiently large increase in the level of exogenous price noise  $\varsigma_X^2$  raises the incentive for information acquisition arbitrarily strongly. This suggests that a discrete change from no signal to the first signal raises the first news subscriber's *ex ante* utility for a large level of exogenous price noise.

<sup>16</sup>For countably many investors, the limit as  $\lambda \rightarrow 1/I$  would be more adequate but does not appear to be tractable.

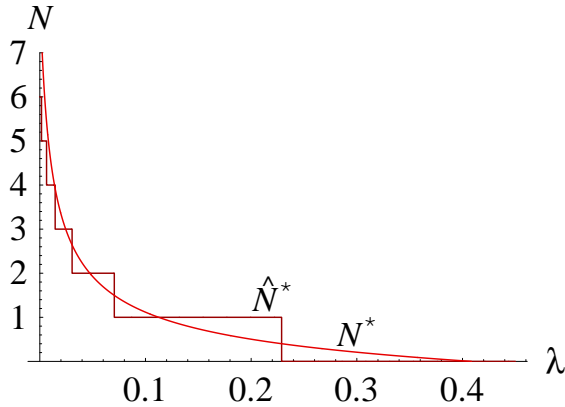


Figure 2: **Optimal Choice of  $N$  and  $\lambda$**

Setting the marginal utility effect of signal (14) equal to zero implicitly defines a number of perfectly divisible signals  $N^*(\lambda, c; \cdot)$ . The acquisition rule  $N^*(\lambda, c; \cdot)$  approximates a news subscriber's optimal choice of signals, up to imperfect divisibility, given the share  $\lambda$  of members in the news subscriber club. In the present framework, acquisition rule  $N^*(\lambda, c; \cdot)$  has no closed form (it is a fourth-order polynomial in  $\lambda$ ). The falling curve in Figure 2 is a plot of condition (14).<sup>17</sup> It shows combinations of  $N$  and  $\lambda$  for which (14) is satisfied. The curve shifts to the South-west when the cost of a signal  $c$  increases. Signal choice has to be discrete, however, so the optimal choice of  $N^*$  given  $\lambda$  is a step function  $\hat{N}^*(\lambda; c)$ . Figure 2 depicts the step-shaped *newspaper acquisition curve* alongside, based on a direct evaluation of the news subscriber's *ex ante* utility (13). Note that  $\hat{N}^*(\lambda, c; \cdot)$  neither depends on initial wealth  $W_0$  nor on information cost  $F$  by CARA utility. The two curves illustrate how condition (14) approximates the exact incentives.

In RICE, every news subscriber must find it weakly preferable to belong to the news subscriber group:  $\mathbb{E}_{pre}^{NS}[U^{NS}(N, \lambda; c, F)] \geq \mathbb{E}_{pre}^{PW}[U^{PW}(N, \lambda)]$ . Similarly, every price watcher's *ex ante* utility must weakly exceed that of news subscribers. So,

$$\mathbb{E}_{pre}^{NS}[U^{NS}(N, \lambda; c, F)] - \mathbb{E}_{pre}^{PW}[U^{PW}(N, \lambda)] = 0 \quad (18)$$

must hold in equilibrium. Given news subscribers' optimal signal choice  $\hat{N}^*(\lambda, c; \cdot)$ , this condition implies an equilibrium share of news subscribers  $\lambda^*(c, F)$ . Condition 18 also implies that the initial wealth of investors within each group must be the same if the same fixed information cost  $F$  applies to every news subscriber. Assumption 11 imposes homogeneous initial wealth on all investors except for the single equity issuer, who alone has no bearing on the information decision (Assumption 7).

<sup>17</sup>The underlying parameter values are  $I = 100$ ,  $\gamma = 1$ ,  $R = 1.1$ ,  $\bar{x} = 1$ ;  $\varsigma_X = 100$ ;  $\sigma_S = 1$ ,  $\bar{\tau}_\theta = 1$ ,  $c = .005$ .

In the present framework, information has to be priced with a two-part tariff. Otherwise no equilibrium in the newspaper market exists as long as at least one investor has an incentive to become a news subscriber. The reason is that a signal to a news subscriber inflicts a strict negative externality on a price watcher through diminishing excess return but must raise the news subscriber's *ex ante* utility by revealed preference. So, indifference between being a price watcher and belonging to the news subscriber group cannot be assured with the marginal cost per newspaper article. In the absence of a fixed information cost and if the conditions for signal acquisition in Theorem 2 hold, all price watchers would strictly prefer news subscription, but  $\lambda = 1$  cannot be an equilibrium.

**Theorem 3** *Under Assumptions 1 through 11 and if the conditions for signal acquisition in Theorem 2 hold, a RICE exists only if the fixed information cost  $F$  is strictly positive.*

**Proof.** Suppose that  $F = 0$  but  $\lambda \geq 1/I$  and  $N^* \geq 1$  in RICE. A news subscriber is free to change  $N$  so  $\mathbb{E}_{pre}^{NS}[U^{NS}(N^*)] \geq \mathbb{E}_{pre}^{NS}[U^{NS}(N=0)]$  by revealed preference. By Theorem 2, there must be at least one price watcher,  $\lambda^* < 1/2$ . By Theorem 1, price watchers face a negative externality so that they suffer a utility loss  $\mathbb{E}_{pre}^{PW}[U^{PW}(N \geq 1)] < \mathbb{E}_{pre}^{PW}[U^{PW}(N=0)]$ . Since,  $\mathbb{E}_{pre}^{NS}[U^{NS}(N=0)] = \mathbb{E}_{pre}^{PW}[U^{PW}(N=0)]$  we can infer that, for small markets,  $\mathbb{E}_{pre}^{NS}[U^{NS}(N \geq 1)] > \mathbb{E}_{pre}^{PW}[U^{PW}(N \geq 1)]$ . So, in equilibrium a strictly positive fixed information cost  $F$  must bring news subscribers' utility down to price watchers' utility. ■

Theorem 3 clarifies that the fixed information cost  $F$  needs to clear the market for membership in the news subscriber group. While the marginal cost for an additional newspaper article  $c$  can be treated as a fundamental of the economy, the fixed information cost  $F$  is an equilibrium outcome.

Figure 3 shows contour plots of condition (18) for various levels of the fixed cost  $F$ .<sup>18</sup> The *indifference contours* depict a news subscriber's  $N$ - $\lambda$  combinations for which the news subscriber attains a specific utility level. Given the vector of economic fundamentals  $(I, \delta, \gamma, W, R, \bar{\mu}_\theta; \bar{x}, \varsigma_X^2; \sigma_S^2, \bar{\tau}_\theta^2; c)$ , a news subscriber's *ex ante* utility varies in  $N$ ,  $\lambda$  and  $F$ . The *indifference contours* do not necessarily satisfy a functional relationship between  $N$  and  $\lambda$ . In fact, most depicted two-group indifference contours are complicated correspondences between  $N$  and  $\lambda$ . Start at  $F = c$ . This high *ex ante* utility level is not attainable for the depicted newspaper acquisition curve, which is based on  $c$ . Reduce a news subscriber's *ex ante* utility to a level corresponding to  $F = 20c$ . This utility level is consistent with the acquisition of one signal by a fraction  $\lambda = .22$  of investors. At  $F = 30c$ , utility is consistent with the acquisition of one or two signals by a fraction  $\lambda = .06$  of investors. These examples

<sup>18</sup>Parameter values are as in Figure 2, see footnote 17 (p. 30). In addition,  $W = 1$ ,  $\bar{\mu}_\theta = 1.3$ ,  $\delta = .9$ .

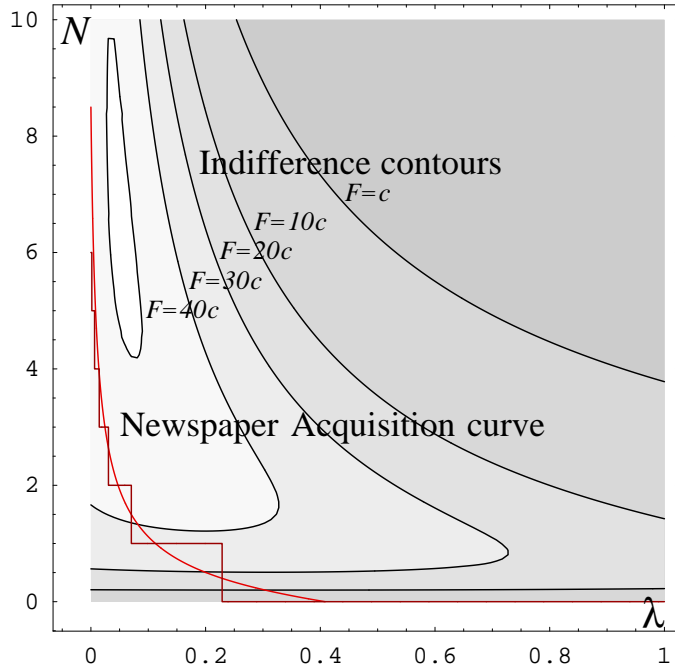


Figure 3: **Equilibrium Combinations of  $N$  and  $\lambda$**

illustrate that multiple equilibrium levels of  $F$  can be associated with equilibrium combinations of  $N$  and  $\lambda$ , which lie on both the newspaper acquisition curve and the indifference contour. In RICE, the equilibrium level of the fixed information cost  $F$  is such that price watchers and news subscribers choose not to change group membership. Theorem 4 summarizes these arguments.

