

Problem set 2

Problem sets are due at the beginning of class on Thursday, January 31.

The data set you need to do this problem set is the same data set that you used for the first problem set: the Excel file named Rhist.xls which can be downloaded from the class web page.

1. Consider a portfolio, named Portfolio20B, which consists of 20% long term T-bonds and 80% large stocks. Construct the series of returns that you would have received on this portfolio for the period 1926-1998 by using Excel to calculate the return to Portfolio20B as a weighted average of the returns to long term bonds and large stocks (with weights .2 on long term bonds and .8 on large stocks). Store your series for the return to Portfolio20B as an additional column in the Excel worksheet that contains the data on the historical returns on the indices. Similarly, consider a portfolio, named Portfolio40B, which consists of 40% long term T-bonds and 60% large stocks, and construct a series for the return to Portfolio40B for the period 1926-1998, storing the series as an additional column in the worksheet.

For each of the following data series: long term T-bonds, large stocks, small stocks, Portfolio20B, Portfolio40B, and T-bills, have Excel calculate the following sample statistics for the 1926-1998 sample period:

- a) mean holding period return (that is, the arithmetic average)
- b) the standard deviation of the holding period return

Report your results for this question by printing out a table with 6 columns (one for each of the six asset return series) and two rows, reporting the estimated mean return on the asset in the first row, and the estimated standard deviation of the return on the asset in the second row. When printing your table, include labels for each of the rows and columns.

2. Calculate the 5x5 matrix that reports the correlation coefficients between long term T-bonds, large stocks, small stocks, Portfolio20B, and Portfolio40B. That is, the five columns represent long term T-bonds, large stocks, small stocks, Portfolio20B, and Portfolio40B, respectively, and the five rows represent long term T-bonds, large stocks, small stocks, Portfolio20B, and Portfolio40B, respectively. In each cell of the matrix, you report the correlation coefficient between the returns to the assets in the associated row and column. The correlation matrix will be symmetric, and will have ones along the main diagonal. Print out the matrix in the form of a table with five columns and five rows, with labels for the rows and columns.

3. Consider two of the asset indices: The “large stock” index (the data for the large stock index is the Standard and Poor 500 stock index) and the long term T-bond index. Use Excel to calculate the efficient frontier which can be achieved with combinations of these two assets (only). You have already calculated the expected return, standard deviation, and correlation coefficient for these two assets. Using the values of these statistics as data inputs, have Excel calculate a table which reports the expected return and the standard deviation of a portfolio which

has the fraction  $z$  in large stocks and the remainder  $(1-z)$  in long term T-bonds, letting the value of  $z$  vary between 0.0 and 1.0 in increments of .02. Your table should have 51 rows (one for each of the values of  $z$  between 0 and 1, inclusive), and 3 columns (the first column reports  $z$ , the second column reports the expected return to the portfolio, and the third column reports the standard deviation of the return to the portfolio). Create a plot of the points in mean-standard deviation space, and have Excel draw in the efficient frontier.

Report your results by printing out both the table and the plot.

4. Among the 51 portfolios you plotted in question 3 are Portfolio20B and Portfolio40B.
  - a) Calculate the efficient frontier which can be achieved with combinations of Portfolio20B and small stocks, using the same procedure as before; i.e., let  $z$  vary between 0.0 and 1.0 in increments of .02 and have Excel calculate the expected return and standard deviation of the return to the portfolio consisting of fraction  $z$  in Portfolio20B and fraction  $(1-z)$  in small stocks. Print out the resulting table (which will have 51 rows and 3 columns).
  - b) Same as part a) but calculate the efficient frontier which can be achieved with combinations of Portfolio40B and small stocks. Print out the resulting table.
  - c) Plot the 3 frontiers (combinations of expected return and standard deviation of return) obtained in parts 3, 4a, and 4b, all on the same mean-standard deviation diagram.

5. Take the T-bill as the risk free asset; that is, use the mean return on T-bills as an estimate of the risk-free rate, and pretend that the standard deviation of the return to T-bills is zero, even though the historical data says otherwise. Take the tables generated in parts 3, 4a, and 4b and augment each table with an additional column which reports, for every value of  $z$ , the slope of the CAL corresponding to the portfolio generated by that value of  $z$ . That is, the additional column of the table will report the slopes of 51 CALs. Print out another copy of the plot of the 3 frontiers that you generated in part 4c. On this second copy, after consulting the tables which report expected return, standard deviation and the slope of the CAL for each value of  $z$ , identify and carefully plot by hand (with a pen) and label the following features:

- a) the risk free rate
- b) the optimal risky portfolio
- c) the minimum variance portfolio
- d) the CAL with the highest reward-to-variability ratio

In question 4, you have constructed portfolios from two particular combinations of long bonds, and large stocks rather than all possible combinations. When question 5 refers to the “optimal risky portfolio”, the “minimum variance portfolio”, and the “CAL with the highest reward-to-variability ratio), it is asking for the best choice within the frontiers that you actually calculated. (In identifying the optimal risky portfolio and its CAL, just find the value of  $z$  among the 51 you evaluated which is closest to the optimal risky portfolio. You do not need to worry about values of  $z$  between the 51 points you evaluated, since you’ve calculated enough points to give a pretty good approximation of the whole frontier.)

Report your results by printing out the plot, with the features a-d drawn in and labeled.