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Structural attribution of observed volatility clustering

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Abstract

Virtually all nonlinear economic models with independent, identically distributed stochastic shocks and time-invariant structural parameters will generate persistent, partially predictable heteroskedasticity ("volatility clustering") in their key dependent variables. This paper offers some examples of this phenomenon, derives i.i.d. shock, time-invariant structural forms which generate various types of observed volatility clustering, and examines the modeling and forecasting implications of such "structural attribution."

JEL Classification: C53; G00; G12

Keywords: Volatility clustering; Induced volatility clustering; Structural attribution; Stochastic volatility; ARCH

1. Introduction

The goal of this paper is to explore ways in which the phenomenon of volatility drift or *volatility clustering*, of the form modeled by ARCH and other stochastic volatility specifications,[1] can arise in, and in turn be modeled by, systems whose

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[1]E.g. Engle et al. (1994), Engle (1995), Ghysels et al. (1996) and Shephard (1996).

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nonstochastic structure is time-invariant and whose stochastic shocks are all zero-mean i.i.d. and consequently homoskedastic. In other words, we study volatility clustering as it relates to the *structural* (i.e. deterministic and causal) properties of a time-invariant economic system which is subject to irreducible "white noise" uncertainty, rather than as an exogenous stochastic property of a single time series variable.

The following section presents some general mechanisms and specific examples of how a time-invariant system subject to i.i.d. shocks can exhibit volatility clustering in its key dependent variables. Section 3 motivates the notion of "structural attribution" of observed volatility clustering by means of a similar but more straightforward exercise, namely the structural attribution of observed *serial correlation* in an economic time series. Section 4 presents i.i.d. shock, time-invariant structural forms which generate several familiar types of volatility clustering, and Section 5 explores the modeling and forecasting implications of such structural attribution.

2. Induced volatility clustering

Volatility clustering can arise from a time-invariant structural form and zero-mean i.i.d. shocks[2] $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ and $\{\tilde{\eta}_t\}_{t=1,2,\ldots}$ in either of two ways. The first involves a multiplicative interaction

$$Y_t = Z_t\tilde{\varepsilon}_t, \tag{1}$$

where Z_t is any stationary or nonstationary *drift variable* with homoskedastic innovation term $\tilde{\eta}_t$, such as the random walk, autoregressive or moving average processes

$$Z_t = \sum_{\tau=1}^{t} \tilde{\eta}_\tau \ \text{ or } \ Z_t = \rho Z_{t-1} + \tilde{\eta}_t \ \text{ or } \ Z_t = \tilde{\eta}_t + \gamma\tilde{\eta}_{t-1} \tag{2}$$

or some combination of these. Past values of the dependent variable Y_t and the drift variable Z_t are assumed to be directly observable and thus part of the information set I_t. Past values of $\tilde{\varepsilon}_t$ and $\tilde{\eta}_t$ are not directly observable, though in some cases they could be estimated or algebraically inferred from the data. We refer to any such process for Y_t as a *drifting coefficient process.* For such a process, the mean and variance of Y_t conditional on the information set I_t are given by

$$\mathrm{E}[Y_t|I_t] = \mathrm{E}[Z_t\tilde{\varepsilon}_t|I_t] = \mathrm{E}[Z_t|I_t]\mathrm{E}[\tilde{\varepsilon}_t|I_t] = 0, \tag{3}$$

$$\mathrm{var}[Y_t|I_t] = \mathrm{E}[Z_t^2\tilde{\varepsilon}_t^2|I_t] = \mathrm{E}[Z_t^2|I_t]\mathrm{E}[\tilde{\varepsilon}_t^2|I_t] = [(\mathrm{E}[Z_t|I_t])^2 + \sigma_{\tilde{\eta}}^2]\sigma_{\tilde{\varepsilon}}^2 \tag{4}$$

and the latter is seen to drift over time with drift in the value of $\mathrm{E}[Z_t|I_t]$. For the three drift processes listed in (2), $\mathrm{E}[Z_t|I_t]$ takes the respective forms $Z_{t-1}, \rho Z_{t-1}$, and $-\Sigma_{\tau=1}^{\infty}(-\gamma)^\tau Z_{t-\tau}$.

[2] Throughout this paper, tildes will be used to denote zero-mean, i.i.d. random variables.

A second way in which volatility clustering can arise from a time-invariant structure and a *single* source of zero-mean i.i.d. shocks $\{\tilde{\eta}_t\}_{t=1,2,\ldots}$ is when a conditionally homoskedastic drift variable Z_t with a form such as (2) has a nonlinear influence on the dependent variable Y_t, either through an explicit or implicit structural relationship of the form

$$Y_t = g(Z_t) \text{ or } h(Y_t) = Z_t. \tag{5}$$

We refer to this as a *drifting input* (or a *drifting implicit input*) *process*. For small values of $\sigma^2_{\tilde{\eta}}$, the conditional mean and variance of Y_t for the relationship $Y_t = g(Z_t)$ can be approximated by

$$\mathrm{E}[Y_t|I_t] \approx g(\mathrm{E}[Z_t|I_t]), \tag{6}$$

$$\mathrm{var}[Y_t|I_t] \approx g'(\mathrm{E}[Z_t|I_t])^2\sigma^2_{\tilde{\eta}}. \tag{7}$$

When $g''(\cdot)\neq 0$ the volatility of Y_t is seen to drift with $\mathrm{E}[Z_t|I_t]$, even for a time-invariant $\sigma^2_{\tilde{\eta}}$. The conditional mean and variance of Y_t for the relationship $h(Y_t) = Z_t$ can be approximated by $\mathrm{E}[Y_t|I_t] \approx h^{-1}(\mathrm{E}[Z_t|I_t])$ and $\mathrm{var}[Y_t|I_t] \approx h'(h^{-1}(\mathrm{E}[Z_t|I_t]))^{-2}\sigma^2_{\tilde{\eta}}$, which similarly yields volatility clustering.

