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The Effect of Zoning on Land Value

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Two types of zoning are identified: externality zoning, which is designed to achieve a Pareto efficient pattern of land use, and fiscal zoning, which is designed to accomplish some other objective. (The latter, for instance, may be aimed at minimizing the tax rate in a community.) The paper shows that it is not in general possible using *a priori* theory to predict the sign (positive or negative) of the effect of either of these forms of zoning on aggregate land value in a community. It is shown, however, that under plausible assumption it can be argued that zoning as currently practiced in many U. S. communities probably has the effect of lowering aggregate land values in the communities doing the zoning.

Most proponents of zoning claim that zoning "protects land values," and many decision-makers in the zoning field apparently believe that their actions lead to maximization of land values. While land values are undoubtedly an important consideration in decisions affecting zoning, little attention has been given to the exact effect of alternate forms of zoning on the price of land. This note attempts to identify, with as much precision as possible, the key parameters which determine the effect of zoning on land prices.

We consider two types of zoning with different objectives—fiscal zoning and externality zoning. The paper demonstrates that both types of zoning can either raise or *lower* aggregate land values.

The following assumptions are made:

(a) Markets for the rental services of land are perfectly competitive.

(b) The price of a parcel of land is equivalent to the suitably discounted present value of the future stream of rents (net of costs) for that parcel.

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[2] The authors' names are listed alphabetically and they share equally responsibility both for the paper's strengths and for its weaknesses. We are indebted to Professors David Pines and Lester Chandler and to an anonymous referee for pointing out errors in previous versions of this paper. We are indebted to Michael Munson for help in preparing the diagrams.

EXTERNALITY ZONING AND FISCAL ZONING

We shall define "externality zoning" as zoning which is in response to the phenomenon that one person's use of land may have external effects—positive or negative—on the uses of neighboring land. The action of the private market may not lead to an economically efficient outcome under these conditions, and standard Pigovian pricing of the externality may be difficult. Coase [1] has suggested one way in which the private market can deal with externalities: Private transfer payments between the conflicting land users can induce one or the other party to modify his economic behavior so that production (or utility) is increased. However, if the transactions costs involved in private market agreements are too high, then zoning may be an effective way of creating an efficient pattern of land use through government regulation. Externality zoning may regulate land use in order to reduce negative externalities below the level which would have occurred in the unregulated market.

For example, if a general class of users (industrial) produces negative externalities which affect another class of users (residential), then externality zoning could restrict industrial users to a number of contiguous parcels of land reserved for such a use. This could reduce the harmful external effects of the industrial uses by minimizing the length of the boundary between conflicting uses.[3]

Furthermore, we define an optimal externality zoning policy as one which produces an economically efficient (Pareto Optimal) allocation of resources. For example, if for simplicity we neglect spatial considerations in the location of industry and assume that land is homogenous in quality, the optimal policy in the above example might segregate industry in a compact area and would locate this area in order to minimize its boundary with residential areas.[4]

"Fiscal zoning" will be defined to mean zoning which creates a different pattern of land use than externality zoning because policy-makers have an objective other than economic efficiency. For example, assume that a suburban community desires local public services of a high quality but also desires a low property tax rate. Such a community might zone vacant land in large lots for high-value single family homes because it believes that owners of expensive homes will pay more property taxes than the cost of providing additional public services to meet their needs. In such circumstances, the possibility that a community's vacant land might be better suited to apartments than to single family homes would make no difference to the community. It would

[3] This discussion assumes that all externalities are of the undesirable sort. At the price of some additional complexity, our analysis could be extended to cover the more general case, but this would not affect our present conclusions.

[4] Zoning in this case enforces segregation of industries and residences.

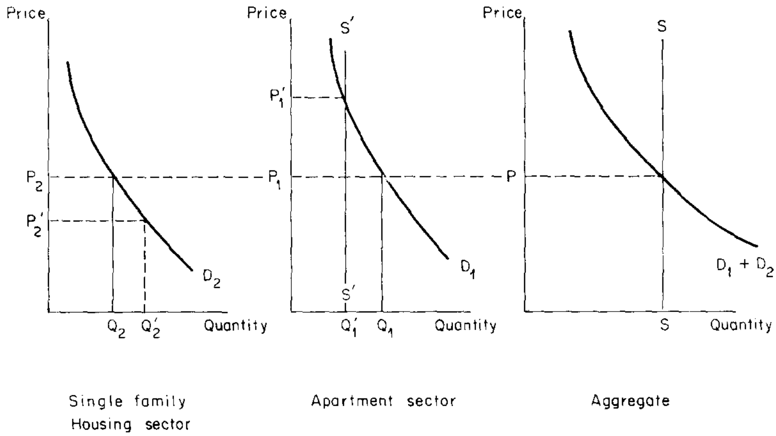

FIG. 1. A two-sector land market.

prefer that the land remain vacant to its being used for apartment buildings, since such a use is seen as a fiscal drain on the community. An optimal fiscal zoning policy can be defined only with reference to the community's objectives but it will not in general lead to the same pattern of land use as would an optimal externality zoning policy.