**Theorem 4** *Under Assumptions 1 through 11 and a two-part tariff for signals, one or countably many two-group RICE exist.*

So, the partial newspaper equilibrium at 9am need not be unique, whereas the partial equilibrium at Wall Street at 10am will be unique given  $N^*$  and  $\lambda^*$ . The number of RICE must be countable because  $N$  and  $I$  are integers. A two-part tariff for signals is a necessary and sufficient condition for RICE existence, where  $F$  clears the market for news subscriber membership under the conditions for signal acquisition in Theorem 2. If the conditions for signal acquisition fail, the unique RICE involves no information acquisition.

Are the equilibria in the newspaper market informationally efficient? Under a Pareto criterion, a benevolent social planner maximizes  $\sum_{i=1}^I \mathbb{E}_{pre}^i[U^i]$  with respect to  $N$  and  $\lambda$ . I call the socially optimal choices of  $N^{**}$  and  $\lambda^{**}$  *informationally efficient*. From preceding analysis we know that, in RICE, at least one investor must be a price watcher. A social planner agrees. For  $\lambda = 1$ , reducing the number of signals exerts a



strictly positive effect on everyone's *ex ante* utility (condition (14) is strictly negative at  $\lambda = 1$  for news subscribers, and price watchers suffer a negative externality from signals at any level of information). So, it cannot be socially desirable that all investors read the same  $N$  newspaper articles, even if newspaper articles are for free.

If economic fundamentals are such that there is no incentive for signal acquisition (the conditions of Theorem 2 fail), then a social planner also agrees with the RICE outcome of no information acquisition. However, if there is a strictly positive incentive for information acquisition (the conditions of Theorem 2 hold), then a social planner desires strictly less information acquisition than occurs in RICE. The reason is that every signal that news subscribers acquire inflicts a strictly negative externality on price watchers through diminishing excess return (Theorem 1). So, when there is at least one news subscriber in RICE, markets provide inefficiently much information in that a benevolent social planner would, for any given  $\lambda$ , allocate strictly less signals if signals were perfectly divisible. Theorem 5 summarizes these insights.

**Theorem 5** *Under Assumptions 1 through 11, an informationally efficient allocation of signals is asymmetric so that at least one investor receives no signal realization. If economic fundamentals are such that information acquisition occurs in RICE, news subscribers acquire more signals than is informationally efficient. If no information acquisition occurs, RICE is informationally efficient.*

## 5 Conclusion

This paper embeds information choice into a canonical model of portfolio choice. Beyond prior work, the framework distinguishes between the share of informed investors in the economy (the extensive margin of information) and the degree of information per informed investor (the intensive margin of information). Investors choose whether to subscribe to a (financial) newspaper that offers private information on asset returns, and the newspaper editor offers the number of newspaper articles (signals) that maximize subscribers' welfare. There is a fundamental tension between the extensive and intensive margin of information. At the intensive margin, subscribers only value additional newspaper articles if there exists a group of less informed investors. So, symmetric information to everyone is neither a market equilibrium nor is it socially optimal. An expansion of the number of subscribers at the extensive margin is socially optimal only if the number of signals to subscribers is reduced.

From an *ex ante* perspective, additional information diminishes the excess return, or equity premium, of a risky asset relative to the opportunity cost of holding a riskfree bond. The reason is that additional information does not alter *ex ante* expected asset payoffs, an instance of the law of iterated expectations. But additional

information raises *ex ante* expected asset price because more information makes the risky asset's *ex ante* expected payoff less uncertain. So investors bid up the risky asset's price—diminishing the equity premium. In incomplete asset markets, the diminishing excess return causes a strict utility loss to investors. The less informed investors who do not purchase information but observe the asset-price realization for information extraction, aggravate this diminishing excess-return effect.

News subscribers' information acquisition inflicts a negative externality on the less informed investors who benefit little with information extraction from asset price compared to the utility loss from a diminished excess return. At low levels of exogenous noise in asset price, collective updating by less informed investors bids up expected asset price so much that no investor wants to subscribe to news. Only when exogenous noise in price is sufficiently large, so that prices are little informative for price watchers, do investors have a strictly positive incentive to become news subscribers. It can never be the case in equilibrium or social optimum, however, that the informed group includes all investors if signals are sold in perfect newspaper copies. In this regard, neither a rational-information choice equilibrium nor a social optimum can be symmetric. In fact, if signals are sold in perfect newspaper copies, the majority of investors must remain uninformed at the extensive margin for information acquisition at the intensive margin to be valuable to news subscribers.

# Appendix

## A Properties of the normal distribution

A rational (Bayesian) investor updates her beliefs using the conditional normal distribution of the payoff given the signal and price realizations. Equilibrium asset price is a function of signals, however, so that rational investors will make use of the following fact.

**Fact 2** Consider a multivariate normal density function  $f((\theta; \mathbf{z}^T) | \mu, \Sigma)$  with  $\mathbf{Z} = (Z_1, \dots, Z_K)^T$ ,  $\mu \equiv (\bar{\mu}_\theta; \mathbb{E}[Z_1], \dots, \mathbb{E}[Z_K])^T$  and

$$\Sigma \equiv \begin{pmatrix} \bar{\tau}_\theta^2 & \text{Cov}(\theta, \mathbf{Z})^T \\ \text{Cov}(\theta, \mathbf{Z}) & \text{Cov}(\mathbf{Z}, \mathbf{Z}^T) \end{pmatrix}.$$

Then the conditional p.d.f. of  $\theta$ , given a vector  $\mathbf{z}$  of realizations of  $\mathbf{Z}$  is normal with

$$f\left(\theta \mid \bar{\mu}_\theta + \text{Cov}(\theta, \mathbf{Z})^T \text{Cov}(\mathbf{Z}, \mathbf{Z}^T)^{-1} (\mathbf{z} - \mathbb{E}[z]), \left[\bar{\tau}_\theta^2 - \text{Cov}(\theta, \mathbf{Z})^T \text{Cov}(\mathbf{Z}, \mathbf{Z}^T)^{-1} \text{Cov}(\theta, \mathbf{Z})\right]^{-1}\right).$$

**Proof.** See Raiffa and Schlaifer (1961, 8.2.1). ■

Fact 1 (p. 12) is a special case of Fact 2 when all signals are conditionally independent.

Apart from this property, three further characteristics of the normal distribution are of use in the present framework.

**Fact 3** For a normally distributed random variable  $z \sim \mathcal{N}(\mu, \sigma^2)$  and an arbitrary constant  $A$ , the expected value of  $e^{-Az}$  is

$$\mathbb{E}[e^{-Az} | \mu, \sigma] = \exp\left\{-A\mu + \frac{A^2}{2}\sigma^2\right\}$$

**Fact 4** For a normally distributed random variable  $z \sim \mathcal{N}(\mu, \sigma^2)$  and an arbitrary constant  $A$ , the expected value of  $z \cdot e^{-Az}$  is

$$\mathbb{E}[ze^{-Az} | \mu, \sigma] = (\mu - A\sigma^2) \exp\left\{-A\mu + \frac{A^2}{2}\sigma^2\right\}.$$

**Proof.** Although Fact 3 is a well-known property, I will prove it again here since Fact 4 follows as a corollary. Note that

$$-\frac{1}{2} \left( \frac{z - (\mu - A\sigma^2)}{\sigma} \right)^2 = -A(z - \mu) - \frac{A^2\sigma^2}{2} - \frac{1}{2} \left( \frac{z - \mu}{\sigma} \right)^2.$$

Thus,

$$\begin{aligned}\mathbb{E} [e^{-Az}] &= \int_{-\infty}^{\infty} e^{-Az} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} dz \\ &= e^{-A\mu + \frac{A^2}{2}\sigma^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-(\mu-A\sigma^2)}{\sigma}\right)^2} dz = e^{-A\mu + \frac{A^2}{2}\sigma^2}.\end{aligned}$$

This proves Fact 3. Similarly,

$$\begin{aligned}\mathbb{E} [ze^{-Az}] &= e^{-A\mu + \frac{A^2}{2}\sigma^2} \int_{-\infty}^{\infty} z \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-(\mu-A\sigma^2)}{\sigma}\right)^2} dz \\ &= e^{-A\mu + \frac{A^2}{2}\sigma^2} [\mu - A\sigma^2],\end{aligned}$$

and Fact 4 follows. ■

Finally, the following fact is useful to derive *ex ante* utility in the case of partly informative prices.

