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A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence

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Building on the model of Sanford J. Grossman and Guy Laroque (1990), this paper provides a model of household consumption and portfolio allocation which incorporates the role of housing as both a consumption good and as a component of wealth. The model captures the following features of the household's problem: (a) utility depends, probably nonseparably, on two distinct goods (nondurable consumption and housing); (b) nondurable consumption can be adjusted costlessly, but housing is subject to an adjustment cost; (c) households face housing price risk in the sense that the relative price of housing varies over time; and (d) in addition to the house, the household can invest in a wide variety of financial assets. This single, reasonably tractable, model generates testable implications for portfolio allocation, risk aversion, asset pricing, and the dynamics of nondurable consumption.

Because the original Grossman and Laroque model considers a utility function in which the durable good is the sole argument, and thus abstracts completely from nondurable consumption, their analysis cannot address either the potential spillover effects of the adjustment costs of the durable good on the dynamics of nondurable consumption, or the implications for portfolio allocation of housing risk arising from variation in the relative price of housing. In addition to generating implications for issues on which the original Grossman and Laroque model was silent, the housing model delivers a strikingly different message concerning asset pricing. That is, in contrast to the Grossman and Laroque result that the consumption-based Capital Asset Pricing Model (consumption-CAPM) fails, the housing model implies that the consumption-CAPM holds.

We assume that the household incurs an adjustment cost when altering the holding of the durable good (or house), although financial assets can be bought and sold costlessly. Consumption of the nondurable good can also be adjusted costlessly. When choosing a new house, the consumer takes into account the fact that the consumption of housing services will be constant at the new level until the subsequent stopping time, when it is again worthwhile to incur the adjustment cost. Thus, the home purchase decision is endogenous and fully rational, but, because of the adjustment cost, infrequent. In this continuous time setting, the household's decision process has a recursive structure; at each instant, the household first decides whether it is optimal to sell the house immediately. On those rare occasions that it is optimal to incur the adjustment cost, the household sells the old house and buys a new one instantaneously. If the household decides that it is not optimal to sell the house immediately, it then determines its optimal holdings of financial assets and optimal level of nondurable consumption conditional on the current housing stock. In essence, because of the adjustment costs associated with the durable good, the current

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housing stock becomes a state variable that affects both the nondurable consumption choice and portfolio allocation.

The analytical model shows that if the covariance matrix of asset returns is block diagonal in the sense that the return to housing is uncorrelated with the returns to financial assets, all households will hold a single optimal portfolio of risky financial assets, despite differences among households in terms of preferences or in terms of the state variables faced.[1] While the state variables do not affect the composition of the optimal risky portfolio, they do affect the household's degree of risk aversion, and therefore the allocation of the portfolio between the optimal risky portfolio and the riskless asset. Unlike the standard model in which utility is a function of a single, nondurable consumption good, the housing model does not imply an exact inverse relationship between the curvature of the utility function and the elasticity of intertemporal substitution (EIS). Under the plausible assumption of limited intratemporal substitutability between the two goods, the model can generate a low EIS of nondurable consumption without assuming a high value of the curvature parameter.

Despite its elegance and power, the standard version of the consumption-CAPM has been notably unsuccessful in explaining the consumption and asset pricing data. As a result, macro and finance papers now routinely invoke utility specifications with habit persistence in order to improve the empirical performance of the model. Along many dimensions, the housing model generates the same implications as the habit-persistence model; both models explain the smoothness of nondurable consumption by introducing an additional state variable to the household's optimization problem. Because the state variable moves slowly (when the state variable is interpreted as the habitual level of consumption) or is unchanged for substantial periods of time (when the state variable is interpreted as the house), both models can generate a low elasticity of intertemporal substitution without requiring a high degree of curvature of the utility function. Both models imply that risk aversion depends on the state variable and is time-varying. However, since the two models differ in the specification of the crucial state variable, it is possible to discriminate between the models empirically. In the final section of the paper, we consider a general utility function which nests the restricted utility functions consistent with the habit-persistence model, the housing model, and the standard model. Using data from the Panel Study of Income Dynamics (PSID) and the American Housing Survey (AHS), we obtain estimates of the parameters of the utility function by estimating the Euler equation for nondurable consumption. The empirical results confirm the finding of Karen Dynan (2000) that very little evidence of habit persistence is found at the household level. Further, the parameter restrictions implied by the habit-persistence model are rejected decisively, while the restrictions imposed by the housing model are not rejected. The estimates imply that the utility function exhibits only a modest degree of curvature, and intratemporal substitutability between housing and nondurable consumption is low.

[1] Joseph Beaulieu (1993) also develops a generalization of Grossman and Laroque (1990) in which the utility function depends on nondurable goods as well as a house. In Beaulieu's model, the relative price of the house in terms of the nondurable good is fixed. Due to the simplifying assumption that the relative price of the two goods is constant, housing is "risky" only because the household may be confronted with paying the adjustment cost; his approach does not allow for housing risk in the form of appreciation or depreciation of the value of the house relative to nondurable goods. Nevertheless, Beaulieu's analysis makes several of the points discussed below; in particular, he points out that adding the durable good (subject to costly adjustment) to the standard consumption-beta model drives a wedge between the elasticity of intertemporal substitution and the reciprocal of the coefficient of relative risk aversion. He also points out that while the Euler equation for nondurable consumption holds in the more general model, the fact that the marginal utility of nondurable consumption depends, at the household level, on the holding of the durable good, aggregation issues will preclude empirical applications of the model based on representative agent specifications.

I. Analytical Model

In an important paper, Grossman and Laroque (1990) analyze optimal consumption and portfolio allocation in a context in which utility is derived solely from an illiquid durable good. They show that even modest transactions costs associated with adjustment of the quantity of the durable good will prevent the household from continuously equating the marginal utility of consumption with the marginal utility of wealth, and therefore cause the consumption-based CAPM to fail. Consumption (that is, consumption of the flow of services from the durable good) and marginal utility are constant for significant periods of time, despite fluctuations in the marginal utility of wealth, because the transactions costs preclude continuous, or even frequent, adjustment of the stock of the durable good.

Flavin and Yamashita (1998) consider a generalization of the Grossman and Laroque model in which current utility is a function of both a durable good, that is, a house, *H*, and a nondurable good, *C*. The nondurable good, *C*, has the ideal attributes of being infinitely divisible and costlessly adjustable. As in Grossman and Laroque, once the household purchases a particular house, no adjustments to the size (or any other attribute such as location) can be made without selling the existing house and incurring an adjustment cost proportional to the value of the house, then purchasing a new house. The household maximizes expected lifetime utility:

$$U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t)\ dt. \tag{1}$$

The instantaneous utility function, $u(H_t, C_t)$, depends on the flow of housing services, which in turn is assumed proportional to the housing stock, *H*.[2] The household's rate of time preference is denoted by δ.

Much of Grossman and Laroque (1990) is devoted to analytical and numerical characterization of the optimal stopping times, $\tau_1, \tau_2, \tau_3, \ldots$, at which the household optimally incurs the adjustment cost and reoptimizes over *H*. In Grossman and Laroque, the stopping times are endogenous in the sense that the household adjusts its holding of the durable good when the stochastic evolution of wealth creates too great a disparity between the existing stock of the durable and the frictionless optimal stock. In addition to the endogenous stopping times modeled by Grossman and Laroque, our version of the model permits "exogenous stopping" in the sense that the adjustment of *H* may be caused by some event which is exogenous with respect to the evolution of wealth. Examples of exogenous events that might induce stopping are: death, in which the house is sold and the proceeds transferred to the heirs; change in job location; retirement; change in marital status; and acquisition or emancipation of children.

Each house is a distinct good, differing from every other house (at a minimum) in terms of its exact location. For the purposes of the analytical model, we assume that the house is not subject to physical depreciation.[3] Using the nondurable good as numeraire, define:

(2) P_t = house price (per square foot) in the household's current market;

P_t' = house price (per square foot) in the region to which the household relocates in the next move.