FISCAL ZONING AND LAND VALUE

We now examine the potential effect of fiscal zoning on land value. Zoning may affect land values directly by affecting the rental price of land or indirectly by affecting the tax rates which then get capitalized in land value. In this section we shall ignore possible tax rate effects and only consider effects on rental prices. The more general case is considered in the following section.

We further assume that there are only two groups of land users in a particular market and that their demand curves for land are not interdependent. One group is apartment producers; the second is producers of single family housing. For purposes of the present discussion, we will consider the land market as an entire urban area rather than one specific municipality in the area. In later sections, we distinguish more carefully between the entire urban land market and the land market within a municipality in the urban area.

Under these assumptions, we can construct the demand curves for land of each of the two user groups, producers of apartments and producers of single family houses, as well as the market demand curve for land. This is shown in Fig. 1. In order to focus specifically on the effects of zoning, we are ignoring land price differences which arise due to different locations in the city. Alternatively, the analysis may be interpreted as applying only to land of a given quality a given distance from the center city.

The supply of land in the market is given by SS.[5] In an unzoned competitive market, equilibrium occurs where the total demand for land by both user groups is equal to total supply. The price of land to each user group is the same: $P_1 = P_2$. Apartment dwellers consume Q_1 land and owners of single family homes consume Q_2 land. The total rental value (V) of land in the market is equal to $P_1Q_1 + P_2Q_2$.

Let us now assume that zoning laws are enacted by all the municipalities which limit the total amount of land available for apartment buildings in the urban area. This is shown by shifting the supply curve to $S'S'$ in the apartment submarket. If $S'S'$ is greater than Q_1, then zoning has no effect on land value. If $S'S'$ is smaller than Q_1, the effect of zoning is to create two separate submarkets for land. Because of the shifts in the supply curve, the price of land used for apartment construction is driven up to P_1', and the quantity of land used for apartment construction drops to Q_1'. The price of land in the single family submarket will also respond to competive pressure. Because of the increased supply of such land, the price of land used for the construction of single family units drops to P_2' and the quantity of land increases to Q_2', where the increase in land available for construction of single family homes is equal to the decrease in land available for construction of apartments. Effectively, the use of land for apartments is taxed by the amount $P_1 - P_1'$; the use of land for single family homes is subsidized by the amount $P_2 - P_2'$. After zoning, the total aggregate rental value of land equals $P_1'Q_1' + P_2'Q_2'$. It is necessary to know the elasticities of demand for land by both user groups in order to predict whether an increase in the total aggregate rental value of land follows the imposition of zoning controls. There is no way to know a priori whether $P_1Q_1 + P_2Q_2$ is greater or smaller than $P_1'Q_1' + P_2'Q_2'$.

The general effect of fiscal zoning on aggregate land value in the market can be derived algebraically. The total value of land before zoning is given as

$$V = P_1Q_1 + P_2Q_2. \tag{1}$$

[5] The SS curve drawn in Fig. 1 assumes an inelastic supply of land available for residential purposes. In some land markets, where considerable quantities of land are used for agriculture, it may be more realistic to assume that the amount of land available to the two-residential uses is highly elastic at a price corresponding to its value in agricultural uses. The analysis below could easily be adapted to this alternative situation. However the results of the analysis would be rather different in this alternative case. Specifically, fiscal zoning would, in the case of perfectly elastic supply at an agricultural price, have the effect of raising aggregate land value. This is the case because if, say, land for apartment uses is restricted, then the value of that land will rise while the value of all other land will remain at its agricultural price. Hence there will be a net rise in aggregate land values. What assumption concerning supply elasticity is most realistic depends upon the specific situation being analyzed. In the case of a relatively small community surrounded by farm land, an assumption of high supply elasticity may be reasonable. If however, the analysis is being applied to the market consisting of land a given distance from the city center, and if most of this land has already been converted away from agricultural uses, then an assumption of an inelastic supply curve may be more appropriate.