**Fact 5** For a normally distributed random variable  $z \sim \mathcal{N}(\mu, \sigma^2)$  and three arbitrary constants  $A, B, D$ , the expected value of  $e^{-\frac{A}{2}(B+Dz)^2}$  is

$$\mathbb{E} \left[ e^{-\frac{A}{2}(B+Dz)^2} \mid \mu, \sigma \right] = \frac{1}{\sqrt{1 + AD^2\sigma^2}} \exp \left\{ -\frac{A(B + D\mu)^2}{2(1 + AD^2\sigma^2)} \right\}.$$

**Proof.** To derive this fact, consider the expectations of  $e^{-A_1z - A_2z^2}$  for two arbitrary constants  $A_1, A_2$ . Note that

$$\begin{aligned}& -\frac{1}{2} \left( \frac{z - \left[ \mu - \left( 1 + 2\frac{A_2}{A_1} \frac{\mu - A_1\sigma^2}{1 + 2A_2\sigma^2} \right) A_1\sigma^2 \right]}{\frac{\sigma}{\sqrt{1 + 2A_2\sigma^2}}} \right)^2 \\ &= -z(A_1 + A_2z) + \frac{\mu(A_1 + A_2\mu) - \frac{A_1^2}{2}\sigma^2}{1 + A_2\sigma^2} - \frac{1}{2} \left( \frac{z - \mu}{\sigma} \right)^2.\end{aligned}$$

Thus,

$$\begin{aligned}\mathbb{E} \left[ e^{-A_1z - A_2z^2} \right] &= \int_{-\infty}^{\infty} e^{-A_1z - A_2z^2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} dz \\ &= \frac{e^{-\frac{\mu(A_1 + A_2\mu) - \frac{A_1^2}{2}\sigma^2}{1 + A_2\sigma^2}}}{\sqrt{1 + 2A_2\sigma^2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \frac{\sigma}{\sqrt{1 + 2A_2\sigma^2}}} e^{-\frac{1}{2} \left( \frac{z - \left[ \mu - \left( 1 + 2\frac{A_2}{A_1} \frac{\mu - A_1\sigma^2}{1 + 2A_2\sigma^2} \right) A_1\sigma^2 \right]}{\frac{\sigma}{\sqrt{1 + 2A_2\sigma^2}}} \right)^2} dz \\ &= \frac{1}{\sqrt{1 + 2A_2\sigma^2}} \exp \left\{ -\frac{\mu(A_1 + A_2\mu) - \frac{(A_1)^2}{2}\sigma^2}{1 + A_2\sigma^2} \right\}.\end{aligned}\tag{A.1}$$

To arrive at Fact 5, observe that

$$\mathbb{E} \left[ e^{-\frac{A}{2}(B+Dz)^2} \right] = e^{-\frac{A}{2}B^2} \mathbb{E} \left[ e^{-\frac{A}{2}(2BDz+D^2z^2)} \right].$$

Then defining  $A_1 \equiv \frac{A}{2}2BD$  and  $A_2 \equiv \frac{A}{2}D^2$ , multiplying (A.1) by  $e^{-\frac{A}{2}B^2}$ , and collecting terms yields Fact 5.  $\blacksquare$

## B Terminal indirect expected utility

Using  $H^i \equiv \exp(-\gamma[(1+R)b^i + Px^i - W_0^i + F^i + cN^i])$  and solving out for  $b^i$  yields demand for the bond

$$b^{i,*} = \frac{1}{1+R} \left( W_0^i - F^i - cN^i - Px^{i,*} - \frac{1}{\gamma} \ln H^{i,*} \right). \quad (\text{B.1})$$

For each unit of the risky asset, bond demand is adjusted by a factor of  $P/(1+R)$  to achieve tomorrow's desired consumption level.

To derive indirect utility (11) in the text, note that (1) simplifies to

$$U^i = -e^{-\gamma(W_0^i - F^i - cN^i)} e^{\gamma(b^i + Px^i)} - \delta e^{-\gamma R b^i} \mathbb{E}^i \left[ e^{-\gamma x^i \theta} \right] \quad (\text{B.2})$$

for CARA utility. By (B.1) (which holds for CARA utility irrespective of the risky asset's distribution), we can write

$$b^{i,*} + Px^{i,*} = \frac{1}{1+R} \left( W_0^i - F^i - cN^i - \frac{1}{\gamma} \ln H^{i,*} + RPx^{i,*} \right),$$

where  $H^{i,*}$  is certain and implicitly given by the first order condition (3-a). Using the above fact and (B.1) in (B.2) yields

$$\begin{aligned} U^{i,*} &= -e^{-\gamma \frac{R}{1+R}(W_0^i - F^i - cN^i)} e^{-\frac{1}{1+R} \ln H^{i,*}} e^{\gamma \frac{R}{1+R} Px^{i,*}} (1 + \delta e^{\ln H^{i,*}} e^{-\gamma x^{i,*} \theta}) \\ &= -\exp \left\{ -\gamma \frac{R}{1+R} (W_0^i - F^i - cN^i) \right\} \left( \frac{e^{\gamma R P x^{i,*}}}{H^{i,*}} \right)^{\frac{1}{1+R}} \left( 1 + \frac{1}{R} \right). \end{aligned}$$

The second step follows by using the first order condition (3-a) to substitute for  $H^{i,*}$ . This establishes (11) in the text.

## C News subscriber pre-posterior beliefs

For a news subscriber and any choice of  $N$ , the pre-posterior joint normal distribution of  $\theta$ ,  $N$  signals, and  $RP$ , i.e. the pre-posterior distribution of  $(\theta; S_1, \dots, S_N; RP)^T$  has a vector of means

$$\bar{\mu}_{NS} = (\bar{\mu}_\theta; \bar{\mu}_\theta, \dots, \bar{\mu}_\theta; \pi_0 + \pi_S N \bar{\mu}_\theta - \pi_X \bar{x})^T$$

and an  $(N+2) \times (N+2)$  variance-covariance matrix

$$\bar{\Sigma}_{NS} = \begin{pmatrix} \bar{\tau}_\theta^2 & \bar{\tau}_\theta^2 \cdot \iota_N^T & \pi_S N \bar{\tau}_\theta^2 \\ \bar{\tau}_\theta^2 \cdot \iota_N & \text{Cov}(\mathbf{S} \cdot \mathbf{S}^T)_N & \pi_S (N \bar{\tau}_\theta^2 + \sigma_S^2) \cdot \iota_N \\ \pi_S N \bar{\tau}_\theta^2 & \pi_S (N \bar{\tau}_\theta^2 + \sigma_S^2) \cdot \iota_N^T & \pi_S^2 N (N \bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \varsigma_X^2 \end{pmatrix}.$$

$\mathbf{S} = (S_1, \dots, S_N)^T$  is the vector of  $N$  signals,  $\iota_N$  denotes an  $N$  vector of ones, and

$$\text{Cov}(\mathbf{S} \cdot \mathbf{S}^T)_N = \begin{pmatrix} \bar{\tau}_\theta^2 + \sigma_S^2 & \bar{\tau}_\theta^2 & \dots & \bar{\tau}_\theta^2 \\ \bar{\tau}_\theta^2 & \bar{\tau}_\theta^2 + \sigma_S^2 & & \bar{\tau}_\theta^2 \\ \vdots & & \ddots & \vdots \\ \bar{\tau}_\theta^2 & \dots & & \bar{\tau}_\theta^2 + \sigma_{NS}^2 \end{pmatrix}.$$

After observing signal realizations  $(s_1, \dots, s_N)$  and  $RP$ , news subscribers apply Fact 2 to this pre-posterior joint normal distribution and obtain a terminal normal distribution of the payoff with conditional mean

$$\mathbb{E}[\theta | RP; s_1, \dots, s_N; \lambda, N] = \mu_{NS} = m_0^{NS} + m_S^{NS} \sum_{j=1}^N s_j + m_{RP}^{NS} RP \quad (\text{C.1})$$

and conditional variance  $\mathbb{V}(\theta | RP; s_1, \dots, s_N; \lambda, N) = \tau_{NS}^2$ , where

$$m_0^{NS} = \frac{\sigma_S^2 \bar{\mu}_\theta}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad (\text{C.2})$$

$$m_S^{NS} = \frac{\bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad (\text{C.3})$$

$$m_{RP}^{NS} = 0, \quad (\text{C.4})$$

$$\tau_{NS}^2 = \frac{\sigma_S^2 \bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}. \quad (\text{C.5})$$

This is Fact 1 (p. 12) because price information is redundant for news subscribers.