A structural model can simultaneously exhibit volatility clustering due to *both* drifting coefficient and drifting input effects, such as the form

$$Y_t = f(Z_t, \tilde{\varepsilon}_t) \tag{8}$$

which can be taken either as a direct structural equation for Y_t, or else as its reduced form equation from an underlying structural model. For small values of $\sigma^2_{\tilde{\eta}}$ and $\sigma^2_{\tilde{\varepsilon}}$, the conditional mean and variance of Y_t can be approximated by

$$\mathrm{E}[Y_t|I_t] \approx f(\mathrm{E}[Z_t|I_t], \mathrm{E}[\tilde{\varepsilon}_t|I_t]) = f(\mathrm{E}[Z_t|I_t], 0), \tag{9}$$

$$\mathrm{var}[Y_t|I_t] \approx f_Z(\mathrm{E}[Z_t|I_t], 0)^2\sigma^2_{\tilde{\eta}} + f_\varepsilon(\mathrm{E}[Z_t|I_t], 0)^2\sigma^2_{\tilde{\varepsilon}}. \tag{10}$$

When $f_{ZZ}(\mathrm{E}[Z_t|I_t],0)\neq 0$, the conditional variance of Y_t is seen to drift with $\mathrm{E}[Z_t|I_t]$ even for time-invariant $\sigma^2_{\tilde{\eta}}$, which is another example of the drifting input process (5). When $f_{Z\varepsilon}(\mathrm{E}[Z_t|I_t], 0)\neq 0$, the conditional variance of Y_t is seen to drift with $\mathrm{E}[Z_t|I_t]$ even for time-invariant $\sigma^2_{\tilde{\varepsilon}}$, which is a generalized version of the drifting coefficient process (1), with $\tilde{\varepsilon}$'s original drifting coefficient Z_t replaced by its drifting partial derivative $f_\varepsilon(\mathrm{E}[Z_t|I_t], 0)$.

2.1. Example: price of a standard commodity

The most basic structural model in economics is the supply-demand model for a standard (flow) commodity. As a simple example, consider a commodity with a deterministic market supply function $Q^{\mathrm{S}}(P_t)$, and market demand function $Q^{\mathrm{D}}(P_t, Z_t) + \tilde{\varepsilon}_t$ in terms of the commodity price P_t, an economic input Z_t (such as income or the price of another good) that follows a conditionally homoskedastic drift processes as in (2), and zero-mean i.i.d. demand shocks $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$. The equilibrium price $P^{\mathrm{e}}_{\mathrm{t}}$ in period t is determined by the market clearing condition

$$Q^{\mathrm{D}}(P_t^{\mathrm{e}}, Z_t) + \tilde{\varepsilon}_t = Q^{\mathrm{S}}(P_t^{\mathrm{e}}). \tag{11}$$

For small departures of Z_t from its conditional mean $\mathrm{E}[Z_t|I_t]$ and small values of $\tilde{\varepsilon}_t$ about its mean of 0, we have

$$\mathrm{d}P_t^{\mathrm{e}} = \frac{Q_Z^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])\mathrm{d}Z_t + \mathrm{d}\tilde{\varepsilon}_t}{Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) - Q_P^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])}. \tag{12}$$

For small values of $\sigma_{\tilde{\eta}}^2$ and $\sigma_{\tilde{\varepsilon}}^2$ the conditional variance of P_t^{e} can accordingly be approximated by

$$\mathrm{var}[P_t^{\mathrm{e}} \mid I_t] \approx \frac{Q_Z^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])^2 \sigma_{\tilde{\eta}}^2 + \sigma_{\tilde{\varepsilon}}^2}{[Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) - Q_P^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])]^2}. \tag{13}$$

It follows that P_t^{e} will exhibit volatility clustering as a result of the drifting value of $\mathrm{E}[Z_t|I_t]$ whenever $Q_{ZZ}^{\mathrm{D}}(\cdot,\cdot)$ or $Q_{PZ}^{\mathrm{D}}(\cdot,\cdot)$ is nonzero, and exhibit volatility clustering as a result of its own drifting conditional mean $E[P_t^{\mathrm{e}}|I_t]$ whenever $Q_{PZ}^{\mathrm{D}}(\cdot,\cdot)$, $Q_{PP}^{\mathrm{S}}(\cdot,\cdot)$ or $Q_{PP}^{\mathrm{D}}(\cdot,\cdot)$ is nonzero.

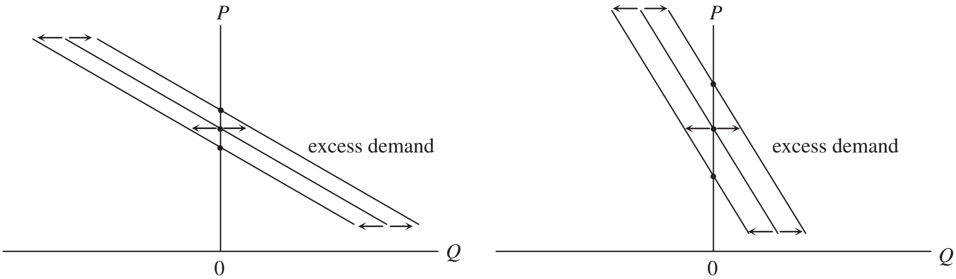

Fig. 1. Induced volatility clustering in P_t^{e} for linear excess demand curve with drifting slope.

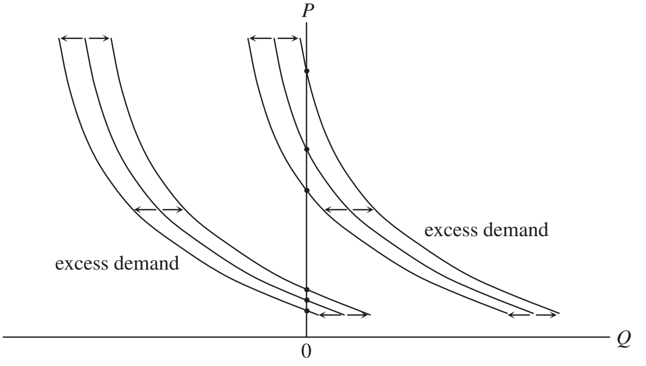

Fig. 2. Induced volatility clustering in P_t^{e} for nonlinear excess demand curve with horizontal drift.

Figs. 1 and 2 illustrate these two types of effects in terms of the market excess demand function $Q^{\mathrm{D}}(P_t, Z_t) + \tilde{\varepsilon}_t - Q^{\mathrm{S}}(P_t)$. In each case, the additive i.i.d. shocks $\tilde{\varepsilon}_t$ imply homoskedastic horizontal shocks in the excess demand curve, as indicated by the equal-length horizontal arrows. Fig. 1 illustrates a pure drifting coefficient effect, where $Q^{\mathrm{D}}_{PP}(\cdot,\cdot)$ and $Q^{\mathrm{S}}_{PP}(\cdot,\cdot)$ are both zero so that the excess demand curve is always linear in P, but $Q^{\mathrm{D}}_{PZ}(\cdot,\cdot)$ is nonzero so that drift in Z_t leads to drift in the *slope* of the excess demand curve, and hence to volatility clustering in P^{e}_t. Fig. 2 illustrates a pure drifting implicit input effect, where $Q^{\mathrm{D}}_{PZ}(\cdot,\cdot)$ is zero so that drift in Z_t leads to a pure horizontal translation of the excess demand curve, but $Q^{\mathrm{D}}_{PP}(\cdot,\cdot)$ and/or $Q^{\mathrm{S}}_{PP}(\cdot)$ are nonzero, so the excess demand curve is nonlinear in P, which again leads to volatility clustering in P^{e}_t.