The change in land value after zoning is calculated by taking a total differential of Eq. (1) and substituting the identity $dQ_1 = -\,dQ_2$.

$$dV = \left[P_1 + Q_1\left(\frac{dP_1}{dQ_1}\right)\right]dQ_1 - \left[P_2 + Q_2\left(\frac{dP_2}{dQ_2}\right)\right]dQ_1. \tag{2}$$

The prezoning price of a parcel of land was the same in either use: therefore $P_1 = P_2$. By substituting this equation in (2) and transforming, we have

$$dV = P\,dQ_1\left[\frac{1}{e_{Q_1,P_1}} - \frac{1}{e_{Q_2,P_2}}\right], \tag{3}$$

where e_{Q_1,P_1} and e_{Q_2,P_2} are in the usual quantity-change-to-price-change demand elasticity form. Suppose, for instance, that Q_2 is the single family housing sector, where available land has been increased, and Q_1 is the apartment housing sector, where available land has been decreased. If demand for land to be employed in apartment construction is more inelastic (or less negative) than demand for land for single family homes, then a decrease in land available for apartment construction will increase aggregate land value in the community. If the converse holds, then zoning will lower aggregate land value.

A similar result holds for the case in which the demands for land by the apartments and single family sectors are interdependent. In this case, it can be shown that the change in total land value as a result of zoning is

$$dV = P\,dQ_1\left[\frac{1}{e_{Q_1,P_1}} + \frac{Q_2}{Q_1}\cdot\frac{1}{e_{Q_1,P_2}} - \frac{1}{e_{Q_2,P_2}} - \frac{Q_1}{Q_2}\cdot\frac{1}{e_{Q_2,P_1}}\right]. \tag{3A}$$

In this case the effect of zoning on land value is still ambiguous. It depends on the elasticities of demand in the two submarkets and on the cross-elasticities of demand weighted by the initial relative land allocation.

These results may appear somewhat counterintuitive. Intuition might suggest that a competitive, unregulated land market would maximize land values. We have shown, however, that it is possible for fiscal zoning to increase aggregate land value. Fiscal zoning enables suppliers of land to act in discriminating monopolist fashion. They can charge a high price in that submarket where demand is inelastic by using zoning to restrict supply and channel the left-over supply into the market with elastic demand. Furthermore, aggregate land value can be increased even when the two submarkets are interrelated if demand for the restricted use (in this case apartments) is sufficiently inelastic.

FISCAL ZONING IN U. S. URBAN LAND MARKETS

Our analysis demonstrates that a priori theory cannot predict the effect of fiscal zoning on land value. However, we have identified some of the key parameters which determine the effect of zoning on land value. In light of this analysis, we now turn to a discussion of the effects of fiscal zoning as it is currently practiced in urban areas of the United States.[6]

There is substantial evidence that the principal motivation for zoning in land markets in the United States is fiscal in nature. In most municipalities. for instance, the amount of land available for apartment buildings is more restricted by zoning legislation than is land available for single family residences. In the following discussion we shall ignore all externality effects, and draw upon our analysis of fiscal zoning.[7]

Because zoning decisions are made locally, it is of interest to examine two issues: (1) the overall effect on an *entire metropolitan area housing market* of the collective zoning decisions of the municipalities in the area, and (2) the effect on land values *within an individual municipality* resulting from its own zoning decisions. We shall begin by considering the first of these questions.

As Eq. (3) shows, the effect on land values of the collective decision of urban municipalities to restrict apartment buildings depends upon the relationships between the elasticities of demand for (a) apartment uses of land and (b) single family uses of land. Because land is a factor input in the production of housing services, the standard theory of derived factor demand is relevant in considering the magnitudes of these elasticities. This theory suggests that, *ceteris paribus*, the demand for land is more inelastic (1) the less there are good substitutes for land in the production of housing, (2); the smaller the ratio of the cost of land to the total cost of housing, (3) the more inelastic is demand for housing, and (4) the more inelastic the supply of other factors used in the production of housing [4, pp. 70–73].

Conditions (1) and (2) point in opposite directions. In (1), the obvious substitute for land is capital. It is likely that land is more substitutable for capital in production of apartments than houses. The heights of apartment buildings can be increased, but the production of single family housing necessarily requires a substantial amount of land per housing unit. Thus, the first consideration suggests that the demand for land for apartment construction might be relatively more elastic than the demand for land for the construction of single family homes. Condition (2) points in the opposite direction since land cost is lower for apartments than for single family houses [3][8].

Condition (3) says that the demand for land will be more inelastic the more inelastic is demand for the final product. Research on price elasticities of

[6] The effect of property taxes on land value is ignored here. It is discussed in the next section.

[7] While we shall not discuss this possibility in detail here, it should be noted that another possible interpretation of the evidence is that observed zoning is really externalities zoning in that apartments may generate negative externalities towards occupants of single family homes.

[8] The data in [3] put the share of land at 13% for elevator apartments and 25% for single family homes.

demand for housing has not settled the question whether the demand for houses is more or less elastic then demand for apartments. DeLeeuw's survey of empirical studies on the subject suggests that the elasticity figure for apartments is a range from -0.7 to -1.4. In comparison, Muth finds the price elasticity for houses to be about -1.0[2].