## D Two-group financial-market equilibrium

A two-group financial market equilibrium is given by matching the coefficients  $\pi_0, \pi_S, \pi_X$  in equation (5) with the according terms in (10). Defining

$$u \equiv \frac{1}{\bar{\tau}_\theta^2} + \left[ (1-\lambda) \frac{\pi_S (\pi_S N - 1)}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \varsigma_X^2} + \lambda \frac{1}{\sigma_S^2} \right] N \quad (\text{D.1})$$

and matching coefficients  $\pi_0, \pi_S, \pi_X$  yields

$$\pi_0 = \frac{1}{u} \left( \frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} - (1-\lambda) \frac{\pi_S N (\pi_0 - \pi_X \bar{x})}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \varsigma_X^2} \right), \quad (\text{D.2})$$

$$\pi_S = \frac{1}{u} \frac{\lambda}{\sigma_S^2}, \quad (\text{D.3})$$

$$\pi_X = \frac{1}{u} \frac{\gamma}{I}. \quad (\text{D.4})$$

Plugging (D.3) and (D.4) into (D.1) and simplifying shows that (D.1) is a linear equation indeed. In general, if there are  $I^{NS}$  (groups) of investors who acquire a strictly positive number of signals,  $u$  is a polynomial of order  $1+2I^{NS}$ . (See Muendler (2002) for the general model.) For two groups,  $u$  has the unique solution

$$u = \frac{1}{\bar{\tau}_\theta^2} + \left( \frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2} \frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2 \sigma_S^2 \varsigma_X^2} \right) \lambda N,$$

and

$$\pi_0 = \frac{[(\lambda I)^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2] \sigma_S^2 \cdot \bar{\mu}_\theta + (\lambda I)(1-\lambda) N \sigma_S^2 \bar{\tau}_\theta^2 \cdot \gamma \bar{x}}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)}, \quad (\text{D.5})$$

$$\pi_S = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left( \frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2} \frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2 \sigma_S^2 \varsigma_X^2} \right) \lambda N} \frac{\lambda}{\sigma_S^2}, \quad (\text{D.6})$$

$$\pi_X = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left( \frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2} \frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2 \sigma_S^2 \varsigma_X^2} \right) \lambda N} \frac{\gamma}{I}. \quad (\text{D.7})$$

## E Normalized excess return

The normalized excess return for investor  $i$  is  $(\mu_i - RP)/\tau_i = \tau_i (\mu_i - RP)/\tau_i^2$ . To simplify the analysis, I separately consider  $\tau_i^2$  and  $(\mu_i - RP)/\tau_i^2$  instead of the normalized excess return itself. I call  $\tau_i^2$  the *updated variance* and  $(\mu_i - RP)/\tau_i^2$  the *key term*.

For a news subscriber, plug (D.5) through (D.7) into  $m_0^{NS}$  (7-a),  $m_S^{NS}$  (7-b), and  $\tau_{NS}^2$  (7-c). This yields  $\tau_{NS}^2$ . Then plug the solutions for  $m_0^{NS}$ ,  $m_{RP}^{NS}$ , and  $\tau_{NS}^2$  along with the solution for  $RP$  (5) into  $(\mu_{NS} - RP)/\tau_{NS}^2$ . Collecting terms and simplifying yields

$$\begin{aligned} \frac{\mu_{NS} - RP}{\tau_{NS}^2} &= \frac{\gamma}{I} \cdot \frac{1}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)} \\ &\cdot \left( (1-\lambda) I \gamma \sigma_S^2 \varsigma_X^2 \cdot \sum_{j=1}^N (S_j - \bar{\mu}_\theta) + (\lambda I^2 N + \gamma \sigma_S^2 \varsigma_X^2) (\sigma_S^2 + N \bar{\tau}_\theta^2) \cdot X \right. \\ &\quad \left. - \lambda I (1-\lambda) I N (\sigma_S^2 + N \bar{\tau}_\theta^2) \cdot \bar{x} \right) \end{aligned} \quad (\text{E.1})$$

and

$$\tau_{NS}^2 = \frac{\sigma_S^2 \bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N} \quad (\text{E.2})$$

as given in (7-c).

For a price watcher, plug (D.5) through (D.7) into  $m_0^{PW}$  (9-a),  $m_{RP}^{PW}$  (9-b), and  $\tau_{PW}^2$  (9-c). This yields  $\tau_{PW}^2$ . Then plug the solutions for  $m_0^{PW}$ ,  $m_{RP}^{PW}$ , and  $\tau_{PW}^2$  along with the solution for  $RP$  (5) into  $(\mu_{PW} - RP)/\tau_{PW}^2$ . Collecting terms and simplifying yields

$$\begin{aligned} \frac{\mu_{PW} - RP}{\tau_{PW}^2} &= \frac{\gamma}{I} \cdot \frac{1}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)} \\ &\cdot \left( -\lambda I \gamma \sigma_S^2 \varsigma_X^2 \cdot \sum_{j=1}^N (S_j - \bar{\mu}_\theta) + \gamma^2 \sigma_S^4 \varsigma_X^2 \cdot X + (\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) \cdot \bar{x} \right) \end{aligned} \quad (\text{E.3})$$

and

$$\tau_{PW}^2 = \frac{[(\lambda I)^2 N \sigma_S^2 + \gamma^2 \sigma_S^4 \varsigma_X^2] \bar{\tau}_\theta^2}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2} \quad (\text{E.4})$$

for price watchers.

## F Moments of the normalized excess return

The pre-posterior moments of the normalized excess return are crucial for information acquisition. Following the preceding Appendix, I separately consider  $\tau_i^2$  and  $(\mu_i - RP)/\tau_i^2$  to simplify derivations. The former is the updated variance and the latter the key term. Since  $\tau_i$  is certain, and both  $\mu_i$  and  $RP$  are normally distributed from a pre-posterior perspective,  $(\mu_i - RP)/\tau_i^2$  is normally distributed.

For a news subscriber, pre-posterior expectation and variance of (E.1) are

$$\hat{\mathbb{E}}_{pre}^{NS} \left[ \frac{\mu_{NS} - RP}{\tau_{NS}^2} \right] = \frac{\gamma}{I} \cdot \frac{[(\lambda I)^2 N + \gamma^2 \sigma_S^4 \varsigma_X^2] (\sigma_S^2 + N \bar{\tau}_\theta^2) \bar{x}}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)}, \quad (\text{F.1})$$

$$\begin{aligned} \hat{\mathbb{V}}_{pre}^{NS} \left( \frac{\mu_{NS} - RP}{\tau_{NS}^2} \right) &= \frac{\gamma^2}{I^2} \cdot \frac{(\sigma_S^2 + N \bar{\tau}_\theta^2) \varsigma_X^2}{[(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)]^2} \\ &\cdot \left( (1-\lambda)^2 I^2 N \gamma^2 \sigma_S^4 \varsigma_X^2 + (\lambda I^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2)^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) \right). \end{aligned} \quad (\text{F.2})$$

For a price watcher, pre-posterior expectation and variance of (E.3) are

$$\hat{\mathbb{E}}_{pre}^{PW} \left[ \frac{\mu_{PW} - RP}{\tau_{PW}^2} \right] = \frac{\gamma}{I} \cdot \frac{[(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2] \bar{x}}{(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)}, \quad (\text{F.3})$$

$$\hat{\mathbb{V}}_{pre}^{PW} \left( \frac{\mu_{PW} - RP}{\tau_{PW}^2} \right) = \frac{\gamma^2}{I^2} \cdot \frac{[(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2] \gamma^2 \sigma_S^4 \varsigma_X^4}{[(\lambda I)^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)]^2}. \quad (\text{F.4})$$

If  $N = 0$ , news subscribers' terms (F.1), (F.2), and (E.2) coincide with the respective price watcher terms (F.3), (F.4), and (E.4), as it should be.



## G Elasticities of moments of the excess return

This Appendix considers mean and variance of the key term (the excess return divided by  $\tau_i^2$ ), and related them to mean and variance of the normalized excess return. The moments of the key term can be derived directly from *ex ante* utility in the preceding Appendices. I denote mean and variance of the key term with hats. I then turn to the equivalents in the text:  $\varepsilon_{\mathbb{E},N}^i \equiv \frac{1}{2}\varepsilon_{\tau^2,N}^i + \hat{\varepsilon}_{\mathbb{E},N}^i$  and  $\varepsilon_{\mathbb{V},N}^i \equiv \varepsilon_{\tau^2,N}^i + \hat{\varepsilon}_{\mathbb{V},N}^i$ .