[2] By choice of units, the factor of proportionality relating housing services to the housing stock is normalized to unity, so that the utility function can be written as a function of the housing stock.

[3] Generalizing the model to allow for a constant rate of depreciation is straightforward. By assuming a depreciation rate of zero, the model is simplified slightly without changing the basic implications of interest.

As in Grossman and Laroque, we abstract from labor income or human wealth, and assume that wealth is held only in the form of financial assets and the durable good. The household can invest in a riskless asset and in any of n risky financial assets. Unlike the durable good, holdings of the financial assets can be adjusted with zero transaction cost.

Thus, wealth is given by:

$$W_t = P_t H_t + B_t + \mathbf{X}_t \ell, \tag{3}$$

where $\mathbf{X}_t = (1 \times n)$ vector of amounts (expressed in terms of the nondurable good) held of the risky assets and ℓ = an $(n \times 1)$ column vector, with each element equal to unity. B_t is the amount held in the form of the riskless asset. All financial assets, including the riskless asset, may be held in positive or negative amounts.[4]

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let $b_{i,t}$ = the value (per share) of the i^{th} risky asset, and assume that asset prices follow an n-dimensional Brownian motion process:

$$db_{i,t} = b_{i,t}\big((\mu_i + r_f)\ dt + d\omega_{i,t}\big). \tag{4}$$

The vector $\boldsymbol{\omega}_{F,t} \equiv (\omega_{1,t}, \omega_{2,t}, \dots, \omega_{n,t})$ follows an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix $\mathbf{\Sigma}$; the corresponding vector of expected excess returns on risky financial assets is $\boldsymbol{\mu} \equiv (\mu_1, \mu_2, \dots, \mu_n)$; and r_f is the riskless rate.

House prices also follow a Brownian motion:

$$dP_t = P_t\big((\mu_H + r_f)\ dt + d\omega_{Ht}\big), \tag{5}$$

$$dP_t' = P_t'\big((\mu_{H'} + r_f)\ dt + d\omega_{H't}\big),$$

where ω_{Ht} and $\omega_{H't}$ are Brownian motions with zero drift, instantaneous variance σ_P^2 and $\sigma_{P'}^2$, respectively, and instantaneous covariance σ_H.

Stacking equations (4) and (5), and defining the $((n + 2) \times 1)$ vector $\mathbf{d}\boldsymbol{\omega}_t$ as

$$\mathbf{d}\boldsymbol{\omega}_t = [d\omega_{1t}, \dots, d\omega_{nt}, d\omega_{Ht}, d\omega_{H't}]^T, \tag{6}$$

the vector $\mathbf{d}\boldsymbol{\omega}_t$ has instantaneous $((n + 2) \times (n + 2))$ covariance matrix $\mathbf{\Omega}$:

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Sigma} & 0 & 0 \\ 0 & \sigma_P^2 & \sigma_H \\ 0 & \sigma_H & \sigma_{P'}^2 \end{bmatrix}. \tag{7}$$

The block diagonality of $\mathbf{\Omega}$ implies that housing prices both in the current market and in the next market are uncorrelated with the returns to financial assets. It is important to note that the block diagonality does *not* require an absence of correlation in regional housing prices; the covariance matrix $\mathbf{\Omega}$ allows for an arbitrary $\sigma_H \equiv \operatorname{cov}(P_t, P_t')$. Because the covariance matrix does

[4] Flavin and Yamashita (1998) consider the model under the alternative assumption that the household must hold nonnegative amounts of all financial assets other than mortgages. Since the household can borrow only in the form of a mortgage, and only up to the value of the house, the house becomes collateral in that model. Hanno Lustig and Stijn Van Nieuwerburgh (2005) also study the role of housing collateral and provide empirical evidence based on aggregate data that a decrease in the ratio of housing collateral to human wealth increases the market price of risk.

not place any restrictions on the cross-sectional correlation of regional housing prices, the model is sufficiently general to incorporate the role of housing investment in providing a hedge against the risk arising from variability in future housing costs. For given σ_P^2 and $\sigma_{P'}^2$, the extent to which home ownership provides a hedge against future housing costs will be increasing in σ_H.[5] Flavin and Yamashita (2002) present empirical evidence that the block diagonality assumed in equation (7) is consistent with data on US house prices and asset returns, using two different sources of data on house prices: the PSID, and the Karl E. Case and Robert J. Shiller (1989) data based on repeat sales transactions prices for four cities (Atlanta, Chicago, Dallas, and San Francisco).[6]

Let $V(H, W, P, P')$ denote the *supremum* of household expected utility, conditional on the current values of the state variables (H, W, P, P'). At every moment, the household considers whether the disparity between the current size house and the frictionlessly optimal size house is sufficiently large to justify paying the transactions cost and reoptimizing over the house. At time $t = 0$, the Bellman equation is

$$V(H_0, W_0, P_0, P_0') = \sup_{\{C_t\},\{\mathbf{X}_t\},\tau} E_0\left[\int_0^\tau e^{-\delta t} u(H_0, C_t)\, dt + e^{-\delta\tau} V(H_\tau, W_\tau, P_\tau, P_\tau')\right], \tag{8}$$

where τ denotes the next stopping time.

Since the quantity of housing will change discontinuously at a stopping time, the notation $H_{\tau-}$ is used to distinguish the quantity of housing immediately prior to the sale from the quantity of housing immediately after the sale, H_τ. At the instant the house is sold, the household pays a transactions cost proportional to the value of the house sold, so that wealth also changes discontinuously. Wealth is denoted $W_{\tau-}$ immediately prior to a sale, and denoted W_τ immediately after. Thus, at a stopping time, τ, wealth evolves according to

$$W_\tau = W_{\tau-} - \lambda P_\tau H_{\tau-}, \tag{9}$$

where λ is the proportional transaction cost.[7] The household faces a "no bankruptcy constraint," $W_t > \lambda P_t H_t$, which says that wealth must always be at least sufficient to pay the transactions cost to sell the current house.

At each instant, the household first decides whether it is optimal to sell the house immediately by comparing the value of the program conditional on selling to the value of the program conditional on not selling, that is, if

$$\sup_{\tilde{H}} V(\tilde{H}, W_{0-} - \lambda P_0 H_{0-}, P_0, P_0') < V(H_{0-}, W_{0-}, P_0, P_0'), \tag{10}$$

[5] The role of homeownership as a hedge against future housing costs is addressed in Francois Ortalo-Magne and Sven Rady (2002) and Todd Sinai and Nicholas Souleles (2005).

[6] The assumption of the block-diagonality of the covariance matrix is also consistent with data from other countries. For example, similar empirical results are provided by Matteo Iacoviello and Ortalo-Magne (2003) for the United Kingdom, by Peter Englund, Min Hwang, and John Quigley (2002) for Sweden, and by David LeBlanc and Christine Lagarenne (2004) for France.

[7] The assumption that selling the old house and purchasing a new one is the only way that the household can adjust the level of housing consumption is obviously a simplification. Some adjustment to the level of housing consumption can be accomplished while staying in the current house, to the extent that the household can expand, remodel, or fail to maintain the house. While acknowledging that recent papers by Ellen McGrattan and James Schmitz (1999), Chris Downing and Nancy Wallace (2001), and Thomas Davidoff (2003) provide empirical evidence that adjustments of this nature are common, we nevertheless assume that any adjustment of the level of housing services requires that the house be sold.

it is not optimal to sell the house at $t = 0$. If, on the other hand, the values on each side of equation (10) are equal, then it is optimal to sell the house, that is, $t = 0$ is a stopping time.