Condition (4) says that the demand for land is more inelastic the more inelastic is the supply of other factors used in the production of housing. Since the other factors used—lumber, labor, plumbing, glass, concrete, etc.—are mostly the same in production of both apartments and houses, this condition is also ambiguous. Thus the theory of derived demand does not suggest an answer to the relative elasticity question. Therefore, the effect of zoning on prices in the entire urban area, taken as a whole, is uncertain.

With regard to the second issue which we have raised—the effects any *individual community's* zoning decisions have on land values *within that community*—it is possible to speculate with greater confidence.

Our analysis of fiscal zoning demonstrates that if most communities in an urban area restrict the land available for apartment uses, two different land markets are created. Zoning pushes the price of land zoned for apartments above the nonzoning equilibrium land price and the price of single family-zoned land below the no-zoning equilibrium price.

It is clear that demand for land in a single community for either of the two kinds of residential use must be much more elastic than is demand for land for that use in the metropolitan area market as a whole. For instance, land for single family use in any one community is highly substitutable for single family land in other communities. Similarly, though land for apartment use is generally in short supply, builders of apartments can easily substitute land in one community for land in another. If land for either use in one community were priced higher than the price prevailing in the metropolitan area generally, large numbers of demanders would shift their demands to other communities. Thus the demand curves for land for either use facing an individual community should be nearly horizontal. (Our implicit assumption is that individual municipalities are sufficiently small that changes in their zoning practices will not significantly affect land prices in the urban market. This assumption seems valid in many urban areas in the United States.)

The collective zoning decisions of municipalities essentially create two distinct submarkets for land with different prices in each submarket. Through its zoning laws, an individual community chooses whether its land will be made available in the single family housing submarket or in the apartment building submarket. If community zoning laws permit apartment construction, community land is made available in a high-price market. However, if community zoning laws permit only single family homes, community land is made available in a low-price market. It would appear to be the case, therefore, that the effect of a *given municipality's* decision to restrict the construction of apartment buildings is to decrease the value of land within that municipality. Land values would be higher if the municipality did not engage in such restrictive zoning.

PROPERTY TAXES, ZONING, AND LAND VALUE

In the preceding analysis we have ignored the effects of the property tax on land value and have concluded that a municipality which excludes apartment buildings expected to generate high tax costs may be acting in a way which is inconsistent with maximizing aggregate land values within the municipality. It can be argued, however, that this conclusion is incorrect because it has been reached only through the unrealistic assumption that taxes do not affect land values. Because, *ceteris paribus*, taxes diminish land values, a municipality which uses fiscal zoning to exclude apartment buildings expected to generate high tax costs may reduce its tax rate and *indirectly* maximize property values.

In this section we identify the key parameters which determine whether it is in the fiscal interests of members of a community to exclude through zoning a potential land use when such a land use would appear to increase the aggregate value of land in the community. To examine this question, we develop a number of equations which show the relationship between the value of land, the property tax, and the rental value of the services of land. Our starting point is the assumption that the price of land is equivalent to the suitably discounted present value of the future stream of rents for land net of costs such as taxes. Specifically, the value of the ith parcel of land in a competitive market is determined by the following equation:

$$V_i = R_i/r - T_i/r, \tag{4}$$

where V_i is the market value of the parcel of land, R_i is the rental value of the lot, T_i is the annual taxes paid on the land and r is the discount rate in the economy. For simplicity we ignore possible future changes in R_i, in r, and in the cost of providing public services, although such changes could be easily taken into account without altering our results.

Summing over all parcels of land in the community, we can derive the equation for aggregate land value (V) in the community:

$$V = \sum R_i/r - \sum T_i/r. \tag{5}$$

If we assume that the taxes raised in the community just cover the costs (C) of providing public services in the community, we can obtain the following relationship by substituting $C = \sum T_i$ and $R = \sum R_i$.

$$V = R/r - C/r. \tag{6}$$

Now let us consider a possible zoning change which permits a given parcel of land to be converted to a higher rent use. Assume that this zoning change will increase the rental value of that parcel of land by ΔR and will increase the total public service costs to the community by ΔC.