**Elasticities of the updated variance and key term moments.** Differentiating news subscribers' moments (E.2), (F.1) and (F.2) with respect to  $N$  yields the elasticities

$$\varepsilon_{\tau^2,N}^{NS} = -\frac{\bar{\tau}_\theta^2 N}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad (\text{G.1})$$

$$\varepsilon_{\mathbb{E},N}^{NS} = -\frac{\gamma^2 \sigma_S^2 \bar{\tau}_\theta^2 \varsigma_X^2 (1-\lambda) N}{(\sigma_S^2 + N\bar{\tau}_\theta^2) [(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]} \cdot \frac{(\lambda I)^2 N^2 \bar{\tau}_\theta^2 - \gamma^2 \sigma_S^2 \varsigma_X^2}{[(\lambda I)^2 N + \gamma^2 \sigma_S^4 \varsigma_X^2]}, \quad (\text{G.2})$$

$$\begin{aligned} \varepsilon_{\mathbb{V},N}^{NS} &= \frac{\gamma^2 \sigma_S^4 \varsigma_X^2 (1-\lambda) N}{(\sigma_S^2 + N\bar{\tau}_\theta^2) [(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]} \\ &\quad \cdot \left( (\lambda I)^2 (1+\lambda) I^2 N (\sigma_S^2 + N\bar{\tau}_\theta^2) (\sigma_S^2 + 2N\bar{\tau}_\theta^2) \right. \\ &\quad \left. + \gamma^2 \sigma_S^2 \varsigma_X^2 I^2 [(1+\lambda)\sigma_S^4 + (2+\lambda(5+\lambda)) N\sigma_S^2 \bar{\tau}_\theta^2 + 6\lambda N^2 \bar{\tau}_\theta^4] \right. \\ &\quad \left. + 2\gamma^4 \sigma_S^4 \bar{\tau}_\theta^2 \varsigma_X^4 (\sigma_S^2 + N\bar{\tau}_\theta^2) \right) / \left( \lambda^2 I^4 N^2 (\sigma_S^2 + N\bar{\tau}_\theta^2) \right. \\ &\quad \left. + I^2 N \gamma^2 \sigma_S^2 \varsigma_X^2 ((1+\lambda^2)\sigma_S^2 + 2\lambda N\bar{\tau}_\theta^2) + \gamma^4 \sigma_S^4 \varsigma_X^4 (\sigma_S^2 + N\bar{\tau}_\theta^2) \right) .. \end{aligned} \quad (\text{G.3})$$

For price watchers, differentiating the moments (E.4), (F.3) and (F.4) with respect to  $N$  yields the elasticities

$$\varepsilon_{\tau^2,N}^{PW} = -\frac{(\lambda I)^2 N^2 \bar{\tau}_\theta^2 [(\lambda I)^2 N + 2\gamma^2 \sigma_S^2 \varsigma_X^2]}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2] [(\lambda I)^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2]}, \quad (\text{G.4})$$

$$\begin{aligned} \varepsilon_{\mathbb{E},N}^{PW} &= \frac{\gamma^2 \sigma_S^2 \bar{\tau}_\theta^2 \varsigma_X^2 \lambda N}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2]} \\ &\quad \cdot \frac{(\lambda I)^2 N^2 \bar{\tau}_\theta^2 - \gamma^2 \sigma_S^4 \varsigma_X^2}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]}, \end{aligned} \quad (\text{G.5})$$

$$\begin{aligned} \varepsilon_{\hat{V},N}^{PW} = & -\frac{\lambda N}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2]} \\ & \cdot \frac{1}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]} \\ & \cdot \left( \lambda^3 I^4 N(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + 2N\bar{\tau}_\theta^2) \right. \\ & \left. + \gamma^2 \sigma_S^4 \varsigma_X^2 \lambda I^2 (\sigma_S^2 + (2 + \lambda)N\bar{\tau}_\theta^2) + 2\gamma^4 \sigma_S^6 \bar{\tau}_\theta^2 \varsigma_X^4 \right). \end{aligned} \quad (\text{G.6})$$

So, the difference between (G.1) and (G.4) is

$$\varepsilon_{\tau^2,N}^{NS} - \varepsilon_{\tau^2,N}^{PW} = \frac{\lambda^2 I^2 N^3 \gamma^2 \sigma_S^2 \bar{\tau}_\theta^4 \varsigma_X^2 - N \gamma^4 \sigma_S^6 \bar{\tau}_\theta^2 \varsigma_X^4}{(\sigma_S^2 + N\bar{\tau}_\theta^2)(\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2)(\lambda^2 I^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2)}.$$

This term is positive iff  $\varsigma_X < \lambda I N \cdot \bar{\tau}_\theta / (\gamma \sigma_S^2)$ .

**Elasticity of the mean raw excess return.** These results at hand, we can evaluate the elasticity of the raw excess return  $\mu_i - RP$  with respect to  $N$ :  $\varepsilon_{\tau^2,N}^i + \varepsilon_{\hat{\mathbb{E}},N}^i$ . For news subscribers and price watchers ( $i = NS, PW$ ).<sup>19</sup>

$$\varepsilon_{\tau^2,N}^i + \varepsilon_{\hat{\mathbb{E}},N}^i = \frac{1}{(\lambda I)^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2} \cdot \frac{\lambda N \bar{\tau}_\theta^2 (\lambda^3 I^4 N^2 + 2\lambda I^2 N \gamma^2 \sigma_S^2 \varsigma_X^4 + \gamma^4 \sigma_S^4 \varsigma_X^4)}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 \varsigma_X^2 (\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)}. \quad (\text{G.7})$$

**Elasticities of the normalized excess return moments.** Consider the elasticities  $\varepsilon_{\mathbb{E},N}^i \equiv \frac{1}{2} \varepsilon_{\tau^2,N}^i + \varepsilon_{\hat{\mathbb{E}},N}^i$  of the mean normalized excess return. For a news subscriber

$$\begin{aligned} \varepsilon_{\mathbb{E},N}^{NS} = \frac{1}{2} \varepsilon_{\tau^2,N}^{NS} + \varepsilon_{\hat{\mathbb{E}},N}^{NS} = & -\frac{1}{2} \frac{N\bar{\tau}_\theta^2}{\sigma_S^2 + N\bar{\tau}_\theta^2} \cdot \left( 1 + \right. \\ & \left. \frac{2\gamma^2(1-\lambda)\sigma_S^2\varsigma_X^2(I^2N^2\lambda^2\bar{\tau}_\theta^2 - \gamma^2\sigma_S^4\varsigma_X^2)}{(\lambda^2I^2N + \gamma^2\sigma_S^2\varsigma_X^2)(\lambda^2I^2N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\varsigma_X^2)} \right). \end{aligned} \quad (\text{G.8})$$

So, for a news subscriber, the elasticity of the pre-posterior variance (G.8) with respect to  $N$  is strictly positive iff

$$\begin{aligned} -\lambda^4 I^4 N^2 (\sigma_S^2 + N\bar{\tau}_\theta^2) - \lambda^2 I^2 N \gamma^2 \sigma_S^2 (2\sigma_S^2 + N(3-\lambda)\bar{\tau}_\theta^2) \varsigma_X^2 \\ + \gamma^4 \sigma_S^4 [(1-2\lambda)\sigma_S^2 - \lambda N\bar{\tau}_\theta^2] \varsigma_X^4 > 0. \end{aligned} \quad (\text{G.9})$$

Thus the necessary and sufficient conditions for (G.8) to be strictly positive are that (i)  $\lambda N\bar{\tau}_\theta^2 < (1-2\lambda)\sigma_S^2$  and (ii)  $\varsigma_X^2 > \zeta^2$ , where  $\zeta^2$  is the positive root of (G.9), a

<sup>19</sup>The following terms were simplified using Mathematica 5. The according notebook file is available online at [www.econ.ucsd.edu/muendler/papers/abs/infgauss.html](http://www.econ.ucsd.edu/muendler/papers/abs/infgauss.html).

second-order polynomial in  $\varsigma_X^2$ . Note that (G.8) is strictly negative for  $\varsigma_X^2 \rightarrow 0$ , and strictly positive for  $\varsigma_X^2 \rightarrow \infty$  iff  $\lambda N \bar{\tau}_\theta^2 < (1-2\lambda)\sigma_S^2$  (see Table 3, p. 26). So there must be a unique positive root of (G.9) if  $\lambda N \bar{\tau}_\theta^2 < (1-2\lambda)\sigma_S^2$ .