Suppose that, at time $t = 0$, the household decides that it is not optimal to sell the house immediately (i.e., $\tau \neq 0$), so that the value function $V(H_0, W_0, P_0, P_0')$ strictly exceeds the maximum value attainable if the house were sold immediately. By continuity, there must be a time interval $(0, s)$ sufficiently small that the possibility of stopping within that interval can be ignored.[8] During such a time interval, wealth evolves according to:

$$(11)\qquad dW_t = [P_t H_0(\mu_H + r_f) + \mathbf{X}_t(\boldsymbol{\mu} + \boldsymbol{\ell} r_f) + r_f B_t - C_t]\,dt + \mathbf{X}_t \mathbf{d}\boldsymbol{\omega}_{F,t} + P_t H_0\, d\omega_{Ht},$$

or, rewriting in order to eliminate the term representing risk-free bonds,

$$(12)\qquad dW_t = [r_f W_t + P_t H_0 \mu_H + \mathbf{X}_t \boldsymbol{\mu} - C_t]\,dt + \mathbf{X}_t \mathbf{d}\boldsymbol{\omega}_{F,t} + P_t H_0\, d\omega_{Ht},$$

and the Bellman equation is

$$(13)\qquad V(H_0, W_0, P_0, P_0') = \sup_{\{\mathbf{X}_t\},\{C_t\}} E_0\left[\int_0^s e^{-\delta t} u(H_0, C_t)\,dt + e^{-\delta s} V(H_0, W_s, P_s, P_s')\right]$$

subject to the budget constraint (12), the process for house prices (5), and the "no bankruptcy constraint." Subtracting $V(H_0, W_0, P_0, P_0')$, dividing by s, and taking the limit as $s \rightarrow 0$ gives:

$$(14)\quad 0 = \lim_{s\rightarrow 0} \sup_{\{\mathbf{X}_t\}\{C_t\}} E_0\left[\frac{1}{s}\int_0^s e^{-\delta t} u(H_0, C_t)\,dt + \frac{1}{s}\left(e^{-\delta s} V(H_0, W_s, P_s, P_s') - V(H_0, W_0, P_0, P_0')\right)\right].$$

Evaluating the integral and using Ito's lemma, equation (14) can be rewritten as

$$(15)\qquad 0 = \sup_{\mathbf{X}_0, C_0}\Bigg\{ u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0') + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H + \mathbf{X}_0 \boldsymbol{\mu} - C_0)$$
$$+ \frac{\partial V}{\partial P} P_0(\mu_H + r_f) + \frac{\partial V}{\partial P'} P_0'(\mu_{H'} + r_f) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}(\mathbf{X}_0 \boldsymbol{\Sigma} \mathbf{X}_0^T + P_0^2 H_0^2 \sigma_P^2)$$
$$+ \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{1}{2}\frac{\partial^2 V}{\partial P'^2} P_0'^2 \sigma_{P'}^2 + \frac{\partial^2 V}{\partial W \partial P} P_0^2 H_0 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P'} P_0 P_0' H_0 \sigma_H$$
$$+ \frac{\partial^2 V}{\partial P \partial P'} P_0 P_0' \sigma_H \Bigg\}.$$

Nondurable consumption satisfies the usual first-order condition

$$(16)\qquad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}.$$

[8] See Grossman and Laroque (1990, 31).

The optimal holding of risky financial assets, stated as shares of wealth, is given by

$$\left(\frac{1}{W_0}\right)\mathbf{X}_0^T = \left[\frac{-\dfrac{\partial V}{\partial W}}{\dfrac{\partial^2 V}{\partial W^2}W_0}\right]\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \tag{17}$$

and the amount held of the riskless asset is

$$B_0 = W_0 - P_0 H_0 - \mathbf{X}_0 \ell. \tag{18}$$

In (17), the expression in square brackets is the reciprocal of the coefficient of relative risk aversion:

$$RRA \equiv -\frac{\dfrac{\partial^2 V(W_t, H_t, P_t, P_t')}{\partial W_t^2}}{\dfrac{\partial V(W_t, H_t, P_t, P_t')}{\partial W_t}} W_t > 0. \tag{19}$$

Note that, because the household's degree of risk aversion depends on the curvature of the value function, behavior toward risk will depend not only on the curvature of the instantaneous utility function, $u(H_t, C_t)$, but also on all of the state variables.[9]

From equation (17), all consumers hold risky assets in exactly the same proportion, despite differences among households in terms of preferences (i.e., in the specification of $u(H, C)$) or in terms of the state variables faced; that is, the mutual fund separation theorem holds. Note that the derivation of equation (17) required the assumption that the covariance matrix is block diagonal; in the absence of this restriction, the mutual fund separation theorem would not hold. Under a covariance matrix that is not block diagonal, risky financial assets could be used to hedge the risk associated with the current house, or to hedge the risk associated with the variability of future house prices. However, under the assumption of block diagonality, since there is no scope for using financial assets to hedge the risk from current or future house prices, the presence of the (risky) housing asset does not induce any "distortion" of the optimal portfolio of risky financial assets, as compared to the risky portfolio implied by the standard model, which abstracts from housing altogether. While the composition of the optimal risky portfolio does not depend on the value of H_t (or the other state variables), the allocation of the overall portfolio between the optimal risky portfolio and the riskless asset will depend on H_t via its effect on risk aversion (equation (19)).

In general equilibrium, the fact that all consumers hold risky assets in the same proportion implies that risk premia are determined by the standard CAPM. Denote the total market value of risky asset i as M_i, and define the $(n \times 1)$ vector $\mathbf{M} \equiv (M_1, M_2, \ldots, M_n)$. Using equation (17) to characterize household demand for each risky asset, and imposing market clearing, implies

$$\boldsymbol{\mu} = \left[\frac{\boldsymbol{M}^T\boldsymbol{\mu}}{\boldsymbol{M}^T\boldsymbol{\Sigma}\boldsymbol{M}}\right]\boldsymbol{\Sigma}\boldsymbol{M}. \tag{20}$$

[9] The property that risk aversion varies with the state is also a feature of the version of the model considered by Grossman and Laroque (1990). In particular, they find that the household is less risk averse (in terms of the allocation of its portfolio between the risky and risk-free asset) shortly before purchasing a new house, and relatively more risk averse immediately after purchasing a new house.

Expressed in more familiar notation, equation (20) can be restated as

$$E(r_i) - r_f = \frac{\text{cov}(r_i - r_f, r_m - r_f)}{\text{var}(r_m - r_f)}\left[E(r_m) - r_f\right]. \tag{21}$$

Asset prices are also consistent with the consumption-beta model; the implications of the traditional CAPM and consumption-beta model exactly coincide in this setting. By denoting the marginal utility of nondurable consumption of household j in period t as

$$\mu(H_{j,t}, C_{jt}) = \frac{\partial u(H_{j,t}, C_{j,t})}{\partial C_{j,t}}, \tag{22}$$

the j^{th} household will satisfy an Euler equation for each risky asset. For the interval $(t, t + s)$, this implies

$$E(r_{i,t+s}) - r_f = \frac{-\text{cov}(r_{i,t+s} - r_f, \mu(H_{j,t+s}, C_{j,t+s}))}{E(\mu(H_{j,t+s}, C_{j,t+s}))}; \tag{23}$$

$$E(r_{m,t+s}) - r_f = \frac{-\text{cov}(r_{m,t+s} - r_f, \mu(H_{j,t+s}, C_{j,t+s}))}{E(\mu(H_{j,t+s}, C_{j,t+s}))}.$$

Even if households are identical in the sense that they have the same preferences (i.e., the same utility function $u(H_t, C_t)$), differences across households in the values of the state variables (including H_t and W_t) will create cross-sectional dispersion in the marginal utility of nondurable consumption, $\mu(H_{j,t}, C_{j,t})$. Nevertheless, since all households are satisfying the Euler equations for nondurable consumption, equation (23) will hold for all households. Rewriting equation (23) to express the risk premium on an individual risky asset in terms of the risk premium on the market portfolio gives

$$E(r_{i,t+s}) - r_f = \frac{\text{cov}(r_{i,t+s} - r_f, \mu(H_{j,t+s}, C_{j,t+s}))}{\text{cov}(r_{m,t+s} - r_f, \mu(H_{j,t+s}, C_{j,t+s}))}\left[E(r_{m,t+s}) - r_f\right]. \tag{24}$$