It is clear from Eq. (6) that the change in total land value in the community (ΔV) resulting from such a zoning change will be

$$\Delta V = \frac{\Delta R - \Delta C}{r}. \tag{7}$$

Next, we consider the change in the tax rate resulting from the zoning change. Before the change in zoning, the tax rate (t_1) needed to generate sufficient revenues to cover total costs in the community was given by the equation

$$t_1 = C/V. \tag{8}$$

After the change in zoning, the tax rate necessary to generate sufficient revenues to cover total public costs is

$$t_2 = (C + \Delta C)/(V + \Delta V). \tag{9}$$

Therefore, the change in the tax rate is given by

$$\Delta t = t_2 - t_1 = (\Delta C \cdot V - C\Delta V)/(V^2 + \Delta V \cdot V). \tag{10}$$

Thus, we have derived equations showing the new aggregate land value and the new tax rate in a community which would result from a change in zoning regulations that permits a higher rent use on a given parcel. We shall assume that residents of the community seek to minimize their tax rate, and, therefore, will vote for a zoning change only if it lowers the tax rate or leaves it unaffected.[9] We are interested in whether such a community will ever choose

[9] Alternatively, one might assume that existing residents seek to minimize their tax *payments* rather than the tax *rate*. Using tax payments as the criterion rather than tax rate would not change the results below since, for an existing land use, tax rate and tax payments always move in the same direction. This can be shown as follows: It is well known that another way of expressing the value of a parcel of land is

$$V_i = R_i/(t + r).$$

(See, for instance, M. Gaffney, Adequacy of land as a tax base, *in* The Assessment of Land Value (D. Holland, Ed.).

Tax payment, therefore, is

$$tV_i = tR_i/(t + r).$$

Taking the derivative of this expression with respect to the tax rate, t, gives

$$\frac{d(tV_i)}{dt} = \frac{R_i}{t + r} - \frac{tR_i}{(t + r)^2} = \frac{tR_i + rt}{(t + r)^2} - \frac{tR_i}{(t + r)^2} = \frac{rt}{(t + r)^2}.$$

This last expression is always positive, and this shows that any change in the tax rate will cause a change of the same sign in the total tax payment on a parcel of land for which the to exclude through zoning a potential land use which would increase the community's aggregate land value.

Mathematically, we pose the question whether aggregate land value can increase, i.e.,

$$\Delta V = (\Delta R - \Delta C)/r > 0, \tag{11}$$

while at the same time the tax rate in the community increases, i.e.,

$$\Delta t = (\Delta C \cdot V - C \cdot \Delta V)/(V^2 + \Delta V \cdot V) > 0. \tag{12}$$

Posing the question in this way, however, makes the answer clear. It may easily be that (11) may hold and that a zoning change will increase land value, while at the same time, the zoning change will necessitate an increase in the tax rate. If we assume that consumer-voters are knowledgeable of potential increases in their tax burdens and that they (or their representatives) vote on the basis of economic self interest, a proposed zoning change which would increase aggregate land value in the community may be rejected by majority vote. (Indeed, in this example it would be rejected unanimously.)

We have shown, then, that in considering a zoning change on a parcel of land, a community which desires to minimize its tax rate may be led to exlcude through zoning a potential land use even though such a land use would increase aggregate land value in the community. The conclusions of the previous sections, therefore, hold in the general case where tax effects are considered.

The analysis above does not imply that the community would vote against *any* proposed zoning change which would increase aggregate land value. It is clear from Eq. (12) that if the change in aggregate land value ΔV, resulting from a zoning change is large enough relative to the change in costs, then the resulting tax rate change will be zero or negative and hence the voters would favor the rezoning. This situation will prevail if the zoning change causes a large enough increase in the value of the parcel in question so that the extra taxes paid on the parcel in its new use fully cover the extra costs generated by the change in use. Mathematically it follows from Eq. (10) that this condition will hold if

$$\Delta C/C \leq \Delta V/V. \tag{13}$$

If this expression holds as an equality, then voters in the community will be indifferent to the zoning change since their fiscal positions will be unaffected. If the equation holds as an inequality, then the voters will positively favor the zoning change since they will directly benefit from it. In the latter case, the increased taxes paid on the rezoned parcel will more than cover the increased costs. A fiscal transfer will thus be made by the owner of the rezoned parcel to the community. This transfer can be used by the residents of the

use—and hence the rental stream—does not change as a result of the zoning change. Hence the analysis below which is done in terms of tax rates could also be done in terms of payments.

community to lower the tax rate generally or to raise the quality of local services at the existing tax rate.

We have shown, therefore, that some, but not all, zoning changes which increase aggregate land value in a community will be accepted by majority voting. A precise cutoff point between those that will and will not be accepted is established by Eq. (13). Finally, the discussion also established the theorem that, if all parcels are identical, any piecemeal zoning change which raises public service costs and which is accepted by majority voting does in fact increase the aggregate land value in a community.

EXTERNALITIES ZONING AND LAND VALUES

So far we have examined possible effects of fiscal zoning on land values. In this section of the paper we consider the effects of externalities zoning. We shall do so by first developing a simple model of a land market with externalities and by then examining the effects of zoning policies which reduce these externalities.

