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ATTITUDES TOWARD RISK: FURTHER REMARKS*

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The purpose of this note is (*i*) to clarify the test results regarding asset integration reported in Binswanger (1981), (*ii*) to clarify some issues regarding the response of risk aversion measures to changes in prospect size and wealth levels, and (*iii*) to correct an erroneous inference regarding relative risk aversion.[1]

I. ASSET INTEGRATION AND THE LINEARITY OF PREFERENCES IN PROBABILITIES

The original analysis in Binswanger (1981) was conducted solely within the expected utility framework, where it is assumed that preferences over prospects are 'linear in the probabilities', and not in terms of any of the various 'non-linear' generalisations of expected utility offered by Kahneman and Tversky (1979), Machina (1982), and others. The expected utility model assumes that the individual's preferences over prospects may be represented by a preference function of the form $\Sigma_i p_i U(W_i)$, where the p_i are the probabilities given in the prospects and the W_i are the corresponding final wealth levels implied by the prospects, so that the probabilities are seen to enter as linear coefficients of the von Neumann–Morgenstern utilities of the final wealth levels. Kahneman and Tversky replace this preference function with one of the form $\Sigma_i \pi(p_i) U(\Delta W_i)$, where the probabilities now enter nonlinearly and the utility index is defined in terms of changes in wealth. Machina posits a general non-linear but differentiable preference function over final wealth distributions and shows that differentiability implies 'local linearity,' so that small deviations from any given prospect may be evaluated in terms of the expectation of some 'local utility function' $U(W; F)$ which depends upon the distribution $F(\cdot)$ over final wealth implied by the current prospect. The assumption that preferences over prospects are defined in terms of their implied distributions over final wealth is referred to by Kahneman and Tversky as 'asset integration.' A final model, which retains linearity in the probabilities but drops asset integration, i.e. $\Sigma_i p_i U(\Delta W_i)$, has been suggested by Markowitz (1952).

Table 1 summarises the assumptions of the above models and of Binswanger's test regarding linearity in the probabilities and asset integration. Since the null hypothesis of Binswanger maintained both these assumptions, his test is therefore a rejection of the *joint* hypothesis of linearity and asset integration.[1]

* The views expressed here are the authors' own and not those of their employers.

[1] This error was first noted by Quizon.

Table 1

Assumptions of the various models and of the empirical test

	Expected utility	Kahneman and Tversky (1979)	Machina (1982; 1983)	Markowitz (1952)	Binswanger test (1981)
Asset integration	yes	no	yes	no	yes
Linearity in the probabilities	yes	no	no	yes	yes

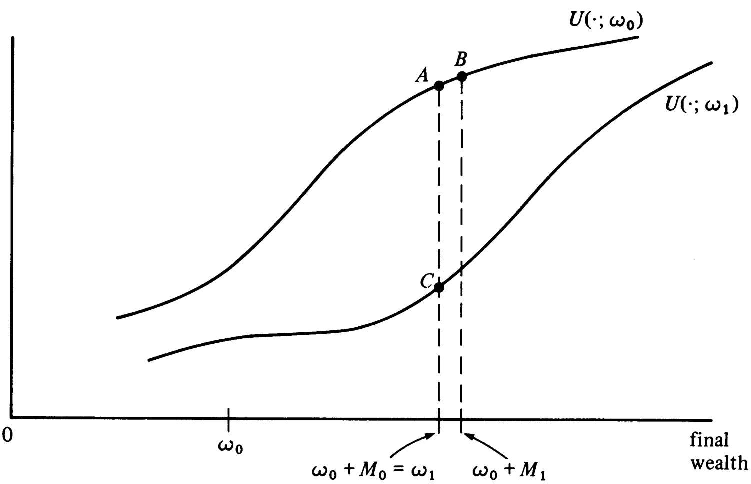

Fig. 1

While therefore inconsistent with the expected utility model, his experimental results are in fact consistent with each of the other three models, although the experimental design is unable to discriminate between them (Machina (1982, pp. 306–8) for example shows how many of the apparent violations of asset integration observed by Markowitz can alternatively be explained by dropping linearity instead - for some violation which cannot be explained in that manner, however, see Kahneman and Tversky (pp. 271–3)).[2]

[1] The test also estimates $\delta \ln Q/\delta \ln \omega$ cross-sectionally, where Q is absolute risk aversion and ω is initial wealth. It thus makes an assumption that there is a commonality across individuals regarding the response of absolute risk aversion to wealth. Note, however, that this does not imply identical utility functions. $\delta \ln Q/\delta \ln \omega$ was estimated in a multivariate framework which allows utility functions to differ according to a number of personal characteristics.

[2] We are here concerned only with interpretation of the test, not with its design of approach which is entirely correct. There is, however, a numerical error on page 879. Let ω be the modal wealth of Rs 13,000 and let ω^* be the level to which wealth must rise in order to result in the same decrease in absolute risk aversion as a change in the game scale from Rs 5 to Rs 50. Then $\Delta \ln \omega = \ln \omega^* - \ln \omega = \ln \omega^* - \ln(13{,}000) = 1{\cdot}794$. Thus, $\omega^* =$ Rs 78,174. The correct statement of the 11th line from the bottom of page 879 therefore is 'the wealth of the (modal) individual must rise by (roughly 500%) to achieve the same (decrease) in risk aversion.' Clearly this correction implies that the rejection of the joint hypothesis of linearity and asset integration is much sharper than previously stated.