Thus, the basic implication of the model is that risk premia on individual assets will be proportional to the risk premium on the market portfolio, and that an asset's beta can be expressed either in terms of the covariance of the asset's return with the marginal utility of consumption (equation (24)) or in terms of the covariance of the asset's return with the market portfolio (equation (21)); in theory, the model provides two alternative ways of obtaining empirical estimates of a unique vector of betas. In practice, of course, either approach to estimating the betas is compromised by serious measurement issues. In terms of the traditional CAPM approach, we do not observe the return on the complete market portfolio and consequently rely on a proxy (such as the return to a broad stock index). In terms of the consumption-beta approach, we do not directly observe the marginal utility of nondurable consumption at the household level. To estimate the risk premia using the consumption-beta approach, we would need to make an assumption about the functional form of the utility function $u(H_t, C_t)$; and to have data on the state variable H_t as well as data on nondurable consumption at the household level. Thus, it is not necessary to conclude that the consumption-beta model should be rejected on the basis of the extensive empirical evidence that the traditional CAPM outperforms the consumption-based CAPM in terms of predicting asset premia. Instead, one can interpret the poor empirical performance of the consumption-beta model as an indication that, in practice, we cannot infer the marginal utility of nondurable consumption with sufficient accuracy to exploit the empirical implications of the model.

None of the preceding analytical results depends on any specific assumptions on the functional form of the utility function. In order to underscore the effect of the adjustment cost on risk aversion, we now assume that the instantaneous utility function is of the CES form:

$$u(H_t, C_t) = \frac{[\gamma H_t^\alpha + C_t^\alpha]^{(1-\rho)/\alpha}}{1-\rho} \qquad \alpha \leq 1, \qquad 0 \leq \gamma, \qquad 1 \neq \rho > 0. \tag{25}$$

The parameter α governs the degree of intratemporal substitutability between housing and nondurable consumption goods; the parameter ρ determines the degree of curvature of the utility function with respect to the composite good.

By an argument parallel to that in Theorem 2.1 of Grossman and Laroque (1990), one can show that the value function $V(H_t, W_t, P_t, P_t')$ is homogeneous of degree $(1-\rho)$ in H and W. If the stock of housing is costlessly adjustable, the value function at t depends on H_t only to the extent that H_t is a component of wealth; H_t does not appear in the value function as a separate state variable. Thus, in the absence of adjustment costs, the value function can be written in the form:

$$V(H_t, W_t, P_t, P_t') = k(P_t, P_t')W_t^{1-\rho}, \tag{26}$$

where $k(P_t, P_t')$ is a function of house prices which does not depend on W_t, which implies

$$RRA \equiv -\frac{\dfrac{\partial^2 V(W_t, H_t, P_t, P_t')}{\partial W_t^2}}{\dfrac{\partial V(W_t, H_t, P_t, P_t')}{\partial W_t}} W_t = \rho. \tag{27}$$

Like the single-good models of optimal consumption and portfolio allocation studied by Robert Merton (1969, 1971), the curvature of the value function is immediately inherited from the curvature of the utility function in a frictionless setting. In the presence of adjustment costs, however, the curvature of the value function, and therefore the coefficient of risk aversion, will depend on the values of the state variables as well as parameters such as ρ. For this reason, the parameter ρ will be referred to as the "curvature parameter" rather than the "risk aversion parameter." Consistent with the implication of (19) that risk aversion will depend on housing as a state variable, Monika Piazzesi, Martin Schneider, and Selale Tuzel (2007) provide empirical evidence that the average share of housing in consumption expenditure helps to forecast excess stock returns.

II. Why Nondurable Consumption Is Smooth

In the standard version of the consumption-beta model, it is assumed that (a) the lifetime utility function is determined within an expected utility framework; (b) the utility function is time-separable; and (c) the utility function depends solely on a single, costlessly adjustable nondurable good. Under these assumptions, the curvature of the utility function immediately determines both risk aversion and the elasticity of intertemporal substitution (EIS). Further, it is an implication of the standard version of the model that the EIS is the reciprocal of the coefficient of relative risk aversion. In response to the large body of empirical work that demonstrated consistent rejection of this implication of the standard model, various authors have considered more general versions of the model by relaxing the assumption of expected utility, Assumption 1, or by relaxing the assumption of time-separable preferences, Assumption 2. In both of these more general specifications, the model no longer has the implication that the EIS is equal to the reciprocal of the coefficient of relative risk aversion. The housing model represents a third approach to generalizing the standard model to allow for a low EIS without requiring an implausibly high degree of risk aversion. Unlike the recursive utility and habit-persistence models, the housing model maintains Assumptions 1 and 2 by using a time-separable expected utility framework, but relaxes Assumption 3 by making the utility function depend on the durable good subject to adjustment costs as well as nondurable consumption.[10]

If utility is a function of a single good, the instantaneous EIS at time t is defined as:[11]

$$EIS \equiv \lim_{s \to t} - \left[\frac{\partial\left(\frac{C_s}{C_t}\right)}{\partial\left(\frac{p_s}{p_t}\right)} \right] \frac{\frac{p_s}{p_t}}{\frac{C_s}{C_t}}, \tag{28}$$

where the relative price of future consumption, C_s, in terms of current consumption, C_t, is given by $p_s/p_t = \exp\left(-\int_t^s r_\tau \, d\tau\right)$. When the utility function is generalized to depend on two goods, we can still characterize the household's willingness to substitute one of the goods (for example, nondurable consumption) across time in response to a change in the interest rate. However, in the two-good case, we can construct two conceptually different elasticities as answers to two different thought experiments. One question that might be posed is: what is the percent change in the ratio of future to current nondurable consumption in response to a 1 percent change in the relative price, holding constant the consumption of the other good (housing)? A second question that could be posed is: what is the percent change in the ratio of future to current nondurable consumption in response to a 1 percent change in the relative price, taking into account the fact that the household's optimal consumption of the other good may also respond to a change in the interest rate? Since the two thought experiments are distinguished by the issue of whether the consumption of the second good is, or is not, held constant, we refer to the resulting elasticities as the "partial EIS" of nondurable consumption and the "total EIS" of nondurable consumption, respectively.

The empirical evidence that households are relatively unresponsive in reallocating consumption across time in response to changes in the interest rate is based on regressions of the growth

[10] The point that an adjustment cost associated with durable goods will in general affect the dynamics of nondurable consumption was made in Ben Bernanke (1985). In the context of the Permanent Income model based on quadratic preferences, Bernanke allows utility to depend on durable goods as well as nondurable goods in a potentially nonseparable way. For tractability, he models the adjustment costs associated with durable goods as a quadratic function of the change in the stock of durables; given the quadratic specification of preferences and adjustment cost, he is able to derive and estimate closed form solutions for the behavior of durable and nondurable consumption goods. Quadratic adjustment costs will induce adjustment dynamics very different from the specification of adjustment costs used here—under the quadratic specification, the adjustment will take the form of a series of small adjustments over a number of periods, while under the specification of adjustment costs used here, the household will maintain a given stock of the durable over a long period and ultimately make a single, large adjustment. When the durable good is interpreted as a house, as in the current paper, modeling the adjustment cost as proportional to the stock seems more plausible than the quadratic function of the change in the stock. In Bernanke's paper, however, "durable goods" refers to durable goods as defined in the NIPA, that is, vehicles, furniture, clothing, etc. Since "durable goods" in his model refers to a collection of smaller individual goods, as opposed to a single indivisible good, the specification of adjustment cost as quadratic in the change in the total stock of durable goods is more plausible. While Bernanke's model allows for nonseparability between durable goods (as defined by the NIPA) and nondurable goods and services, his empirical results indicate that the restriction implied by separability cannot be rejected.