Assume that there are only two land uses, residential uses which do not generate externalities and industrial uses which produce externalites affecting residential uses.

For simplicity, we shall continue to assume that land at all locations in the community is equally desirable for residential purposes. This might be true, for instance, of a suburban community some distance from the central business district of a metropolitan area. In such a community the cost of commuting may not be substantially influenced by exactly where in the suburban community a worker resides. Hence the worker may be largely indifferent to the specific location of his home. We shall also make the simplifying assumption that, because of topographic or political borders, the city is rectangular in shape.

Assume that only residential land within 2000 ft of the industry is affected by the externality but that the effect is uniform within those 2000 ft. Assume that the demand for land for industrial use is completely inelastic at I_D square feet and that the industry ships its products from a depot in the center of the community so that there is a small saving in transportation cost to being located near the center. Finally, assume that all families are identical, have the same incomes and seek to maximize the folllwing utility function:

$$U = [\alpha(L_f + L_c/E)^{-\rho} + (1 - \alpha)G^{-\rho}]^{-1/\rho},[10] \qquad (14)$$

where L_c is land affected by externalities, L_f is land far enough away from the

[10] The constant elasticity of substitution form was chosen because its properties are well known and because it allows for the possibility of a price elasticity of demand which is not equal to one. The analysis could easily be done with other specifications of the utility function.

externalities not to be affected by them, G is all other goods and services, and E is a cardinal index of the level of externalities produced and is defined such that $E \geq 1$.

Consider a nonzoning equilibrium in such a world. There will be a tendency for the industrial land uses to cluster in the center of the community as shown in the diagram below. They will tend toward the center because transportation costs are minimized there. Furthermore, in a frictionless market, they will

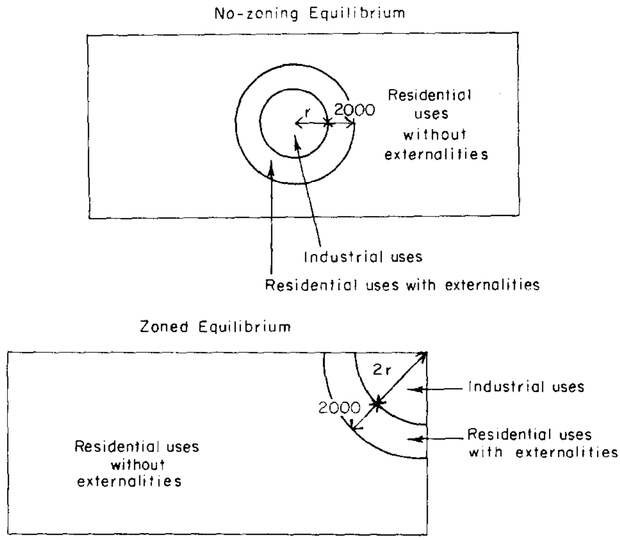

Diagram 1

arrange themselves on a circular plot of land of radius r such that their demand for land I_D equals πr^2. Since by assumption externalities spread 2000 ft beyond the border of the industrial use, there will be $[\pi(2000 + r)^2 - \pi r^2]$ square feet of externality-affected land available for residential use. Call this S_c for supply of close-in land. Finally if S_{tot} is the total supply of residential land available in the community (i.e., S_{tot} = total land available $- I_D$), we can define S_f as the total supply of residential land not affected by externalities, and it must be the case that $S_f = S_{\text{tot}} - S_c$.

Suppose now that we decide to zone the community in such a way as to reduce the amount of land which will be affected by the negative externalities. It is easy to show that this can be done by zoning a quarter circle of land of radius $2r$ at one of the corners of the community for industrial use.[11] This

[11] The area of a quarter circle of radius $2r$ is given by $\frac{1}{4}\pi(2r)^2 = \pi r^2$ which is just the same industrial area as we had in our previous unzoned equilibrium. (In some situations, slightly different zoning patterns might be more effective in reducing the amount of residential land affected by externalities. For instance, if the rectangular city is sufficiently long and

will means that S_c', the area of the boundary residential land within 2000 ft of the industrial uses, is $[\frac{1}{4}\pi(2000 + 2r)^2 - \frac{1}{4}\pi(2r)^2]$, which is clearly less than S_c, the previous supply of close-in residential land affected by externalities. This in turn must mean the S_f', the amount of land for residential uses in the zoned equilibrium which is not affected by externalities, is greater than S_f, the amount of such land in the nonzoned case.