II. THE BEHAVIOUR OF RISK AVERSION

The failure of either asset integration, linearity, or both to hold does not imply that one cannot measure risk aversion at any wealth level in terms of the curvature of the appropriate local utility function. Instead it implies that any locally measured utility function is not necessarily independent of the existing level of wealth. In Fig. 1, for example, a small change in the payoff of a prospect from M_0 to M_1 essentially leads the individual from point A to point B on the local utility function $U(\cdot\,; \omega_0)$, where ω_0 is initial wealth. A large change in initial wealth from ω_0 to ω_1, on the other hand, would result in the individual being on a new local utility function $U(\cdot\,; \omega_1)$ (e.g. at point C).

Table 2

The effect of wealth on partial risk aversion

Game Level	Mean $\delta \ln S/\delta \ln \omega$	95% Confidence Interval
Rs 0·50	−0·6839	(−1·5359, 0·1681)
Rs 5·00	−0·3188	(−0·9380, 0·3004)
Rs 50·00	−0·0813	(−0·6401, 0·4775)

The experiment clearly allows one essentially to measure the curvature of the local utility function at the subjects' current wealth levels. Since absolute, partial and relative risk aversion are all different transformations of the same curvature measure, they can be measured experimentally even if asset integration and/or linearity do not hold. It is thus possible to estimate how these measures vary with initial wealth by use of cross sectional regressions, as done in the experiment.

However, if asset integration or linearity do not hold, only the behaviour of partial risk aversion can be inferred from a single local utility function- the value of the partial risk aversion coefficient at point A determines the response of an individual with wealth ω_0 to a small change in the payoff M_0. On the other hand, the value of the absolute and relative risk aversion measures at point A do not imply anything about the individual's behaviour should initial wealth rise from ω_0 to ω_1, since this large change in initial wealth would cause the local utility function to shift, so that risk aversion is now determined by the curvature of $U(\cdot\,; \omega_1)$.

Thus, the only meaningful measurements from the individual responses themselves (as opposed to the cross sectional regressions) are those concerning the behaviour of partial risk aversion as the payoffs change but initial wealth is held constant. Binswanger's (1980) experiment found that partial risk aversion was increasing in the payoffs.

The response of partial risk aversion to wealth changes, on the other hand, is based on the cross sectional regression of responses on wealth and other individual characteristics.[1] Table 2 reports the elasticities of partial risk aversion S with respect to wealth found at the three real payoff levels of the experiment (in the original paper only the one at the Rs 5·00 level was reported).

III. THE RESPONSE OF RELATIVE RISK AVERSION WITH RESPECT TO WEALTH: A CORRECTION

In the earlier paper, this response was investigated prior to the consideration of asset integration, and the assumptions of linearity and asset integration were maintained implicitly. Even under these assumptions the inference of declining relative risk aversion (p. 874 and p. 888 fn. 1) was incorrect. It was based on a misinterpretation of the numbers in panel V of Table 1 (the numbers themselves are correct). Recall that relative risk aversion traces the behaviour of an individual when both wealth and the scale of the payoffs change. The erroneous inference was based, however, on a comparison of the behaviour of individuals with constant wealth when only the payoff scale was changed. If for the moment we maintain the assumptions of linearity and asset integration, the experiment still allows us to make an inference concerning relative risk aversion. Let $W = \omega + M$ where ω is initial wealth and M is the certainty equivalent of the prospect (X, p). If wealth and the outcomes of a gamble increase proportionately then $kW \simeq k(\omega + M)$. Defining relative risk aversion $R = -WU''/U'$ and absolute risk aversion $Q = -U''/U'$ and differentiating gives:

$$\frac{dR}{dW} = Q + W\frac{dQ}{dW}. \tag{1}$$

Dividing both sides of (1) by Q we obtain

$$\frac{1}{Q}\frac{dR}{dW} = 1 + \frac{W}{Q}\frac{dQ}{dW}. \tag{2}$$

Note from Table 1 of the original paper that $Q \geqslant 0$ except for alternative F. Also, because M is usually small relative to ω,

$$\frac{W}{Q}\frac{dQ}{dW} \simeq \frac{\delta \ln Q}{\delta \ln \omega}. \tag{3}$$

$\delta \ln Q/\delta \ln \omega$ has been estimated in a simultaneous equation system for the Rs. 5·00 game to be $-0{\cdot}3188$ (see Table 2 of the original paper). Thus

$$\frac{dW}{dR} = Q(1 - 0{\cdot}3188) \geqslant 0, \tag{4}$$

and we come to an inference of increasing relative risk aversion.

This inference is based in the results of the Rs. 5·00 game, but it also holds for the other game levels and all values within two standard errors of the coefficient estimates. Again note that it only has meaning if we make the twin

[1] We thus assume that there is interpersonal comparability of these responses. However, this does not imply that individuals are assumed to have identical utility functions, as the regressions contained other personal characteristics.

assumptions of linearity and asset integration, however, despite the evidence to the contrary.

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REFERENCES

Binswanger, Hans P. (1980). 'Attitudes toward risk: experimental measurement in rural India.' *American Journal of Agricultural Economics*, vol. 62 (August), pp. 395–407.

—— (1981). 'Attitudes toward risk: theoretical implications of an experiment in rural India.' ECONOMIC JOURNAL, vol. 91 (December), pp. 867–90.

Kahneman, Daniel and Tversky, Amos (1979). 'Prospect theory: an analysis of decision under risk.' *Econometrica*, vol. 47 (March), pp. 263–91.

Machina, Mark (1982). '"Expected utility" analysis without the independence axiom.' *Econometrica*, vol. 50 (March), pp. 277–323.

—— (1983). 'Generalized expected utility analysis and the nature of observed violations of the independence axiom.' In Bernt Stigum and Fred Wenstøp (Eds.) (1983). *Foundations of Utility and Risk Theory with Applications*. Dordrecht, Holland: D. Reidel.

Markowitz, Harry (1952). 'The utility of wealth.' *Journal of Political Economy*, vol. 60 (April), pp. 151–8.