Moreover, for a news subscriber, the pre-posterior mean of the normalized expected excess return monotonically increases in supply-noise  $\varsigma_X^2$  if  $\varsigma_X^2$  is sufficiently large:

$$\partial \varepsilon_{\mathbb{E},N}^{NS} / \partial \varsigma_X^2 = \frac{2\lambda^2 I^2 N \gamma^2 (1-\lambda) \sigma_S^2 (\sigma_S^2 + N \tau^2)}{[I^2 N \lambda^2 + \gamma^2 \sigma_S^2 \varsigma_X^2]^2 [I^2 N \lambda^2 (\sigma_S^2 + N \tau^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + N \lambda \tau^2) \varsigma_X^2]^2} \cdot \left( 2\lambda^2 I^2 N \gamma^2 \lambda^2 \sigma_S^4 \varsigma_X^2 + \gamma^4 \sigma_S^4 (2\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^4 - \lambda^4 I^4 N^3 \bar{\tau}_\theta^2 \right).$$

This derivative is strictly positive if

$$\varsigma_X^2 > \frac{-\lambda^2 I^2 N \gamma^2 \sigma_S^4 + \sqrt{\lambda^4 I^4 N^2 \gamma^4 \sigma_S^4 (\sigma_S^4 + 2N \sigma_S^2 \bar{\tau}_\theta^2 + \lambda N^2 \bar{\tau}_\theta^4)}}{\gamma^4 \sigma_S^4 (2\sigma_S^2 + \lambda N \bar{\tau}_\theta^2)}.$$

Consider the elasticity  $\varepsilon_{\mathbb{E},N}^{PW} \equiv \frac{1}{2} \varepsilon_{\tau^2,N}^{PW} + \varepsilon_{\hat{\mathbb{E}},N}^{PW}$  of the mean normalized excess return for a price watcher

$$\varepsilon_{\mathbb{E},N}^{PW} = \frac{1}{2} \varepsilon_{\tau^2,N}^{PW} + \varepsilon_{\hat{\mathbb{E}},N}^{PW} = -\frac{1}{2} \frac{\lambda N \bar{\tau}_\theta^2}{\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2} \cdot \left( \frac{\lambda^5 I^6 N^3 (\sigma_S^2 + N \bar{\tau}_\theta^2) + \lambda^3 I^4 N^2 \gamma^2 \sigma_S^2 (3\sigma_S^2 + N(2-\lambda) \bar{\tau}_\theta^2) \varsigma_X^2}{\lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^2} + \frac{2\lambda(1+\lambda) I^2 N \gamma^4 \sigma_S^6 \varsigma_X^4 + 2\gamma^6 \sigma_S^8 \varsigma_X^6}{\lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^2} \right) < 0. \quad (\text{G.10})$$

There was no difference in the elasticity of the mean raw excess return between news subscribers and price watchers. So, the difference between (G.8) and (G.10) is  $\varepsilon_{\mathbb{E},N}^{NS} - \varepsilon_{\mathbb{E},N}^{PW} = -\frac{1}{2} (\varepsilon_{\tau^2,N}^{NS} - \varepsilon_{\tau^2,N}^{PW})$ , which is given by (G.7).

Turn to the elasticities  $\frac{1}{2} \varepsilon_{\mathbb{V},N}^i \equiv \frac{1}{2} \varepsilon_{\tau^2,N}^i + \frac{1}{2} \varepsilon_{\hat{\mathbb{V}},N}^i$  of the normalized excess return variance. For a news subscriber

$$\frac{1}{2} \varepsilon_{\mathbb{V},N}^{NS} = \frac{1}{2} \varepsilon_{\tau^2,N}^{NS} + \frac{1}{2} \varepsilon_{\hat{\mathbb{V}},N}^{NS} = -\frac{1}{2} \frac{N}{\sigma_S^2 + N \bar{\tau}_\theta^2} \left( \bar{\tau}_\theta^2 - \left\{ \left( \gamma^2 (1-\lambda) \sigma_S^4 \varsigma_X^2 \left[ \lambda^2 I^4 N (1+\lambda) (\sigma_S^2 + N \bar{\tau}_\theta^2) (\sigma_S^2 + 2N \bar{\tau}_\theta^2) + I^2 \gamma^2 \sigma_S^2 ((1+\lambda) \sigma_S^4 + N(2+\lambda(5+\lambda)) \sigma_S^2 \bar{\tau}_\theta^2 + 6\lambda N^2 \bar{\tau}_\theta^4) \varsigma_X^2 + 2\gamma^4 \sigma_S^4 \bar{\tau}_\theta^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) \varsigma_X^4 \right] \right) / \left( \left[ \lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^2 \right] \left[ \lambda^2 I^4 N^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) + I^2 N \gamma^2 \sigma_S^2 ((1+\lambda^2) \sigma_S^2 + 2\lambda N \bar{\tau}_\theta^2) \varsigma_X^2 + \gamma^4 \sigma_S^4 (\sigma_S^2 + N \bar{\tau}_\theta^2) \varsigma_X^4 \right] \right) \right\} \right). \quad (\text{G.11})$$

So, for a news subscriber, the elasticity of the pre-posterior variance (G.11) with respect to  $N$  is strictly positive iff

$$\begin{aligned}
& -\lambda^4 I^6 N^3 \bar{\tau}_\theta^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) - \lambda^2 I^4 N \gamma^2 \sigma_S^2 \left[ \lambda N \bar{\tau}_\theta^2 (3N \bar{\tau}_\theta^2 + 3\lambda \sigma_S^2) - (1-\lambda^2) \sigma_S^4 \right] \zeta_X^2 \\
& - I^2 \gamma^4 \sigma_S^4 \left[ \lambda N \bar{\tau}_\theta^2 (3\lambda N \bar{\tau}_\theta^2 + \lambda(5+\lambda) \sigma_S^2 - 3\sigma_S^2) - (1-\lambda^2) \sigma_S^4 \right] \zeta_X^4 \\
& + \gamma^6 \sigma_S^6 \bar{\tau}_\theta^2 \left[ (1-2\lambda) \sigma_S^2 - \lambda N \bar{\tau}_\theta^2 \right] \zeta_X^6 > 0.
\end{aligned} \tag{G.12}$$

Condition (G.12) is a third-order polynomial in  $\zeta_X^2$ . Analysis shows that condition (G.12) has a unique real root  $\bar{\zeta}^2$  (and two imaginary roots). Note that (G.11) is strictly negative for  $\zeta_X^2 \rightarrow 0$ , and strictly positive for  $\zeta_X^2 \rightarrow \infty$  iff  $\lambda N \bar{\tau}_\theta^2 < (1-2\lambda) \sigma_S^2$  (see Table 3, p. 26). So the unique real root  $\bar{\zeta}^2$  of (G.12) must be positive if  $\lambda N \bar{\tau}_\theta^2 < (1-2\lambda) \sigma_S^2$  and negative otherwise. Hence, the necessary and sufficient conditions for (G.11) to be strictly positive are that (i)  $\lambda N \bar{\tau}_\theta^2 < (1-2\lambda) \sigma_S^2$  and (ii)  $\zeta_X^2 > \bar{\zeta}^2$ , where  $\bar{\zeta}^2$  is the unique real root of (G.12).

Subtracting (G.8) from (G.11) and simplifying shows that  $\frac{1}{2} \varepsilon_{\mathbb{V},N}^{NS} > \varepsilon_{\mathbb{E},N}^{NS}$  for any parameter combination,

$$\frac{1}{2} \varepsilon_{\mathbb{V},N}^{NS} - \varepsilon_{\mathbb{E},N}^{NS} = \textit{To Be Inserted} > 0. \tag{G.13}$$

This implies that  $\bar{\zeta}^2 < \zeta^2$ .

Consider the elasticity  $\frac{1}{2} \varepsilon_{\mathbb{V},N}^{PW} \equiv \frac{1}{2} \varepsilon_{\tau^2,N}^{PW} + \frac{1}{2} \varepsilon_{\hat{\tau}_\theta,N}^{PW}$  of the normalized excess return variance for a price watcher

$$\begin{aligned}
\frac{1}{2} \varepsilon_{\mathbb{V},N}^{PW} &= \frac{1}{2} \varepsilon_{\tau^2,N}^{PW} + \frac{1}{2} \varepsilon_{\hat{\tau}_\theta,N}^{PW} = -\frac{1}{2} \frac{\lambda N}{\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \zeta_X^2}. \\
& \cdot \frac{\lambda^3 I^4 N (\sigma_S^2 + 3N \bar{\tau}_\theta^2) + \lambda I^2 \gamma^2 \sigma_S^2 (\sigma_S^2 + N(4+\lambda) \bar{\tau}_\theta^2) \zeta_X^2 + 2\gamma^4 \sigma_S^4 \bar{\tau}_\theta^2 \zeta_X^4}{\lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \zeta_X^2} < 0.
\end{aligned} \tag{G.14}$$

The difference between (G.11) and (G.14) is strictly positive:

$$\begin{aligned}
\frac{1}{2} \varepsilon_{\mathbb{V},N}^{NS} - \frac{1}{2} \varepsilon_{\mathbb{V},N}^{PW} &= \frac{1}{2} \frac{N}{\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \zeta_X^2} \cdot \left( \right. \\
& \frac{\lambda^4 I^6 N^2 (\sigma_S^2 + 2N \bar{\tau}_\theta^2) + \lambda^2 I^4 N \gamma^2 \sigma_S^2 [2\sigma_S^2 + N(3+2\lambda) \bar{\tau}_\theta^2] \zeta_X^2}{\lambda^2 I^4 N^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) + I^2 N \gamma^2 \sigma_S^2 [(1+\lambda^2) \sigma_S^2 + 2\lambda N \bar{\tau}_\theta^2] \zeta_X^2 + \gamma^4 \sigma_S^4 (\sigma_S^2 + N \bar{\tau}_\theta^2) \zeta_X^4} \\
& \left. + \frac{I^2 \gamma^4 \sigma_S^4 (\sigma_S^2 + 4\lambda N \bar{\tau}_\theta^2) \zeta_X^4 + \gamma^6 \sigma_S^6 \bar{\tau}_\theta^2 \zeta_X^6}{\lambda^2 I^4 N^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) + I^2 N \gamma^2 \sigma_S^2 [(1+\lambda^2) \sigma_S^2 + 2\lambda N \bar{\tau}_\theta^2] \zeta_X^2 + \gamma^4 \sigma_S^4 (\sigma_S^2 + N \bar{\tau}_\theta^2) \zeta_X^4} \right)
\end{aligned} \tag{G.15}$$