Thus we have shown how zoning can reduce the supply of residential land affected by externalities and correspondingly increase the supply of land which is not affected. But is this change economically efficient? Clearly the answer depends upon the magnitude of the increased transportation costs for the industrial uses. It is clear that the zoning increases the utility of residential users by reducing the total amount of externalities by which they are affected. This gain to residential users, however, is at least partially offset by additional resource costs of transportation imposed on the industries who must, by assumption now, ship their product into the center of the community. Clearly whether the increase costs outweight the increased utility depends on the magnitude of the costs. For the rest of this analysis, however, we shall assume that transportation costs are sufficiently small so that the zoning change described above is economically efficient.

Summarizing our discussion up to this point, we have characterized a world in which industrial uses cluster together and in which there is capitalization of externalities in lower land values. We have described an economically efficient zoning scheme in such a world. We shall now discuss whether it can be determined theoretically that such a zoning scheme would tend to raise or lower aggregate land value in the community. To consider this question we must introduce equilibrium land prices into the model.

In assuming a typical family utility function of the form given in expression (14) above, we have implicitly assumed that it takes relatively more of the externality-affected land than of the nonaffected land to provide a given amount of utility. In particular, expression (14) implicitly makes the assumption that the marginal utility of land close to industry is proportional to but smaller than the marginal utility of land farther from the externality.[12] An acre of close land provides only $1/E$ of the marginal utility of the more desirable land. It follows from this that in market equilibrium, the rents of the two kinds of land must be in a ratio of $1:E$ to each other. If this were not the case, then it would be cheaper at the margin for all families to obtain a given amount of utility from one type of land than from the other and all families

narrow, the most effective zoning pattern might be to zone of one whole end of it for the industrial use rather than one corner. The results presented in the paper do not depend on the exact shape of the externality-minimizing zoning pattern. All that is necessary for the discussion is that there be some zoning pattern which—like the one suggested in the paper—will reduce the amount of residential land affected by the externalities.)

[12] This proportionality assumption is not necessary to obtain the results reached below, but it substantially facilitates the analysis.

would want to consume only that one kind of land causing disequilibrium in the market. For example, if $E = 2$, then from the point of view of giving a family utility, 200 ft^2 of externality-affected land is completely equivalent to 100 square feet of the land farther from the industrial uses. And since 200 ft^2 of one kind of land is equivalent to 100 ft^2 of another, it is clear that in equilibrium their prices must be in the ratio of 1:2 (or more generally, $1:E$). If this were not the case, all consumers would want to buy one kind of land or the other, and this would cause prices to change.

For simplicity we will assume that any single family either buys all L_c land or all L_f land (i.e., the same family does not buy combinations of the two). In equilibrium, families consuming land affected by externalities will consume exactly E times the amount of land consumed by families living on land unaffected by externalities. This follows from the fact that L_c enters the utility function simply as a proportionately differently scaled L_f and from the fact, noted in the previous paragraph, that the difference in scaling between L_c and L_f is completely offset by a proportional difference in the price. If the consumers of L_f maximize their utility at price P_f and at an L_f equal to, say, 1000 ft^2, then it must be the case the consumers of L_c (who have by assumption the same utility function and the same income) must maximize their utility by consuming E times 1000 ft^2 at a price $P_c = P_f/E$. It also follows that the total amount of money spent on land will be the same for a consumer of L_c as for a consumer of L_f.

With some additional notation, we can now draw upon the above observations to mathematically characterize equilibrium for the residential land market in the model. Let N equal the total number of families in the community and K equal the number which, in equilibrium, consume land not affected by externalities. (N is exogenous to the model but K is endogenous.) From the utility function specified as expression (14) above it is possible to derive the demand curve for land for one of the K families which live on land which is not affected by externalties. We shall express this demand curve as $P_f = P_f(L_f)$, where P_f is the market price of land far from externalities and L_f is the amount demanded by a typical consumer of such land. Remembering that S_f and S_c are defined as the market supplies of the two kinds of land and that S_{tot} is defined as the total amount of residential land (i.e., $S_{tot} = S_c + S_f$), we can characterize equilibrium in our model with the following equation system:

$$
\begin{aligned}
KL_f &= S_f, \\
(N - K)L_c &= S_c, \\
L_c &= EL_f, \\
P_f &= P_f(L_f), \\
P_c &= P_f/E, \\
S_c &= S_{tot} - S_f.
\end{aligned} \tag{15}
$$

N, E, S_f, and S_{tot} are exogenous to the market equilibrating process, so the above equations are a system of six equations in six endogenous variables, K, S_c, L_c, L_f, P_c, and P_f. Given N and S_{tot} which are exogenous to the system and an S_f which is determined by zoning policy as discussed above, these equations can be solved for equilibrium values of the endogenous variables. In particular,

$$K = S_f \cdot NE/(S_{\text{tot}} - S_f + S_f \cdot E) \tag{16}$$

and

$$L_f = [(E - 1)S_f + S_{\text{tot}}]/NE. \tag{17}$$

Before we can discuss the impact of zoning on land values, we must also consider the market for industrial property. We have assumed that the demand for industrial use is perfectly inelastic at I_D. The equilibrium rental price for this land will be P_c—the same price as neighboring residential land which is affected by the externalities. This is because industrial users must compete for the land with alternative users and the potential alternative use near the industrial uses is residential use affected by externalities. In order to bid land away from such residential uses, the industrial users must pay P_c for the land.