2.2. Example: return on a financial asset

Consider an asset whose cash value in its terminal period T will be given by a nonlinear function $\pi(\varepsilon_1 + \cdots + \varepsilon_T)$ of the accumulation of a sequence of zero-mean i.i.d. "news" variables $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots,T}$, which are realized and observed one period at a time. Define $Z_t = \sum_{\tau=1}^{t-1} \varepsilon_\tau$ as the news available at the start of period t, that is, before the realization of $\tilde{\varepsilon}_t$. Assume that the discount rate is zero, and that the price of the asset at the end of period t (that is, after the realization of $\tilde{\varepsilon}_t$) is given by the expectation of its terminal value, conditional on the news to date:

$$P_t(Z_t + \varepsilon_t) = \mathrm{E}_{\tilde{\varepsilon}_{t+1},\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \varepsilon_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)]. \tag{14}$$

Since this also implies

$$P_{t-1}(Z_t) = \mathrm{E}_{\tilde{\varepsilon}_t,\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \tilde{\varepsilon}_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)], \tag{15}$$

the asset's one-period gross rate of return viewed from the start of period t is given by the random variable

$$R_t(Z_t, \tilde{\varepsilon}_t) = \frac{P_t(Z_t + \tilde{\varepsilon}_t)}{P_{t-1}(Z_t)} = \frac{\mathrm{E}_{\tilde{\varepsilon}_{t+1},\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \tilde{\varepsilon}_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)]}{\mathrm{E}_{\tilde{\varepsilon}_t,\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \tilde{\varepsilon}_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)]}. \tag{16}$$

Note that while $\tilde{\varepsilon}_t$ appears as an actual random variable in the left and middle terms of (16) as well as in the *numerator* of the right term, it is expected out in the *denominator* of the right term. By the Law of Iterated Expectations and the fact that Z_t is a sufficient statistic for the information set I_t at the start of period t, the conditional mean of the gross return is given by

$$\mathrm{E}[R_t(Z_t, \tilde{\varepsilon}_t)|I_t] = \mathrm{E}_{\tilde{\varepsilon}_t}[R_t(Z_t, \tilde{\varepsilon}_t)] = \frac{\mathrm{E}_{\tilde{\varepsilon}_t,\tilde{\varepsilon}_{t+1},\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \tilde{\varepsilon}_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)]}{\mathrm{E}_{\tilde{\varepsilon}_t,\ldots,\tilde{\varepsilon}_T}[\pi(Z_t + \tilde{\varepsilon}_t + \tilde{\varepsilon}_{t+1} + \cdots + \tilde{\varepsilon}_T)]} = 1 \tag{17}$$

which comes as no surprise, given our assumptions of expectation-based pricing and zero discounting. Expanding about $\tilde{\varepsilon}_t = 0$, the conditional variance of the return is approximated by

$$\mathrm{var}[R_t(Z_t,\tilde{\varepsilon}_t)|I_t] = \mathrm{var}[R_t(Z_t,\tilde{\varepsilon}_t)] \approx \frac{[\mathrm{E}_{\tilde{\varepsilon}_{t+1},\ldots,\tilde{\varepsilon}_T}[\pi'(Z_t+\tilde{\varepsilon}_{t+1}+\cdots+\tilde{\varepsilon}_T)]]^2\sigma^2_{\tilde{\varepsilon}}}{[\mathrm{E}_{\tilde{\varepsilon}_t,\ldots,\tilde{\varepsilon}_T}[\pi(Z_t+\tilde{\varepsilon}_t+\tilde{\varepsilon}_{t+1}+\ldots+\tilde{\varepsilon}_T)]]^2}. \tag{18}$$

In spite of the homoskedasticity and serial independence of the news variables $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots,T}$, the conditional variance of the gross return (as well as the conditional variance of P_t itself) is seen to drift with the drift in Z_t, through both the numerator and denominator of (18).

Although (16) is similar to the general specification (8) in that it also depends on both a homoskedastic drift variable Z_t and an i.i.d. shock $\tilde{\varepsilon}_t$, it differs from (8) in two respects. The first difference is that the drift variable Z_t in $R_t(Z_t,\tilde{\varepsilon}_t)$ is not the accumulation of *separate variables* $\{\tilde{\eta}_t\}$ but rather, the accumulation of past values of $\tilde{\varepsilon}_t$ itself. But since $\tilde{\varepsilon}_t$ is independent of its past values this difference is not essential, and like (8), the specification (16) exhibits volatility clustering from both a drifting input effect and a drifting coefficient effect: R_t is seen to exhibit volatility clustering from a drifting input effect through its numerator $P_t(Z_t+\tilde{\varepsilon}_t)$, and since this numerator is divided by the drifting predetermined variable $P_{t-1}(Z_t)$, R_t also exhibits induced volatility due to the drifting coefficient $1/P_{t-1}(Z_t)$. These two effects correspond to Z_t's influence on the conditional variance formula through the numerator and denominator of (18), respectively.

A second difference between (16) and (8), which may seem to contradict our goal of obtaining volatility clustering from i.i.d. shocks and a time-invariant structure, is that (16) involves a *time-dependent* return formula $R_t(\cdot,\cdot)$, so it is no surprise that it would generate a time-dependent unconditional variance path $\mathrm{var}[R_1|I_0],\ldots,\mathrm{var}[R_T|I_0]$. Two remarks are in order: First, the formulas $R_1(\cdot,\cdot),\ldots,R_T(\cdot,\cdot)$ are *not* structural, but rather, are derived from the one true structural formula of the model, namely the terminal cash value function $\pi(\cdot)$, via the pricing formula (14). Second, it is important to note that even if a model *does* imply a time-dependent unconditional variance path, it may or may not exhibit volatility clustering. Volatility clustering is *not* defined as a *time-dependent unconditional variance path* $\mathrm{var}[R_t|I_0]$, but rather, as *serially correlated departures in the conditional variance* $\mathrm{var}[R_t|I_t]$ *from its predicted path* $\mathrm{E}[\mathrm{var}[R_t|I_t]|I_0]$.[3] In other words, it is the persistent drift in the conditional variance $\mathrm{var}[R_t|I_t]$ *about* its (constant or nonconstant) predicted path $\mathrm{E}[\mathrm{var}[R_t|I_t]|I_0]$ that constitutes volatility clustering in R_t, and which is implied by the dependence of $\mathrm{var}[R_t|I_t]$ upon the drifting variable Z_t in (18).