[11] For example, see Olivier Blanchard and Stanley Fischer (1989, 40).

rate of nondurable consumption on the interest rate. Since the quantity of housing is not controlled in the econometric sense, in these regressions the slope coefficient obtained by regressing the growth rate of nondurable consumption on the interest rate is, empirically, the answer to the question: to what extent are households willing to substitute nondurable consumption intertemporally in response to a change in the interest rate, taking into account that the household's consumption of the other good may also respond to the change in the interest rate? That is, the slope coefficient in the bivariate specification is an estimate of the total EIS, rather than the partial EIS.

To characterize the total EIS of nondurable consumption in this model, consider the Euler equation for nondurable consumption in a continuous time setting with no uncertainty:

$$\exp\{-\delta(s-t)\}\frac{\mu(H_s,C_s)}{\mu(H_t,C_t)} = \frac{p_s}{p_t} = \exp\left\{-\int_t^s r_\tau\, d\tau\right\}, \tag{29}$$

where the marginal utility of nondurable consumption is denoted as $\mu(H_t,C_t)$. Using a Taylor series expansion for the marginal utility of nondurable consumption gives

$$\mu(H_s,C_s) = \mu(H_t,C_t) + \frac{\partial^2 u(H_t,C_t)}{\partial C_t^2}(C_s - C_t) + \frac{\partial^2 u(H_t,C_t)}{\partial C_t \partial H_t}(H_s - H_t) + \psi(s,t). \tag{30}$$

Since the approximation error, $\psi(s,t)$, will vanish when we take the limit to obtain the instantaneous elasticity, we use just the linear terms to rewrite (29) as

$$1 + \frac{\dfrac{\partial^2 u(H_t,C_t)}{\partial C_t^2}}{\mu(H_t,C_t)}(C_s - C_t) + \frac{\dfrac{\partial^2 u(H_t,C_t)}{\partial C_t \partial H_t}}{\mu(H_t,C_t)}(H_s - H_t) \approx \exp\{\delta(s-t)\}\frac{p_s}{p_t}. \tag{31}$$

Solving (31) for the growth rate of nondurable consumption gives

$$\frac{C_s - C_t}{C_t} \approx a\left[\exp\{\delta(s-t)\}\frac{p_s}{p_t} - 1\right] + b\left(\frac{H_s - H_t}{H_t}\right), \tag{32}$$

$$\text{where } a = \frac{\dfrac{\partial u(H_t,C_t)}{\partial C_t}}{C_t\dfrac{\partial^2 u(H_t,C_t)}{\partial C_t^2}} \quad \text{and } b = \frac{-\dfrac{\partial^2 u(H_t,C_t)}{\partial C_t \partial H_t}}{\dfrac{\partial^2 u(H_t,C_t)}{\partial^2 C_t^2}}\frac{H_t}{C_t}.$$

For the total EIS of nondurable consumption, take the total derivative of C_s/C_t with respect to p_s/p_t:

$$\frac{d\left(\dfrac{C_s}{C_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)} = a\exp\{\delta(s-t)\} + b\frac{d\left(\dfrac{H_s}{H_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)}. \tag{33}$$

Note that the right-hand side of (33) depends on the derivative $d(H_s/H_t)/d(p_s/p_t)$, which reports the extent to which the household's consumption of housing in the small time interval (t,s) responds to a change in the interest rate. This derivative, and therefore the total EIS of nondurable consumption, depends critically on the presence or absence of an adjustment cost on housing. If both goods are costlessly adjustable, the consumption of the two goods will satisfy the condition that the intratemporal marginal rate of substitution is equal to the relative price. Using $\tilde{\theta}_t$ to denote the price of housing services relative to nondurable consumption, this implies, for the CES utility function,[12]

$$H_t = \left[\frac{\gamma}{\tilde{\theta}_t}\right]^{1/(1-\alpha)} C_t = \theta_t C_t \qquad \text{where } \theta_t = \left[\frac{\gamma}{\tilde{\theta}_t}\right]^{1/(1-\alpha)}. \tag{34}$$

Thus, in the absence of adjustment costs on housing (and assuming CES utility), the consumption of housing services moves in sync with nondurable consumption:

$$\frac{d\left(\dfrac{H_s}{H_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)} = \frac{\theta_s}{\theta_t}\frac{d\left(\dfrac{C_s}{C_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)}, \tag{35}$$

which implies

$$\frac{d\left(\dfrac{C_s}{C_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)} = \frac{a}{1 - b\dfrac{\theta_s}{\theta_t}}\exp\{\delta(s-t)\}. \tag{36}$$

Thus, in the absence of adjustment costs, the (instantaneous) total EIS is

$$\textit{Total EIS}(\lambda{=}0) \equiv \lim_{s\to t}\frac{-a\exp\{\delta(s-t)\}}{1 - b\dfrac{\theta_s}{\theta_t}}\,\frac{\dfrac{p_s}{p_t}}{\dfrac{C_s}{C_t}} = \frac{-a}{1-b} = \frac{-\dfrac{\partial u(H_t,C_t)}{\partial C_t}}{\dfrac{\partial^2 u(H_t,C_t)}{\partial C_t^2}C_t + \dfrac{\partial^2 u(H_t,C_t)}{\partial C_t \partial H_t}H_t}. \tag{37}$$

In the presence of a nonconvex adjustment cost on housing, however, the intratemporal marginal condition does not hold. If we consider a time t such that it is not optimal to sell the house immediately, we can specify a small time interval (t,s) such that the probability hitting a bound within that interval can be ignored. In this case, $H_s = H_t$ and

$$\frac{d\left(\dfrac{H_s}{H_t}\right)}{d\left(\dfrac{p_s}{p_t}\right)} = 0. \tag{38}$$

[12] Since the argument of the utility function is the service flow from housing, the relative price depends on the user cost of housing services, not the price of the asset.

This implies that, in the presence of a nonconvex adjustment cost on housing, the total EIS is

$$\textit{Total EIS}(\lambda>0) \equiv \lim_{s \to t} \; - a \exp\{\delta(s-t)\} \frac{\dfrac{p_s}{p_t}}{\dfrac{C_s}{C_t}} = -a = \frac{-\dfrac{\partial u(H_t, C_t)}{\partial C_t}}{\dfrac{\partial^2 u(H_t, C_t)}{\partial C_t^2} C_t}. \tag{39}$$

If we take a positive cross derivative of utility with respect to the two goods as the plausible case, an adjustment cost on housing reduces the responsiveness of nondurable consumption to the interest rate, as measured by the total EIS:

$$\textit{Total EIS}(\lambda>0) = \frac{-\dfrac{\partial u(H_t, C_t)}{\partial C_t}}{\dfrac{\partial^2 u(H_t, C_t)}{\partial C_t^2} C_t} < \textit{Total EIS}(\lambda=0) = \frac{-\dfrac{\partial u(H_t, C_t)}{\partial C_t}}{\dfrac{\partial^2 u(H_t, C_t)}{\partial C_t^2} C_t + \dfrac{\partial^2 u(H_t, C_t)}{\partial C_t \partial H_t} H_t}. \tag{40}$$

To understand the intuition behind the result that the household's willingness to substitute nondurable consumption across time depends on the presence or absence of an adjustment cost on the other good, consider the limiting case of Leontief preferences, i.e., $u(C,H) = (1-\rho)^{-1}[\min(\gamma H, C)]^{1-\rho}$. Assume that housing is subject to a nonconvex adjustment cost, and consider a household for whom it is not optimal to sell the house and buy a new one this instant. When the household last reoptimized over its consumption of housing services, it chose the bundle $\gamma H_0 = C_0$. In response to a fall in the interest rate, the household would like to reallocate some nondurable consumption from the future to the present, but an increase in nondurable consumption beyond C_0 would generate no gain in utility as long as housing is equal to H_0. Thus, under Leontief utility, nondurable consumption will be constant at the old level, C_0, and the total EIS of nondurable consumption is zero.