We have now examined equilibrium conditions in both residential and industrial land markets in our model. It follows from our observations that the total rental value of land in the community is given by

$$V = P_f S_f + P_c S_c + P_c I_D. \tag{18}$$

I_D is exogenous to the model, and we have seen above that equilibrium P_f, P_c, and S_c values can all be expressed as functions of S_f which can be controlled through zoning policy. Therefore, we can analyze the way that the proposed zoning change would affect land value, V, by computing the derivative of V with respect to S_f. This can be done as follows: Substituting from Eqs. (15) into Eq. (18) and manipulating, we get

$$V = NL_f P_f + P_f I_D/E. \tag{19}$$

Remembering that P_f is a function of L_f and that L_f is a function of S_f and differentiating, we get

$$\begin{aligned}\frac{dV}{dS_f} &= \left\{N\frac{d[L_f \cdot P_f(L_f)]}{dL_f} + \frac{I_D}{E}\frac{dP_f}{dL_f}\right\}\frac{dL_f}{dS_f} \\ &= \frac{dL_f}{dS_f}\left[NP_f\left(\frac{1}{e_{L_f,P_f}} + 1\right) + \frac{I_D}{E}\frac{dP_f}{dL_f}\right],\end{aligned} \tag{20}$$

where e_{L_f,P_f} is the price elasticity of demand of a typical individual's demand curve for land which is unaffected by externalities.

The term outside of the brackets in Eq. (20) is positive. Within the brackets, the sign of the second term is always negative since the derivative in that term is simply the slope of the typical individual's demand curve for land which is free of externalities. However, the sign of the first term within the brackets cannot be determined a priori since it depends whether the value of e_{L_f, P_f} is greater than or less that minus one. If e_{L_f, P_f} is more negative than minus one, i.e., if demand for land is quite elastic, then it is quite possible that the positive first term within the brackets could outweigh the negative second term, thus causing the expression and the entire derivative to be positive.

This shows therefore that we cannot determine *a priori* whether the economically efficient zoning change discussed above which increases S_f will lead to higher or lower aggregate land value. Whether it will or not depends on the value of consumer elasticity of demand for land. Thus we have shown that it is not possible to determine *a priori* whether economically efficient zoning will raise or lower land values.

In concluding, it may be useful to indicate at a more intuitive level what the model above shows. We have created a model in which Pareto Optimal zoning effectively increases the amount of land available by increasing the relative share of the total acreage which is not affected by externalities. The model shows that if demand for land is inelastic, it is possible that such zoning might cause aggregate land value to drop. Intuitively, before the zoning there is a scarcity of good land and this scarce land commands a relative high price. Pareto Optimal zoning reduces this effective scarcity and, depending on demand elasticities, may reduce aggregate land value.

CONCLUSIONS

We have identified two important types of zoning—externality zoning and fiscal zoning—and have shown that it is not possible using *a priori* theory to predict what effects either of the two types of zoning will have on land values. In the case of fiscal zoning, however, we drew a distinction between the effects of zoning on land values in an *entire* urban market, and its effects on land values in a specific municipality *within* the market, and we suggested that fiscal zoning, as it is currently practiced in most areas, probably has the effect of reducing individual municipalities' land values below what they would otherwise be. In support of this contention we showed that the residents of a community, voting in their economic self interest, might reject a zoning change which would increase the community's aggregate land value.

In the case of externalities zoning, we showed that the effect on land values of Pareto Optimal zoning depends upon the exact nature of consumers' utility functions. This was demonstrated for a model in which not all residential land is affected by externalities and in which different parcels of land therefore sell for different prices.

REFERENCES

1. R. H. Coase, The problem of social cost, *J. Law Econ.* **3**, 1–44 (1960).
2. F. DeLeeuw, The demand for housing: A review of cross-section evidence *Restats*, **53**, 1–10 (1971); R. Muth, The demand for non-farm housing, *in* "The Demand For Durable Goods" (A. Harberger, Ed.) Chicago, University of Chicago Press (1960).
3. President's Committee on Urban Housing, "Report: A Decent Home," p. 118. Washington, D.C., (1968)
4. A. Rees, "The Economics of Trade Unions," Chicago, Univ. of Chicago Press (1963).