By assuming an infinite horizon, we can construct an asset pricing model that exhibits both induced volatility clustering and time-invariant pricing and return functions $P(\cdot,\ldots,\cdot)$ and $R(\cdot,\ldots,\cdot)$: Consider an orchard with overlapping cohorts of trees, where each tree yields fruit for $L+1$ periods, and the productivity (net with respect to some average) of trees planted in period t is $\tilde{\varepsilon}_t$. Because of scale effects in

[3]Thus, even though a pure random walk $Z_t = \Sigma_{\tau=1}^{t}\tilde{\eta}_\tau$ has a time-dependent unconditional variance path $\mathrm{var}[Z_t|I_0] \equiv t\sigma^2_{\tilde{\eta}}$, it exhibits no volatility clustering, since its conditional variance $\mathrm{var}[Z_t|I_t]$ never departs from its predicted value $\mathrm{E}[\mathrm{var}[Z_t|I_t]|I_0] = \sigma^2_{\tilde{\eta}}$.

processing and marketing, total profits in period t is given by a nonlinear function $\pi(\varepsilon_{t-L}+\cdots+\varepsilon_t)$, so the market value of the firm at the end of period t is given by the discounted conditional expectation

$$\begin{aligned} P(\varepsilon_{t-L},\ldots,\varepsilon_t) = {} & \pi(\varepsilon_{t-L}+\cdots+\varepsilon_t) \\ & + \delta \mathrm{E}_{\tilde{\varepsilon}_{t+1}}[\pi(\varepsilon_{t-L+1}+\cdots+\varepsilon_t+\tilde{\varepsilon}_{t+1})] \\ & + \delta^2 \mathrm{E}_{\tilde{\varepsilon}_{t+1},\tilde{\varepsilon}_{t+2}}[\pi(\varepsilon_{t-L+2}+\cdots+\varepsilon_t+\tilde{\varepsilon}_{t+1}+\tilde{\varepsilon}_{t+2})] \\ & \vdots \\ & + \delta^L \mathrm{E}_{\tilde{\varepsilon}_{t+1},\tilde{\varepsilon}_{t+2},\ldots,\tilde{\varepsilon}_{t+L}}[\pi(\varepsilon_t+\tilde{\varepsilon}_{t+1}+\tilde{\varepsilon}_{t+2}+\cdots+\tilde{\varepsilon}_{t+L})] \\ & + \sum_{\tau=L+1}^{\infty} \delta^\tau \mathrm{E}_{\tilde{\varepsilon}_{t+\tau-L},\ldots,\tilde{\varepsilon}_{t+\tau}}[\pi(\tilde{\varepsilon}_{t+\tau-L}+\tilde{\varepsilon}_{t+\tau-L+1}+\cdots+\tilde{\varepsilon}_{t+\tau})]. \end{aligned} \tag{19}$$

Since the productivity shocks $\tilde{\varepsilon}_t$ are i.i.d., the pricing function $P(\cdot,\ldots,\cdot)$ is time-invariant, and the one-period gross rate of return viewed from the start of period t, namely

$$R(\varepsilon_{t-L-1},\ldots,\varepsilon_{t-1},\tilde{\varepsilon}_t) = \frac{P(\varepsilon_{t-L},\ldots,\varepsilon_{t-1},\tilde{\varepsilon}_t)}{P(\varepsilon_{t-L-1},\ldots,\varepsilon_{t-1})} \tag{20}$$

is similarly time-invariant. Since nonlinearity of $\pi(\cdot)$ implies that $\partial P(\varepsilon_{t-L},\ldots,\varepsilon_{t-1},\varepsilon_t)/\partial\varepsilon_t|_{\varepsilon_t=0}$ will drift with each of the L moving sums $\varepsilon_{t-L}+\cdots+\varepsilon_{t-1}, \varepsilon_{t-L+1}+\cdots+\varepsilon_{t-1},\ldots,\varepsilon_{t-2}+\varepsilon_{t-1}$ and ε_{t-1} (which determine the predictable component of profits in the current and each of the next L–1 periods), and $1/P(\varepsilon_{t-L-1},\ldots,\varepsilon_{t-1})$ will drift with the moving sums $\varepsilon_{t-L-1}+\cdots+\varepsilon_{t-1}, \varepsilon_{t-L}+\cdots+\varepsilon_{t-1},\ldots,\varepsilon_{t-2}+\varepsilon_{t-1}$ and ε_{t-1}, we have that R_t again exhibits volatility clustering due to both drifting input effects in its numerator and a drifting coefficient effect from its denominator.

2.3. Induced covariance clustering and joint volatility clustering

Give a pair of variables $\{Y_{1t}\}_{t=1,2,\ldots}$ and $\{Y_{2t}\}_{t=1,2,\ldots}$, we can ask the following two questions about their joint behavior:[4]

1. Do Y_{1t} and Y_{2t} exhibit *covariance clustering*—that is, does their conditional covariance $\mathrm{cov}[Y_{1t},Y_{2t}|I_t]$ exhibit drift in the sense of serially correlated departures about its predicted path?
2. Do Y_{1t} and Y_{2t} exhibit *joint volatility clustering*—that is, do their conditional variances $\mathrm{var}[Y_{1t}|I_t]$ and $\mathrm{var}[Y_{2t}|I_t]$ exhibit contemporaneously correlated departures from their predicted paths?

[4] We avoid the term "covolatility clustering" on the grounds that it is ambiguous: if it is taken to mean "clustering of covolatility" then it corresponds to what we term "covariance clustering." On the other hand, if it is taken to mean "correlation of volatility clustering" then it corresponds to what we term "joint volatility clustering."