## H Utility responses to signal acquisition

The derivative of *ex ante* utility (13) with respect to  $N$ , given  $\lambda$ , is

$$\begin{aligned}
-\frac{1+R}{\mathbb{E}_{pre}^i[U^{i,*}]} \frac{\partial \mathbb{E}_{pre}^i[U^{i,*}]}{\partial N} &= -\gamma R c \\
&+ \frac{1}{N} \frac{\tau_i^2 \left( \hat{\mathbb{E}}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i^2} \right] \right)^2}{1 + \frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)} \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \varepsilon_{\hat{\mathbb{E}}, N}^i \right) \\
&+ \frac{1}{N} \frac{\frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)}{1 + \frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)} \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \frac{1}{2} \varepsilon_{\hat{V}, N}^i \right) \\
&\cdot \frac{(1+R) + \tau_i^2 \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right) - \tau_i^2 \left( \hat{\mathbb{E}}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i^2} \right] \right)^2}{1 + \frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)},
\end{aligned} \tag{H.1}$$

where  $\varepsilon_{\tau^2, N}^i$  is the elasticity of  $\tau_i^2$ ,  $\varepsilon_{\hat{\mathbb{E}}, N}^i$  the elasticity of  $\hat{\mathbb{E}}_{pre}^i[(\mu_i - RP)/\tau_i^2]$  and  $\varepsilon_{\hat{V}, N}^i$  the elasticity of  $\hat{V}_{pre}^i((\mu_i - RP)/\tau_i^2)$ , with respect to  $N$ .

As in the text,  $E_i(\lambda, N)$ ,  $V_i(\lambda, N)$  and  $\Delta_i(\lambda, N)$  are

$$E_i(\lambda, N) \equiv \frac{1}{N} \frac{\tau_i^2 \left( \hat{\mathbb{E}}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i^2} \right] \right)^2}{1 + \frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)}, \tag{H.2}$$

$$V_i(\lambda, N) \equiv \frac{1}{N} \frac{\frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right)}{\left[ 1 + \frac{\tau_i^2}{1+R} \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right) \right]^2}, \tag{H.3}$$

$$\Delta_i(\lambda, N) \equiv (1+R) + \tau_i^2 \hat{V}_{pre}^i \left( \frac{\mu_i - RP}{\tau_i^2} \right) - \tau_i^2 \left( \hat{\mathbb{E}}_{pre}^i \left[ \frac{\mu_i - RP}{\tau_i^2} \right] \right)^2. \tag{H.4}$$

For news subscribers

$$E_{NS} = \textit{To Be Inserted} \tag{H.5}$$

$$E_{NS} \cdot \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \varepsilon_{\hat{\mathbb{E}}, N}^i \right) = \textit{To Be Inserted} \tag{H.6}$$

$$V_{NS} \cdot \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \frac{1}{2} \varepsilon_{\hat{V}, N}^i \right) = \textit{To Be Inserted} \tag{H.7}$$

$$\Delta_{NS} = \textit{To Be Inserted} \tag{H.8}$$

with a threshold value

$$\Delta_{NS} > 0 \Leftrightarrow \bar{x}^2 < (\bar{x}_{cutoff}^{\Delta, NS})^2. \quad (\text{H.9})$$

And for price watchers

$$E_{PW} = \textit{To Be Inserted} \quad (\text{H.10})$$

$$E_{PW} \cdot \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \varepsilon_{\mathbb{E}, N}^i \right) = \textit{To Be Inserted} \quad (\text{H.11})$$

$$V_{PW} \cdot \left( \frac{1}{2} \varepsilon_{\tau^2, N}^i + \frac{1}{2} \varepsilon_{\mathbb{V}, N}^i \right) = \textit{To Be Inserted} \quad (\text{H.12})$$

$$\Delta_{PW} = \textit{To Be Inserted} \quad (\text{H.13})$$

with a threshold value

$$\Delta_{PW} > 0 \Leftrightarrow \bar{x}^2 < (\bar{x}_{cutoff}^{\Delta, PW})^2 = \textit{To Be Inserted}. \quad (\text{H.14})$$

## I Negative externality: Proof of Theorem 1

Define  $a \equiv \mathbb{E}_{pre}^{PW}[\cdot]^2 / \bar{x}^2 > 0$  (see (F.3) above), define  $b \equiv \mathbb{V}_{pre}^{PW}(\cdot) / (1+R) > 0$ ,  $d \equiv \varepsilon_{\mathbb{E}, N}^{PW} / N < 0$  and  $g \equiv \frac{1}{2} \varepsilon_{\mathbb{V}, N}^{PW} / N < 0$ , where the signs of  $d$  and  $g$  follow from (G.4), (G.10) and (G.14). In equilibrium (definition 1), a price watcher's *ex ante* utility strictly increases with signals iff

$$\frac{a}{1+b} \frac{d+b(d-g)}{1+b} \bar{x}^2 + \frac{(1+R)bg}{1+b} > 0,$$

which depends on the sign of the factor  $d+b(d-g)$ . By (F.3), (G.4), (G.10) and (G.14),

$$\begin{aligned} d+b(d-g) = & - \left( \lambda \bar{\tau}_\theta^2 \left[ \lambda^7 I^8 N^4 (1+R) (\sigma_S^2 + N \bar{\tau}_\theta^2)^2 + 2\lambda^5 I^6 N^3 (1+R) \gamma^2 \sigma_S^2 (\sigma_S^2 + N \bar{\tau}_\theta^2) \cdot \right. \right. \\ & \cdot (2\sigma_S^2 + N \bar{\tau}_\theta^2) \varsigma_X^2 + \lambda^3 I^4 N^2 \gamma^4 \sigma_S^4 \left( (5 + (2-\lambda)\lambda + R(5+2\lambda)) \sigma_S^4 + 2\lambda N \sigma_S^2 \bar{\tau}_\theta^2 (2(2-\lambda) \right. \\ & \left. \left. + 2R(1+\lambda)) + \lambda N^2 (1+R) (2-\lambda) \bar{\tau}_\theta^4 \right) \varsigma_X^4 + 2\lambda I^2 N \gamma^6 \sigma_S^8 \left( (1+R + 2(1+R)\lambda - \lambda^2) \sigma_S^2 \right. \right. \\ & \left. \left. + \lambda N (2-\lambda + R(2+\lambda)) \bar{\tau}_\theta^2 \right) \varsigma_X^6 + \gamma^8 \sigma_S^{10} \left( (2+2R-\lambda) \sigma_S^2 + 2\lambda N R \bar{\tau}_\theta^2 \right) \varsigma_X^8 \right] \Bigg) / \\ & \left( 2(1+R) (\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2) (\lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^4 \varsigma_X^2) \right. \\ & \left. \left[ \lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^2 \right]^2 \right) < 0. \end{aligned}$$

So, because  $\bar{x}^2 > 0$ , a price watcher's *ex ante* utility strictly decreases in the number of signals.

## J Information acquisition: Proof of Theorem 2

Define  $A \equiv \mathbb{E}_{pre}^{NS}[\cdot]^2 / \bar{x}^2 > 0$  (see (F.1) above), define  $B \equiv \mathbb{V}_{pre}^{NS}(\cdot) / (1+R) > 0$ ,  $D \equiv \varepsilon_{\mathbb{E},N}^{NS} / N$  and  $G \equiv \frac{1}{2} \varepsilon_{\mathbb{V},N}^{NS} / N$ , where the signs of  $D$  and  $G$  are ambiguous (see (G.1), (G.8) and (G.11)). In equilibrium (definition 1), a news subscriber's marginal *ex ante* utility gain from an additional signal is strictly positive iff

$$\frac{A}{1+B} \frac{D+B(D-G)}{1+B} \bar{x}^2 + \frac{(1+R)BG}{1+B} > 0,$$

which depends on the sign and size of the factor  $D+B(D-G)$ . By (F.1), (G.1), (G.8) and (G.11),