Conversely, retain the assumption of Leontief preferences, but assume that both goods are costlessly adjustable. Given the kink in the indifference curve, the household will always consume the two goods in fixed proportions, so that we can think of a single composite good G_t which consists of C_t units of the nondurable good and $H_t = \gamma^{-1} C_t$ units of housing. In response to a decline in the interest rate, the household increases consumption of both goods in their fixed proportions, i.e., increases consumption of the composite good. A regression of the growth rate of the quantity of the composite good on the interest rate would yield a coefficient of ρ^{-1}. Further, the Leontief structure implies that the three goods (nondurable consumption, housing services, and the composite good) will all have the same growth rate. Thus, a regression of the growth rate of nondurable consumption on the interest rate would also yield a coefficient of ρ^{-1}.

The Leontief case is obviously extreme, but a similar effect arises with a general nonseparable utility function. A nonconvex adjustment cost on the durable good implies that any increases in the nondurable good will occur without a concomitant increase in the durable good. In this case, the total EIS of nondurable consumption is simply the (negative) inverse of the curvature of the utility function with respect to C_t, H_t held constant. However, if both goods are costlessly adjustable, an intratemporal first-order condition will hold, and the household will increase its current consumption of both goods in response to a decline in the interest rate. If the two goods are complements (i.e., the cross derivative is positive), the concomitant increase in H_t raises the marginal utility of nondurable consumption at any level of C_t and partially offsets the decline in the marginal utility of nondurable consumption.

Evaluating equation (40) for the CES utility function given in equation (25), the total EIS becomes

$$\text{(41)}\quad \textit{Total EIS} = \frac{-1}{(1-\rho-\alpha)\left[\dfrac{C_t^{\alpha}}{\gamma H_t^{\alpha} + C_t^{\alpha}}\right] + (\alpha - 1) + \phi_t(1-\rho-\alpha)\left[1 - \dfrac{C_t^{\alpha}}{\gamma H_t^{\alpha} + C_t^{\alpha}}\right]},$$

$$\text{where } \phi_t = \begin{Bmatrix} 1 \; \textit{if}\; \lambda = 0 \\ 0 \; \textit{if}\; \lambda > 0 \end{Bmatrix}.$$

Using the notation $\kappa_t = C_t^{\alpha}/(\gamma H_t^{\alpha} + C_t^{\alpha})$, note that

$$\text{(42)}\qquad 0 \le \kappa_t = \frac{C_t^{\alpha}}{\gamma H_t^{\alpha} + C_t^{\alpha}} \le 1,$$

so that, depending on the presence of adjustment costs associated with housing, the total EIS is

$$\text{(43a)}\qquad \textit{Total EIS}(\lambda{>}0) = \frac{1}{(1-\kappa_t)(1-\alpha) + \kappa_t \rho};$$

$$\text{(43b)}\qquad \textit{Total EIS}(\lambda{=}0) = \frac{1}{\rho}.$$

By considering various special cases of the model, one can identify several sets of assumptions under which the total EIS is simply equal to the reciprocal of the curvature parameter, ρ. This familiar special case arises under any of the following assumptions:

- Utility depends only on nondurable consumption, i.e., $\gamma = 0$;

- Utility depends on both goods, but is separable, i.e., $\alpha = 1 - \rho$;

- Utility depends nonseparably on two goods, but both are costlessly adjustable, i.e., $\lambda = 0$.

Because the vast majority of consumption models in the literature fit into one of the three sets of assumptions, it is not surprising that many people use the term “EIS” as synonymous with “the reciprocal of the curvature parameter.”[13]

The housing model developed in Section I invokes a fourth set of assumptions: utility depends nonseparably on nondurable consumption and on housing, nondurable consumption is costlessly adjustable, but housing is subject to a nonconvex adjustment cost ($\lambda > 0$). Under these assumptions, the total EIS of nondurable consumption is given by equation (43a) and depends on both the curvature parameter, ρ, and the parameter governing the intratemporal substitutability of the two goods, α. Note that depending on the degree of intratemporal substitutability of the two goods, the housing model may, or may not, predict a low EIS of nondurable consumption. If the two goods are perfect substitutes ($\alpha = 1$), the effect of adjustment costs on housing is to increase the EIS of nondurable consumption. Conversely, if the two goods are complements in the sense

[13] For example, Masao Ogaki and Carmen Reinhart (1998) estimate a CES utility function equivalent to (29), and use the term “elasticity of intertemporal substitution” to refer to the inverse of the curvature parameter, ρ^{-1}.

that $\alpha < 1 - \rho$, the effect of the adjustment cost is to reduce the EIS of nondurable consumption.[14] In the next section, we estimate, using household-level data, the Euler equation for nondurable consumption in order to test the model and provide estimates of the crucial parameters of the utility function, α and ρ.

III. The Housing Model as a "Structural" Interpretation of Habit Persistence

Models of habit persistence provide another approach for breaking the tight relationship between the elasticity of intertemporal substitution and risk aversion. In particular, papers by Andrew Abel (1990), John Campbell and John Cochrane (1998, 1999), George Constantinides (1990), Wayne Ferson and Constantinides (1991), John Heaton (1995), and Suresh Sundarson (1989) examine the macroeconomic and asset pricing implications of a variety of models incorporating preferences that exhibit habit persistence. Of the many models of habit persistence contained in the literature, the model posed by Constantinides (1990) provides a convenient comparison to the housing model, as Constantinides considers the effects of habit persistence in an infinite horizon, continuous time model which, like the housing model, incorporates a portfolio decision and abstracts from labor income. That is, Constantinides considers the lifetime utility function:

$$U = E_0 \int_0^{\infty} e^{-\delta t} \frac{(C_t - X_t)^{1-\rho}}{1-\rho}\, dt. \tag{44}$$

In this specification, the habitual level of consumption, X_t, can be interpreted as the subsistence level of consumption in the sense that marginal utility becomes infinite at $C_t = X_t$. Constantinides shows that the non–time separable utility specification in (44) implies that the EIS will be time-varying and a function of the state variable representing the habitual level of consumption, X_t, in addition to the curvature parameter, ρ:

$$EIS = \frac{-\mu(C_t, X_t)}{C_t \dfrac{\partial \mu(C_t, X_t)}{\partial C_t}} = \frac{1 - \dfrac{X_t}{C_t}}{\rho}. \tag{45}$$

Because X_t is the subsistence level of consumption, the specification of the utility function in equation (44) implies that $X_t/C_t < 1$ and therefore that habit persistence reduces the elasticity of intertemporal substitution. Constantinides also shows that risk aversion is not constant over time, as in the time-separable case, but instead varies with the ratio of the two state variables in the value function. That is, the degree of relative risk aversion at time t is given by

$$RRA = -\frac{\dfrac{\partial^2 V(W_t, X_t)}{\partial W_t^2}}{\dfrac{\partial V(W_t, X_t)}{\partial W_t}} W_t = \frac{\rho}{1 - g\left[\dfrac{X_t}{W_t}\right]}, \tag{46}$$

[14] Larry Epstein and Stanley Zin (1989, 1991) provide another way of breaking the tight link between risk aversion and intertemporal substitution by dispensing with the expected utility framework and assuming a more general, recursive utility specification in which two different parameters govern preferences concerning risk and preferences concerning intertemporal substitution.

where g is a positive constant that depends on the interest rate and the parameters that govern the strength of habit persistence. Thus, in contrast to the time-separable case, in which relative risk aversion is constant and completely determined by the curvature of the utility function, ρ, in the presence of habit persistence, the household's degree of relative risk aversion is an increasing function of the ratio of habit to wealth. Note that the habit-persistence model, like the housing model, implies that the household's current choices (with respect to nondurable consumption and portfolio composition) will depend not only on current wealth, but also on the path of wealth. That is, in a comparison of two households that are identical in terms of their preferences and current wealth, but differ in terms of the historical path of wealth, the two households may differ in terms of their optimal level of nondurable consumption and their optimal portfolio composition because the households may face different values of the state variables (habit or housing stock).[15]

The habit-persistence model and the housing model have a long list of common features: both retain the expected utility framework, both explain the smoothness of nondurable consumption by introducing an additional state variable, and both imply that a household with stable preferences will nevertheless display variation over time in the degree of relative risk aversion and the elasticity of intertemporal substitution. In light of the many parallel implications of the two models, the housing model might be thought of as a "structural"[16] model of behavior that looks like habit persistence at the aggregate level.