To see that either of these phenomena can occur without the other, let $\{\tilde{\omega}\}_{t=1,2,\ldots}$, $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ and $\{\tilde{\eta}_t\}_{t=1,2,\ldots}$ be zero-mean, unit-variance i.i.d. shock variables, and Z_t be a drift variable with values in [0,1] and which is in the information set I_t. Since the pair of variables $Y^*_{1t} = \sqrt{Z_t}\tilde{\omega}_t + \sqrt{1-Z_t}\tilde{\varepsilon}_t$ and $Y^*_{2t} = \sqrt{Z_t}\tilde{\omega}_t + \sqrt{1-Z_t}\tilde{\eta}_t$ satisfy $\mathrm{E}[Y^*_{1t}|I_t] = \mathrm{E}[Y^*_{2t}|I_t] = 0$, $\mathrm{cov}[Y^*_{1t}, Y^*_{2t}|I_t] = Z_t$ and $\mathrm{var}[Y^*_{1t}|I_t] = \mathrm{var}[Y^*_{2t}|I_t] = 1$, they exhibit covariance clustering, but neither joint nor individual volatility clustering. Conversely, since the variables $Y^{**}_{1t} = Z_t\tilde{\varepsilon}_t$ and $Y^{**}_{2t} = Z_t\tilde{\eta}_t$ satisfy $\mathrm{E}[Y^{**}_{1t}|I_t] = \mathrm{E}[Y^{**}_{2t}|I_t] = 0$, $\mathrm{var}[Y^{**}_{1t}|I_t] = \mathrm{var}[Y^{**}_{2t}|I_t] = Z_t^2$ and $\mathrm{cov}[Y^{**}_{1t}, Y^{**}_{2t}|I_t] = 0$, they exhibit both individual and joint volatility clustering, but not covariance clustering.

As with volatility clustering, both covariance clustering and joint volatility clustering can arise in structural systems which are time-invariant and whose shocks are all zero-mean i.i.d. In the supply-demand example of Section 2, since the market clearing quantity Q_t^{e} is a deterministic function $Q^{\mathrm{S}}(\cdot)$ of market clearing price P_t^{e}, Eq. (12) implies that for small departures of Z_t from its conditional mean $\mathrm{E}[Z_t|I_t]$ and small values of $\tilde{\varepsilon}_t$ about its mean of 0, we have

$$\mathrm{d}Q_t^{\mathrm{e}} = Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t])\frac{Q_Z^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])\mathrm{d}Z_t + \mathrm{d}\tilde{\varepsilon}_t}{Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) - Q_P^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])}. \tag{21}$$

For small values of $\sigma^2_{\tilde{\eta}}$ and $\sigma^2_{\tilde{\varepsilon}}$ the conditional variance of Q_t^{e} and conditional covariance of P_t^{e} and Q_t^{e} can be approximated by

$$\mathrm{var}[Q_t^{\mathrm{e}} \mid I_t] \approx Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t])^2 \frac{Q_Z^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])^2\sigma^2_{\tilde{\eta}} + \sigma^2_{\tilde{\varepsilon}}}{[Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) - Q_P^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])]^2}, \tag{22}$$

$$\mathrm{cov}[P_t^{\mathrm{e}}, Q_t^{\mathrm{e}} \mid I_t] \approx Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) \frac{Q_Z^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])^2\sigma^2_{\tilde{\eta}} + \sigma^2_{\tilde{\varepsilon}}}{[Q_P^{\mathrm{S}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t]) - Q_P^{\mathrm{D}}(\mathrm{E}[P_t^{\mathrm{e}}|I_t], \mathrm{E}[Z_t|I_t])]^2}. \tag{23}$$

Since drift in $\mathrm{E}[Z_t|I_t]$ affects $\mathrm{var}[P_t^{\mathrm{e}}|I_t]$, $\mathrm{var}[Q_t^{\mathrm{e}}|I_t]$ and $\mathrm{cov}[P_t^{\mathrm{e}},Q_t^{\mathrm{e}}|I_t]$, P_t^{e} and Q_t^{e} will generally exhibit both covariance clustering and joint volatility clustering. Since the partial derivative $Q_P^{\mathrm{D}}(\cdot,\cdot)$ is negative and the derivative $Q_P^{\mathrm{S}}(\cdot)$ is positive, the drifting conditional covariance $\mathrm{cov}[P_t^{\mathrm{e}},Q_t^{\mathrm{e}}|I_t]$ is always positive.[5] However, whether the conditional variances $\mathrm{var}[P_t^{\mathrm{e}}|I_t]$ and $\mathrm{var}[Q_t^{\mathrm{e}}|I_t]$ tend to drift in the same direction as each other or in opposite directions will depend on the signs and magnitudes of the second-order derivatives of the demand and supply functions. The following figures illustrate this for a simple demand function of the form $Q^{\mathrm{D}}(P_t) + Z_t + \tilde{\varepsilon}_t$. In Fig. 3, where demand is convex in price and supply is linear, $\mathrm{var}[P_t^{\mathrm{e}}|I_t]$ and $\mathrm{var}[Q_t^{\mathrm{e}}|I_t]$ both unambiguously rise/fall as the demand curve drifts to the right/left. In Fig. 4, where demand is convex in price and supply is concave, $\mathrm{var}[P_t^{\mathrm{e}}|I_t]$ unambiguously rises/falls as demand drifts to the right/left, but the behavior of $\mathrm{var}[Q_t^{\mathrm{e}}|I_t]$ will depend on the exact magnitudes of $Q_{PP}^{\mathrm{S}}(\cdot)$ and $Q_{PP}^{\mathrm{D}}(\cdot,\cdot)$, via (22).

[5]This is natural since we assume a drifting stochastic demand curve and a deterministic supply curve.

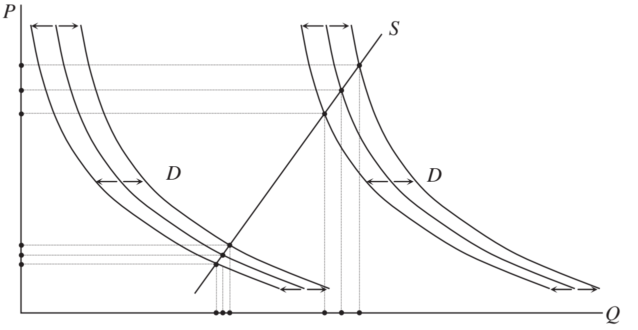

Fig. 3. Induced individual and joint volatility clustering in P_t^e and Q_t^e.

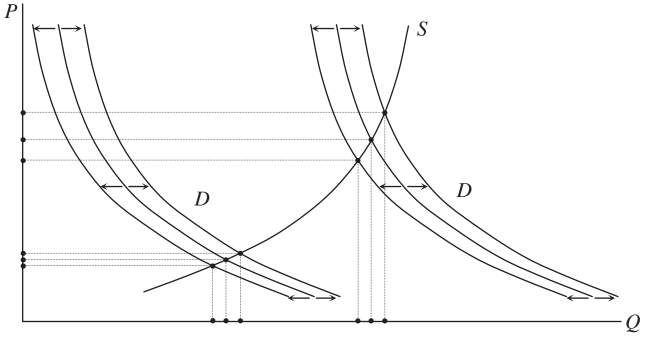

Fig. 4. Induced volatility clustering in P_t^e, but not necessarily volatility clustering in Q_t^e or joint volatility clustering.