$$\begin{aligned} D+B(D-G) &= - \left( \bar{\tau}_\theta^2 \left[ \lambda^6 I^6 N^3 (1+R) (\sigma_S^2 + N \bar{\tau}_\theta^2)^2 \right. \right. \\ &\quad + 3 \lambda^4 I^4 N^2 (1+R) \gamma^2 \sigma_S^2 (\sigma_S^2 + N \bar{\tau}_\theta^2)^2 \varsigma_X^2 + \lambda^2 I^2 N \gamma^4 \sigma_S^4 \cdot \\ &\quad \cdot \left( (2+R+2(1+R)\lambda - \lambda^2) \sigma_S^4 + 2N(2+R+2(1+R)\lambda - \lambda^2) \sigma_S^2 \bar{\tau}_\theta^2 \right. \\ &\quad \left. \left. + N^2 (2 + \lambda(2+R(4-\lambda) + \lambda)) \bar{\tau}_\theta^4 \right) \varsigma_X^4 + \gamma^6 \sigma_S^6 \left( (2-\lambda)\lambda(\sigma_S^2 + N \bar{\tau}_\theta^2) \right)^2 \right. \\ &\quad \left. \left. - R(\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \left( (1-2\lambda)\sigma_S^2 - \lambda N \bar{\tau}_\theta^2 \right) \right) \varsigma_X^6 \right] \Bigg) / \\ &\quad \left( 2(1+R)(\sigma_S^2 + N \bar{\tau}_\theta^2) (\lambda^2 I^2 N + \gamma^2 \sigma_S^2 \varsigma_X^2) \cdot \right. \\ &\quad \left. \cdot \left[ \lambda^2 I^2 N (\sigma_S^2 + N \bar{\tau}_\theta^2) + \gamma^2 \sigma_S^2 (\sigma_S^2 + \lambda N \bar{\tau}_\theta^2) \varsigma_X^2 \right]^2 \right). \end{aligned}$$

## K Final wealth maximization

In a final wealth maximization framework as in Grossman and Stiglitz (1980), investors maximize  $U^i = \mathbb{E} [u(C_1^i) | RP, \{s_1, \dots, s_N\}]$  with respect to  $x^i$ .  $C_0^i = 0$  and  $\delta = 1$  without loss of generality in this context. The same first order condition as below results for risky-asset demand (4). Indirect utility  $U^{*,i} = \mathbb{E} [u(C_1^{*,i}) | RP, \{s_1, \dots, s_N\}]$  becomes

$$\mathbb{E}_{pre}^i [U^{i,*}] = -\kappa^i \cdot e^{\gamma R(F^i + cN^i)} \cdot \mathbb{E}_{pre}^i \left[ \exp \left\{ -\frac{1}{2} \left( \frac{\mu_i - RP}{\tau_i} \right)^2 \right\} \right],$$

where  $\kappa^i \equiv \exp\{-\gamma RW_0^i\} > 0$ , similar to (12). Apply Fact 5 (in Appendix A, p. 36) to indirect utility above to obtain (pre-posterior) *ex ante* utility

$$\mathbb{E}_{pre}^i[U^{i,*}] = -\kappa^i \cdot \exp\{\gamma R(F^i + cN^i)\} \cdot \frac{1}{\sqrt{1 + \mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right)}} \exp\left\{-\frac{1}{2} \frac{\left(\mathbb{E}_{pre}^i\left[\frac{\mu_i - RP}{\tau_i}\right]\right)^2}{1 + \mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right)}\right\},$$

similar to (13). Appendix F (p. 40) reports the parametric expressions for the pre-posterior mean  $\mathbb{E}_{pre}^i[(\mu_i - RP)/\tau_i]$  and variance  $\mathbb{V}_{pre}^i((\mu_i - RP)/\tau_i)$ .

Taking the derivative and multiplying it by the positive factor  $-1/\mathbb{E}_{pre}^i[U^{i,*}]$  for clarity yields

$$\begin{aligned} -\frac{1}{\mathbb{E}_{pre}^i[U^{i,*}]} \frac{\partial \mathbb{E}_{pre}^i[U^{i,*}]}{\partial N} &= -\gamma R c \mathbf{1}(i = NS) \\ &\quad + E_i(\lambda, N) \cdot \varepsilon_{\mathbb{E}, N}^i(\lambda, N) \\ &\quad + V_i(\lambda, N) \cdot \frac{1}{2} \varepsilon_{\mathbb{V}, N}^i(\lambda, N) \cdot \Delta_i(\lambda, N), \end{aligned}$$

where  $\varepsilon_{\mathbb{E}, N}^i$  is the elasticity of the mean  $\mathbb{E}_{pre}^i[(\mu_i - RP)/\tau_i]$  and  $\varepsilon_{\mathbb{V}, N}^i$  is the elasticity of the variance  $\mathbb{V}_{pre}^i((\mu_i - RP)/\tau_i)$  with respect to  $N$ . The definitions of the terms  $E_i(\lambda, N)$ ,  $V_i(\lambda, N)$ , and  $\Delta_i(\lambda, N)$  are now

$$\begin{aligned} E_i(\lambda, N) &\equiv \frac{1}{N} \frac{\left(\mathbb{E}_{pre}^i\left[\frac{\mu_i - RP}{\tau_i}\right]\right)^2}{1 + \mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right)}, \\ V_i(\lambda, N) &\equiv \frac{1}{N} \frac{\mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right)}{\left[1 + \mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right)\right]^2}, \\ \Delta_i(\lambda, N) &\equiv 1 + \mathbb{V}_{pre}^i\left(\frac{\mu_i - RP}{\tau_i}\right) - \left(\mathbb{E}_{pre}^i\left[\frac{\mu_i - RP}{\tau_i}\right]\right)^2. \end{aligned}$$

It can be shown that the signs of terms in Table 2 are the same as before and that now  $\bar{x}_{cutoff}^{\Delta, NS} = \bar{x}_{cutoff}^{\Delta, PW}$ . The terms in Table 3 and the proofs of Theorems 1 and 2 carry over, merely setting the factor  $1 + R$  to one whenever it appears. In summary and contrary to the case of a fully revealing asset price, where intertemporal optimization and the choice of portfolio size matters crucially (Muendler 2007), final wealth maximization does not obstruct the qualitative insights in the present framework.



## References

- ADMATI, ANAT R., and PFLEIDERER, PAUL (1987): “Viable Allocations of Information in Financial Markets,” *Journal of Economic Theory*, 43(1), 76–115
- (1988): “Selling and Trading on Information in Financial Markets,” *American Economic Review*, 78(2), 96–103
- BARLEVY, GADI, and VERONESI, PIETRO (2000): “Information Acquisition in Financial Markets,” *Review of Economic Studies*, 67(1), 79–90
- DIAMOND, DOUGLAS W. (1985): “Optimal Release of Information by Firms,” *Journal of Finance*, 40(4), 1071–94
- DIAMOND, DOUGLAS W., and VERRECCHIA, ROBERT E. (1991): “Disclosure, Liquidity, and the Cost of Capital,” *Journal of Finance*, 46(4), 1325–59
- EASLEY, DAVID, HVIDKJAER, SOEREN, and O’HARA, MAUREEN (2002): “Is Information Risk a Determinant of Asset Returns?,” *Journal of Finance*, 57(5), 2185–2221
- GROSSMAN, SANFORD J., and STIGLITZ, JOSEPH E. (1980): “On the Impossibility of Informationally Efficient Markets,” *American Economic Review*, 70(3), 393–408
- HELLWIG, MARTIN F. (1980): “On the Aggregation of Information in Competitive Markets,” *Journal of Economic Theory*, 22(3), 477–98
- LI, LODE, MCKELVEY, RICHARD D., and PAGE, TALBOT (1987): “Optimal Research for Cournot Oligopolists,” *Journal of Economic Theory*, 42(1), 140–66
- MORRIS, STEPHEN, and SHIN, HYUN SONG (2002): “Social Value of Public Information,” *American Economic Review*, 92(5), 1521–34
- MOSCARINI, GIUSEPPE, and SMITH, LONES (2001): “The Optimal Level of Experimentation,” *Econometrica*, 69(6), 1629–44
- MUENDLER, MARC-ANDREAS (2002): “Essays on International Trade, Growth and Finance,” Ph.D. thesis, University of California, Berkeley
- (2007): “The Possibility of Informationally Efficient Markets,” *Journal of Economic Theory*, 133(1), 467–483
- RAIFFA, HOWARD, and SCHLAIFER, ROBERT (1961): *Applied Statistical Decision Theory*, Studies in managerial economics. Graduate School of Business Administration, Harvard University, Boston, Massachusetts
- RAITH, MICHAEL (1996): “A General Model of Information Sharing in Oligopoly,” *Journal of Economic Theory*, 71(1), 260–88
- TONG, HUI (2005): “Disclosure Standards and Market Efficiency: Evidence from Analysts’ Forecasts,” Bank of England, unpublished manuscript

- VELDKAMP, LAURA L. (2006): "Media Frenzies in Markets for Financial Information," *American Economic Review*, 96(3), 577
- VERRECCHIA, ROBERT E. (1982): "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, 50(6), 1415-30
- (2001): "Essays on Disclosure," *Journal of Accounting and Economics*, 32(1-3), 97-180