In order to estimate the parameters of the utility function, and test the housing model against the habit-persistence model, we consider a utility function that nests both models. Generalized to allow for habit persistence in nondurable consumption, the CES utility function for household i becomes

$$(47)\qquad u(C_{i,t}, C_{i,t-1}, H_{i,t}) = \frac{[(C_{i,t} - dC_{i,t-1})^{\alpha} + \gamma H_{i,t}^{\alpha}]^{(1-\rho)/\alpha}}{1-\rho} \qquad \alpha \leq 1,\ 1 \neq \rho > 0,\ \gamma \geq 0.$$

If $\gamma = 0$ and $\alpha = 1$, the utility function in (47) is a simple habit persistence specification, with the stock of habit proportional to last period's nondurable consumption. A positive value of d indicates habit persistence in the sense that the utility associated with a given level of current nondurable consumption is decreasing in the previous level of consumption. A negative value of d indicates that the consumption good, although physically nondurable, exhibits durability in the utility flow in the sense that consumption of the nondurable good generates utility in both the current and subsequent periods.

Under the assumption that the nondurable consumption good is costlessly adjustable, the Euler equation for nondurable consumption holds. Since a priori there is no reason to rule out roles for both state variables, $C_{i,t-1}$ and $H_{i,t}$, we estimate the Euler equation implied by the utility function in (47), then test the restrictions imposed by the various nested models: housing, habit persistence, or the standard model with neither habit persistence nor habit. The Euler equation for nondurable consumption is

$$(48)\qquad 1 = \beta E_t\left[\frac{(C_{i,t+1} - dC_{i,t})^{\alpha-1}Q_{i,t+1} + \beta d(C_{i,t+2} - dC_{i,t+1})^{\alpha-1}Q_{i,t+2}}{(C_{i,t} - dC_{i,t-1})^{\alpha-1}Q_{i,t} + \beta d(C_{i,t+1} - dC_{i,t})^{\alpha-1}Q_{i,t+1}}(1 + r_{i,t+1})\right],$$

[15] In contrast, the generalized model of Epstein and Zin (1990) with recursive preferences implies that optimal consumption and portfolio composition will depend on current wealth, but not on the path of wealth.

[16] Pun intended.

where $Q_{i,t} = \left[(C_{i,t} - dC_{i,t-1})^{\alpha} + \gamma H_{i,t}^{\alpha}\right]^{[1-(\alpha+\rho)]/\alpha}$, β is the discount factor, and $r_{i,t+1}$ is the real after-tax asset return from t to $t + 1$.[17]

The Euler equation is estimated with data from the PSID, which contains data on housing in addition to the food consumption data used by many authors as a proxy for nondurable consumption.[18] That is, data on household food expenditure, defined as the sum of food expenditure at home and the value of food stamps (deflated by the Consumer Price Index (CPI) for food at home) plus food eaten out (deflated by the CPI for food away from home), was used to represent nondurable consumption, $C_{i,t}$. The after-tax real interest rate, $r_{i,t}$, is defined as

$$r_{i,t} = (1 - \tilde{\tau}_{i,t})R_t - \pi_t\,, \tag{49}$$

where $R_{i,t}$ is the nominal interest rate on one-year Treasury bills, $\tilde{\tau}_{i,t}$ is the household's marginal tax rate, and π_t is the inflation rate as measured by the CPI.

The PSID provides data on the value of owner-occupied houses and annual rents paid by renters. However, as an argument of the utility function, the housing variable, $H_{i,t}$, reflects some measure of the physical *quantity* of housing consumed, rather than the *value* of housing consumed. In principle, one could start with the PSID data on the value of the house (as reported by the respondent) and attempt to deflate the house value with an index of housing prices. In practice, there is substantial cross-sectional variation in housing prices within regions or cities, as well as across regions or cities. Since the region-wide price index provides only a crude approximation of the house price inflation within a particular neighborhood, deflating by the region-wide index would produce data that (inaccurately) indicate that even families who reside in the same physical house nevertheless are consuming different quantities of housing in different years. For this reason, we use a measure of housing consumption that is based on physical characteristics of the house, rather than attempting to deflate the reported house value by a price index. Of the many different metrics one could use to measure the quantity of housing, we use the simplest quantity measure: square footage.[19]

While the PSID does not provide data on the square footage of homes, it does report, for both homeowners and renters, the number of rooms. To impute the square footage of the homes of PSID respondents, we first used data from the American Housing Survey (AHS) to estimate a model of square footage as a function of number of rooms and other housing variables common to both the AHS and the PSID. That is, using data from the AHS, we estimated an equation explaining the size of the home (in square feet) as a function of dummy variables representing whether the household was (a) located in a suburb, (b) located in a non-SMA region, (c) a renter,

[17] Because of differences in marginal tax rates, $r_{i,t+1}$ varies across households.

[18] Based on NIPA data for 1930–2002, the annual growth rate of total nondurable consumption expenditures and the growth rate of food consumption have a correlation coefficient of 0.9. Thus, even though food consumption represents slightly less than half of total nondurable consumption expenditures, it seems to be a reasonable proxy for nondurable consumption.

[19] If the objective were to construct a measure of the quantity of housing at a single point in time, we recognize that the approach of deflating the house value by a regional price index would provide a better measure of real housing consumption because the house value will reflect many attributes other than square footage, such as location and construction materials. For this application, however, we are particularly interested in comparing the behavior of nondurable consumption across two periods in which housing consumption did not change, against the behavior of nondurable consumption across two periods in which housing consumption did change. A simple physical measure of housing consumption like square footage has the important property that measured housing consumption is constant as long as the family stays in the same house. Compared to the true (unobserved) quantity of housing, the data on imputed square footage are contaminated with several types of measurement error (first, because they abstract from the quality dimension and, second, because some households who stay in the same home may nevertheless substantially alter their housing consumption through remodeling). However, we argue that the instrumental variables used in the estimation are uncorrelated with the measurement error.

TABLE 1—COMPARISON OF HOUSING, HABIT PERSISTENCE, AND STANDARD MODELS

		Restricted forms		
	Unrestricted form	Housing	Habit	Standard
Subjective discount factor (β)	0.98	0.98	0.98	0.98
Total number of observations	25,421	25,421	25,421	25,421
Parameters:				
Intratemporal substitution (α)	−6.485	−6.668	1	1
	(1.751)	(1.689)		
Habit formation (d)	0.007	0	0.009	0
	(0.006)		(0.007)	
Curvature (ρ)	1.846	1.799	7.520	7.778
	(0.267)	(0.244)	(2.804)	(2.301)
Weight on housing (γ)	1.039	1.015	0	0
	(0.310)	(0.287)		
Implied total EIS of C		0.131	0.132	0.129
Hypothesis tests [p-values]:				
$\alpha = 1$	[0.00]	[0.00]		
$\rho = 1$	[0.00]	[0.00]	[0.00]	[0.00]
$\alpha = 1 - \rho$	[0.00]	[0.00]		
Overidentifying restrictions	[0.38]	[0.42]	[0.13]	[0.06]
		0.880	13.771	14.760
LR test statistic		[0.35]	[0.00]	[0.00]

Notes: Asymptotic standard errors are in parentheses. Probability values for hypothesis tests are in brackets. Sample period is 1975 to 1985. The EIS is calculated using the point estimates of the parameters and the 1974–1987 sample averages of the variables. The subjective discount factor of 0.98 was imposed, not estimated.