2.4. Other examples

We are not the first to present structural models that generate volatility clustering. Other researchers who have done so include:

- Brock and LeBaron (1996), who develop an asymmetric-information, adaptive-beliefs model of asset pricing and volume, in which volatility clustering arises from traders experimenting with different belief systems based on past profits and expected future profits.[6]
- Evans and Ramey (1995), who develop a boundedly rational, adaptive-learning model of asset pricing, in which underlying regime switches trigger expectations adjustments that in turn lead to volatility clustering.
- Cabrales and Hoshi (1996), who develop a heterogeneous-beliefs asset pricing model in which changes in the distribution of wealth over time lead to volatility clustering, as well as a separate model which generates the same effect from heterogeneous risk preferences.

[6]See also Brock (1987) on some related theoretical and econometric issues.

- Den Haan and Spear (1998), who develop a heterogeneous-agent, incomplete markets model of interest rates, in which financial frictions lead to volatility clustering which is correlated with both the borrowing/lending rate spread and the business cycle.
- de Fontnouvelle (2000), who develops a costly information model of asset trading, in which agents' time-varying information acquisition strategies serves to generate volatility clustering in both price and trading volume.
- Timmermann (2001), who develops an imperfect-information model of asset pricing, in which large revisions in agents' parameter estimates in the periods following each structural break leads to volatility clustering in prices.

Although each of these models was designed to generate volatility clustering as one its implications, they differ from our own analysis and examples both in their greater levels of specificity (each being directed at a particular empirical phenomenon) and their greater levels of complexity (involving asymmetric information, rational or boundedly rational beliefs and learning, or heterogeneous agents). The purpose of our own analysis and examples is to identify the general sources of induced volatility clustering—namely drifting coefficient or drifting input effects—and show how induced volatility clustering can emerge in even the simplest of nonlinear supply-demand systems, or homogeneous-agent/rational expectations asset pricing models.

3. Structural attribution of observed serial correlation

If the residuals of a regression equation $Y_t = \alpha + \beta X_t + u_t$ are found to exhibit serial correlation of the MA(1) form $u_t = \tilde{\varepsilon}_t + \gamma\tilde{\varepsilon}_{t-1}$ for some γ and i.i.d. series $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$, then it is possible to fully "structurally attribute" this serial correlation by simply expressing the complete model as

$$Y_t = \alpha + \beta X_t + \tilde{\varepsilon}_t + \gamma\tilde{\varepsilon}_{t-1}, \tag{24}$$

that is, by assuming that the underlying shock variables are i.i.d. rather than serially correlated, and that each shock variable $\tilde{\varepsilon}_t$ not only affects Y_t but also has structural effect γ upon Y_{t+1}.

If the serial correlation in $Y_t = \alpha + \beta X_t + u_t$ is instead found to take the AR(1) form $u_t = \rho u_{t-1} + \tilde{\varepsilon}_t$ for some $\rho \in [0, 1)$ and i.i.d. $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$, then it can be fully structurally attributed by expressing the complete model in either of the equivalent forms

$$Y_t = \alpha + \beta X_t + \sum_{\tau=0}^{\infty} \rho^{\tau}\tilde{\varepsilon}_{t-\tau}, \tag{25}$$

$$Y_t = (1-\rho)\alpha + \beta X_t + \rho Y_{t-1} - \rho\beta X_{t-1} + \tilde{\varepsilon}_t, \tag{25$'$}$$

where the first form posits that each of the i.i.d. shock variables $\tilde{\varepsilon}_t$ affects Y_t and also has geometrically diminishing structural effects ρ, ρ^2,... on its subsequent values

$Y_{t+1}, Y_{t+2}, \ldots$, and the second form posits that Y_t and X_t both directly structurally affect Y_{t+1}. Since each of the models (24), (25), (25)′ involves only time-invariant parameters and zero-mean i.i.d. shocks $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$, we say that each of these models serves to *fully structurally attribute* the observed serial correlation in Y_t. Using standard techniques, most ARMA processes for the residual term u_t can be treated in a similar manner.[7] Although these i.i.d. shock structural models are of course observationally equivalent to the form $Y_t = \alpha + \beta X_t + u_t$ with its specified form of serial correlation, there is a sense in which they can be said to constitute a more complete explanation of the observed statistical properties of the variable Y_t.

4. Structural attribution of observed volatility clustering

If a variable Y_t or the residuals in a regression equation for Y_t are observed to exhibit volatility clustering, then in many cases it will be similarly possible to structurally attribute this phenomenon by means of a time-invariant structural model with zero-mean i.i.d. shocks. As with the structural attribution of observed serial correlation, the specific form of the structural model will depend upon the specific form of the observed volatility clustering, such as whether (and how) the conditional variance of u_t is related to Y_t, related to any explanatory variables X_t, related to other observable variables Z_t, or related to its own past values $u_{t-1}, u_{t-2}, \ldots$ Such dependence is useful, since it implies that the statistical volatility clustering properties of a variable can provide hints in the search for an underlying structural or causal model of that variable. We consider the following examples:

4.1. Simple autoregressive volatility

If the zero-mean residuals in a regression equation $Y_t = \alpha + \beta X_t + u_t$ are found to exhibit volatility clustering of the simple form $\text{var}(u_t|I_t) = \rho u_{t-1}^2$, they can be generated by a process of the form

$$u_t = \tilde{\varepsilon}_t \prod_{\tau=1}^{t-1} |\tilde{\varepsilon}_\tau| = \tilde{\varepsilon}_t \left| \tilde{\varepsilon}_{t-1} \prod_{\tau=1}^{t-2} |\tilde{\varepsilon}_\tau| \right| = \tilde{\varepsilon}_\tau |u_{t-1}| \tag{26}$$

for zero-mean i.i.d. $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ with $\text{var}(\tilde{\varepsilon}_t) = \rho$, so that by defining $Z_t = \ln(|u_t|)$, this form of volatility clustering can be fully structurally attributed by an i.i.d. shock, time-invariant system of the form

$$\begin{aligned} &Y_t = \alpha + \beta X_t + \exp(Z_{t-1})\tilde{\varepsilon}_t, \\ &Z_t = Z_{t-1} + \ln(|\tilde{\varepsilon}_t|), \\ &\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots} \text{ zero-mean i.i.d. with } \text{var}(\tilde{\varepsilon}_t) = \rho, \end{aligned} \tag{27}$$

where Y_t is seen to exhibit induced volatility clustering via the drifting coefficient $\exp(Z_{t-1})$, for the conditionally homoskedastic drift variable Z_t.

[7] See, for example, Sargan (1980).

4.2. ARCH

More generally, if the residuals in $Y_t = \alpha + \beta X_t + u_t$ are found to exhibit volatility clustering of the standard *ARCH* (*autoregressive conditional heteroskedasticity*) form $\mathrm{var}(u_t|I_t) = c + \rho u_{t-1}^2$, they can be generated by a process of the form[8]

$$\begin{aligned} u_t &= \tilde{\varepsilon}_t\sqrt{(c/\rho) + (c/\rho)\sum\nolimits_{s=1}^{t-1}\prod\nolimits_{\tau=s}^{t-1}\tilde{\varepsilon}_\tau^2} \\ &= \tilde{\varepsilon}_t\sqrt{(c/\rho) + \left[(c/\rho)\tilde{\varepsilon}_{t-1}^2 + (c/\rho)\tilde{\varepsilon}_{t-1}^2\sum\nolimits_{s=1}^{t-2}\prod\nolimits_{\tau=s}^{t-2}\tilde{\varepsilon}_\tau^2\right]} \\ &= \tilde{\varepsilon}_t\sqrt{(c/\rho) + \tilde{\varepsilon}_{t-1}^2\left[(c/\rho) + (c/\rho)\sum\nolimits_{s=1}^{t-2}\prod\nolimits_{\tau=s}^{t-2}\tilde{\varepsilon}_\tau^2\right]} \\ &= \tilde{\varepsilon}_t\sqrt{(c/\rho) + u_{t-1}^2} \end{aligned} \tag{28}$$

for zero-mean i.i.d. $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ with $\mathrm{var}(\tilde{\varepsilon}_t) = \rho$, so that by the substitution $Z_t = u_t^2$, this form of volatility clustering can be fully structurally attributed by an i.i.d. shock, time-invariant system of the form

$$\begin{aligned} &Y_t = \alpha + \beta X_t + \tilde{\varepsilon}_t\sqrt{(c/\rho) + Z_{t-1}}, \\ &Z_t = \tilde{\varepsilon}_t^2[(c/\rho) + Z_{t-1}], \\ &\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots} \text{ zero-mean i.i.d. with } \mathrm{var}(\tilde{\varepsilon}_t) = \rho, \end{aligned} \tag{29}$$

where Y_t exhibits induced volatility clustering via the drifting coefficient $\sqrt{(c/\rho) + Z_{t-1}}$, for a variable Z_t that is subject to both homoskedastic additive and homoskedastic multiplicative drift.

4.3. Long memory processes

A different generalization of the simple autoregressive volatility form is if the residuals in $Y_t = \alpha + \beta X_t + u_t$ are found to exhibit volatility clustering of the *long memory process* form $\mathrm{var}(u_t|I_t) = |u_{t-1}|^{2\rho}$. In such a case, they can be generated by a process of the form

$$u_t = \tilde{\varepsilon}_t\prod_{\tau=1}^{t-1}|\tilde{\varepsilon}_\tau|^{\rho^{(t-\tau)}} = \tilde{\varepsilon}_t\left[|\tilde{\varepsilon}_{t-1}|\prod_{\tau=1}^{t-2}|\tilde{\varepsilon}_\tau|^{\rho^{(t-1-\tau)}}\right]^\rho = \tilde{\varepsilon}_t|u_{t-1}|^\rho \tag{30}$$

for zero-mean i.i.d. $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ with $\mathrm{var}(\tilde{\varepsilon}_t) = 1$, so that by the substitution $Z_t = \ln(|u_t|)$, this form of volatility clustering can be fully structurally attributed by an i.i.d. shock, time-invariant system of the form

$$\begin{aligned} &Y_t = \alpha + \beta X_t + \exp(\rho Z_{t-1})\tilde{\varepsilon}_t, \\ &Z_t = \rho Z_{t-1} + \ln(|\tilde{\varepsilon}_t|), \\ &\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots} \text{ zero-mean i.i.d. with } \mathrm{var}(\tilde{\varepsilon}_t) = 1, \end{aligned} \tag{31}$$

[8]The following form can be equivalently written as $u_t = \mathrm{sgn}(\tilde{\varepsilon}_t)\left[(c/\rho)\Sigma_{s=1}^t\Pi_{\tau=s}^t\tilde{\varepsilon}_\tau^2\right]^{1/2}$.

where Y_t exhibits induced volatility clustering via the drifting coefficient $\exp(\rho Z_{t-1})$, for the conditionally homoskedastic autoregressive drift variable Z_t.

4.4. Factor-dependent volatility

If the predicted mean of Y_t conditional upon some univariate variable X_t, and its predicted or observed variance, are found to be linked by relationships of the form

$$\mathrm{E}[Y_t|X_t] \equiv G(X_t), \quad \mathrm{var}[Y_t|X_t] \equiv \gamma G'(X_t)^2, \tag{32}$$

then Y_t can be *approximately structurally attributed* by the time-invariant, i.i.d. shock, drifting-input form

$$Y_t = G(X_t + \tilde{\varepsilon}_t), \quad \mathrm{E}[\tilde{\varepsilon}_t] = 0, \quad \mathrm{var}(\tilde{\varepsilon}_t) = \gamma, \tag{33}$$

where the attribution is approximate since the structural form (33) only approximately implies the conditional mean and variance formulas (32).

In other cases, however, such structural attribution can be exact. For example, if the conditional mean and variance relationships should be found to take the separate quadratic forms

$$\mathrm{E}[Y_t|X_t] = \alpha + \beta X_t + \gamma X_t^2, \quad \mathrm{var}[Y_t|X_t] = k_1 + k_2 X_t + k_3 X_t^2, \tag{34}$$

then we can obtain exact structural attribution by the drifting-input structural form $Y_t \equiv G(X_t + \tilde{\varepsilon}_t)$, for the quadratic function

$$G(X) = \alpha - k_3/(4\gamma) + \beta X + \gamma X^2 \tag{35}$$

and zero-mean i.i.d. shocks $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ which satisfy

$$\begin{aligned} \sigma_{\tilde{\varepsilon}}^2 = \mathrm{E}[\tilde{\varepsilon}_t^2] &= k_3/(4\gamma^2) \\ \mathrm{E}[\tilde{\varepsilon}_t^3] &= k_2/(4\gamma^2) - \beta k_3/(4\gamma^3) \\ \mathrm{E}[\tilde{\varepsilon}_t^4] &= k_1/\gamma^2 - \beta k_2/(2\gamma^3) + \beta^2 k_3/(4\gamma^4) + k_3^2/(16\gamma^4). \end{aligned} \tag{36}$$

Observe that in contrast to the functional dependence between the conditional mean and variance relationships in (32), the coefficients α, β and γ in the conditional mean function and coefficients k_1, k_2 and k_3 in the conditional variance function of (34) bear no necessary linkage to each other beyond the weak inequalities implied by the nonnegativity of $\mathrm{E}[\tilde{\varepsilon}_t^2]$ and $\mathrm{E}[\tilde{\varepsilon}_t^4]$ in (36).