(d) living in a mobile home, and on a third-order polynomial in the number of rooms. Separate models were estimated for each of the four regions (Northeast, Midwest, South, and West). The regional models estimated from the AHS data, reported in the Data Appendix (available at http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.1.474), were then used to generate estimated square footage data for each PSID household.

Estimation of the Euler equation was by GMM, for the 1975–1985 sample period.[20] Consistent estimation requires the use of instruments that are correlated with the true (unobserved) values of food consumption and housing services, but uncorrelated with both the measurement error in observed $C_{i,t}$ and $H_{i,t}$ and the expectation error in the Euler equation. The absence of correlation with the expectation error was ensured by lagging the instrumental variables (the growth rate in real household income, the change in total annual hours worked by all family members, and the growth rate of housing square footage) by two periods relative to the Euler equation; that is, the instruments reflected changes from $t - 2$ to $t - 1$ for the Euler equation linking marginal utility in t to $t + 1$.[21]

Table 1 reports parameter estimates for four versions of the model.[22] The most general version (labeled "unrestricted") allows for effects from both housing and habit persistence. In addition to

[20] Because the food questions were not asked in 1973, or in 1988–1989, the food data are available only for 1974–1987. After allowing for required leads and lags, this left a sample period of 1975–1985.

[21] Note that because our measure of housing services is an imputation of square footage, which by construction is constant as long as the household remains in the same house or apartment, the instrument for growth rate of housing square footage will be nonzero only if the household moves between $t - 2$ and $t - 1$. It seems plausible to assume that this instrument is correlated with the actual (unobserved) change in housing square footage between $t - 2$ and $t - 1$, but uncorrelated with the measurement error in imputed square footage for t or $t + 1$, and uncorrelated with the measurement error in food consumption. Similarly, it seems reasonable to assume that the growth rate of real household income and the change in total annual hours worked between $t - 2$ and $t - 1$ are uncorrelated with the measurement errors in housing square footage and in food consumption.

[22] In the absence of an adjustment cost (i.e., $\lambda = 0$), the data should satisfy a second Euler equation (for the marginal utility of housing services) and should satisfy an intratemporal first order-condition (equation (34)). Statistical rejection

restricted specifications for the housing model and the habit-persistence model, Table 1 reports results for a restricted version of the model with neither housing nor habit persistence (labeled "standard"). For each version of the model, the implied total EIS of nondurable consumption is calculated from the point estimates of the parameters and sample averages of the data. (That is, the implied total EIS of nondurable consumption is calculated using equation (43a) for the housing model, equation (45) for the habit-persistence model, and ρ^{-1} for the standard model.) The various versions of the model all generate essentially the same value of the total EIS of about 0.13, but differ in the mapping between the EIS and the underlying preference parameters. In the standard model, of course, a low total EIS of 0.13 is interpreted as an implication of a fairly high value of the curvature parameter ($\rho = 7.8$). In the habit-persistence model, the estimate of the parameter d, which reflects the importance of habit in the utility function, is indistinguishable from zero, both in terms of its magnitude ($d = 0.009$), and in terms of statistical significance. Since the data do not attribute a quantitatively significant role to habit persistence, the estimate of the curvature parameter of 7.5 is essentially the same as in the standard model. In the specification for the housing model, the estimate of the intratemporal substitution parameter, α, is –6.7, and reasonably precisely estimated. The null hypothesis of perfect intratemporal substitutability between the two goods ($H_0 : \alpha = 1$) is rejected at high confidence levels. The estimate of the curvature parameter, ρ, is 1.8. While the estimated value of the curvature parameter is only modestly greater than unity, it is sufficiently precisely estimated to reject the log specification of the utility function (i.e., the null hypothesis that $\rho = 1$).[23] In the housing model, the reciprocal relationship between the EIS and the curvature parameter does not hold in general, but will hold in the special case of separable utility (i.e., when $\alpha = 1 - \rho$). However, the parameter restriction $\alpha = 1 - \rho$ is also rejected at high confidence levels. Further, the finding that the estimated value of α (-6.7) is smaller than the estimate of $1 - \rho$ (-0.8) attributes the low EIS of nondurable consumption to the substantially imperfect substitutability between the two goods, rather than to a high degree of curvature of the utility function with respect to the composite good.[24]

The last two columns of Table 1 report the likelihood ratio test statistic, and the associated probability value, of each of the three restricted models against the general model. Both the standard model and the habit-persistence model are decisively rejected, while the housing model survives with a probability value of only 0.35.

of either of these additional first-order conditions constitutes a rejection of the frictionless version of the model.

[23] Ogaki and Reinhart (1998) estimate the parameters of a nonseparable utility function equivalent to (25). In terms of data, Ogaki and Reinhart use aggregate per capita data on the services of durables as defined by the NIPA and nondurable consumption minus clothing. Despite the fact that Ogaki and Reinhart use a very different dataset (aggregate versus household-level data, NIPA definitions of nondurable consumption, and services of durables versus food consumption and housing), they estimate the curvature parameter, ρ, to be in the range 2.22 to 3.12, which is very close to our estimate of 1.8. Their estimate of the intratemporal substitutability parameter, α, of 0.15 is substantially larger than our estimate of -6.7, which presumably reflects the fact that there is considerably smaller intratemporal substitutability between food and housing than there is between nondurables (as a category) and durables (as a category).

[24] Raj Chetty and Adam Szeidl (2004) also provide a model that appeals to adjustment costs associated with some goods (called "commitment goods") to explain consumption dynamics which look like habit persistence at the aggregate level. In contrast to our analysis, which relies crucially on the nonseparability of the utility function, Chetty and Szeidl's analysis assumes that the utility function is separable between the "commitment good" (e.g., housing) and the noncommitment good (e.g., food). As a result of the assumed separability, in their model the dynamic behavior of the noncommitment good is exactly the same as in the standard model; for example, the noncommitment good does not exhibit habit persistence, and has an EIS equal to the inverse of the curvature parameter. In the "consumption commitments" model, total aggregate consumption displays dynamics similar to a habit-persistence model in the sense that aggregate housing consumption is a slowly moving state variable that mimics external habit formation, and marginal utility depends on the gap between total consumption (aggregate consumption of food and housing) and habitual consumption (aggregate consumption of housing). Thus, in the Chetty and Szeidl model, the presence of adjustment costs on the commitment good explains why aggregate housing consumption and aggregate total consumption are "smoother" than implied by the completely frictionless case, but does not explain the smoothness of the noncommitment component of consumption.

IV. Conclusions

Despite the quantitative importance of housing as a component of the household budget, and of the household portfolio, the dominant models in macroeconomics and in finance typically ignore housing entirely, and build their optimization problems on a utility function that takes as its argument a single, costlessly adjustable, nondurable good. This simplifying assumption, though drastic, would be reasonable if (a) abstracting from housing did not appreciably alter the implications of the model, and (b) the more plausible specification in which housing is treated as a separate good, imperfectly substitutable with nondurable consumption, were intractable. The paper provides a generalization of the important, but highly stylized, model of Grossman and Laroque (1990), and identifies the conditions under which the model remains tractable in a setting sufficiently general to incorporate variation in the price of housing relative to the nondurable good. The required assumption (that housing price risk is uncorrelated with the returns to financial assets) seems to be reasonably consistent with the data.

The housing model differs substantially from the standard model, but delivers many of the same implications as the habit-persistence model, because the assumption that housing is subject to a nonconvex adjustment cost causes the current house to become one of the state variables that affect the household's optimal level of nondurable consumption and optimal portfolio allocation. While the housing model and the habit-persistence model are both theoretically capable of explaining why nondurable consumption is "smooth," without invoking an implausibly large degree of risk aversion, empirical tests using household-level data strongly favor the housing model.

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