4.5. Covariance clustering

If the residuals in the pair of regression equations $Y_{1t} = \alpha_1 + \beta_1 X_{1t} + u_{1t}$ and $Y_{2t} = \alpha_2 + \beta_2 X_{2t} + u_{2t}$ are found to exhibit simple autoregressive covariance clustering of the form $\mathrm{cov}[u_{1t}, u_{2t}|I_t] = \gamma u_{1,t-1} u_{2,t-1}$, but no individual or joint volatility clustering, so that $\mathrm{var}[u_1 t|I_t] = \sigma_{u_1}^2$ and $\mathrm{var}[u_2 t|I_t] = \sigma_{u_2}^2$, they can be generated by a process of the form

$$u_{1t} = \sigma_{u_1}\left[\sqrt{(\gamma u_{1,t-1}u_{2,t-1})/(\sigma_{u_1}\sigma_{u_2})}\tilde{\omega}_t + \sqrt{1-(\gamma u_{1,t-1}u_{2,t-1})/(\sigma_{u_1}\sigma_{u_2})}\tilde{\varepsilon}_t\right],$$
$$u_{2t} = \sigma_{u_2}\left[\sqrt{(\gamma u_{1,t-1}u_{2,t-1})/(\sigma_{u_1}\sigma_{u_2})}\tilde{\omega}_t + \sqrt{1-(\gamma u_{1,t-1}u_{2,t-1})/(\sigma_{u_1}\sigma_{u_2})}\tilde{\eta}_t\right] \quad (37)$$

for the zero-mean, unit-variance, i.i.d. shocks $\{\tilde{\omega}_t\}_{t=1,2,\ldots}$, $\{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}$ and $\{\tilde{\eta}_t\}_{t=1,2,\ldots}$. Defining $Z_t = (\gamma u_{1t}u_{2t})/(\sigma_{u_1}\sigma_{u_2})$, we can fully structurally attribute this process by the i.i.d. shock, time-invariant system

$$Y_{1t} = \alpha_1 + \beta_1 X_{1t} + \sigma_{u_1}\sqrt{Z_{t-1}}\tilde{\omega}_t + \sigma_{u_1}\sqrt{1-Z_{t-1}}\tilde{\varepsilon}_t,$$
$$Y_{2t} = \alpha_2 + \beta_2 X_{2t} + \sigma_{u_2}\sqrt{Z_{t-1}}\tilde{\omega}_t + \sigma_{u_2}\sqrt{1-Z_{t-1}}\tilde{\eta}_t,$$
$$Z_t = \gamma\left[Z_{t-1}\tilde{\omega}_t^2 + (1-Z_{t-1})\tilde{\varepsilon}_t\tilde{\eta}_t + \sqrt{Z_{t-1}}\sqrt{1-Z_{t-1}}\tilde{\omega}_t(\tilde{\varepsilon}_t+\tilde{\eta}_t)\right],$$
$$\{\tilde{\omega}_t\}_{t=1,2,\ldots}, \{\tilde{\varepsilon}_t\}_{t=1,2,\ldots}, \{\tilde{\eta}_t\}_{t=1,2,\ldots} \text{ zero-mean, unit variance, i.i.d.} \quad (38)$$

5. Modeling and forecasting implications

The fact that we can construct structural models which generate different specific forms of observed volatility clustering in a variable or variables does not imply that the causal variables and causal relationships in these models necessarily exist. Rather, it is our hope that with a more complete understanding of the relationship between specific features of structural models and specific forms of volatility clustering, increasingly sophisticated statistical specifications of observed volatility clustering in a variable or variables can be used to suggest possible structural explanations of those variables, which can then be formulated and tested in the standard manners.

When such structural explanations can be satisfactorily established, they can in turn serve to help better predict volatility. Although the examples of structural attribution in the previous section all involved time-invariant forms, the parameters of structural economic models often change in predictable ways—for example, as a result of announced changes in tax rates or other policy variables. Such predicted structural changes will imply well-defined predicted changes ("breaks") in the volatility clustering properties of their key variables, which could not have been predicted from any statistical volatility clustering specification based solely on current and past values of those variables.

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References

Brock, W.A., 1987. Nonlinearity in Finance and Economics, Manuscript. Department of Economics, University of Wisconsin.

Brock, W.A., LeBaron, B.D., 1996. A dynamic structural model for stock return volatility and trading volume. Review of Economics and Statistics 78, 94–110.

Cabrales, A., Hoshi, T., 1996. Heterogeneous beliefs, wealth accumulation, and asset price dynamics. Journal of Economic Dynamics and Control 20, 1073–1100.

de Fontnouvelle, P., 2000. Information dynamics in financial markets. Macroeconomic Dynamics 4, 139–169.

Den Haan, W.J., Spear, S.A., 1998. Volatility clustering in real interest rates: theory and evidence. Journal of Monetary Economics 41, 431–453.

Engle, R.F., 1995. ARCH Models. Oxford University Press, Oxford.

Engle, R.F., Bollerslev, T., Nelson, D.B., 1994. ARCH models. In: Engle, R.F., McFadden, D.L. (Eds.), Handbook of Econometrics, vol. IV. North-Holland, Amsterdam, pp. 2961–3031.

Evans, G.W., Ramey, G., 1995. Underparametrized Learning, Regime Switching and Asset Pricing, Manuscript. Department of Economics, University of California, San Diego.

Ghysels, E., Harvey, A., Renault, E., 1996. Stochastic volatility. In: Madalla, G.S., Rao, C.R. (Eds.), Handbook of Statistics, vol. 14. Elsevier Science, Amsterdam, pp. 119–191.

Sargan, J.D., 1980. Some tests of dynamic specification for a single equation. Econometrica 48, 879–897.

Shephard, N., 1996. Statistical aspects of ARCH and stochastic volatility. In: Cox, D.R., Hinkley, D.V., Barndorff-Nielsen, O.E. (Eds.), Time Series Models in Econometrics, Finance and Other Fields. Chapman and Hall, London, pp. 1–67.

Timmermann, A., 2001. Structural breaks, incomplete information and stock prices. Journal of Business and Economic Statistics 19, 299–314.