Hold-Up and Durable Trading Opportunities

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Abstract

This paper examines a contractual setting with unverifiable investment and a durable trading opportunity, in which trade can take place in any one of an infinite number of periods. The contractual setting features cross-investment, meaning that the seller's investment affects the buyer's benefit of trade. Two different trade technologies are studied, one in which the seller has the individual action that consummates trade and one in which the buyer has the action that consummates trade. The set of outcomes supported in the durability setting is shown to be equivalent to the set supported in the related setting without durability. Thus, rather than durability, it is the technology of investment and trade — in particular, whether investment and trade actions are *divided* or *unified* (Buzard and Watson 2009) — that plays the critical role in determining whether the seller can be induced to invest at the efficient level. The issue of multiple equilibrium is analyzed and it is shown that particular non-stationary contracts can achieve unique implementation. The modeling exercise thus qualifies the recent view that durability may contribute to the hold-up problem.

1 Introduction

The hold-up problem arises in situations in which contracting parties can renegotiate their contract between the time they make unverifiable relation-specific investments and the time at which they can trade.[1] For instance, consider a simple model of trade between a buyer and a seller. First, the parties write a contract specifying the terms of trade. Second, the seller chooses a level of investment, which is observed by the buyer but is unverifiable to the external enforcer (i.e., court). Next the parties can renegotiate their contract. Finally, the

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[1] Che and Sákovics (2008) provide a short overview of the hold-up problem, which was first described by Klein, Crawford and Alchian (1978), and Williamson (1975,1977). Analysis was provided by Grout (1984), Grossman and Hart (1986), and Hart and Moore (1988).

parties have the opportunity to trade and the external enforcer compels monetary transfers conditional on the verifiable trade actions. Assume that the seller's investment affects only the buyer's benefit of trade — a setting that presents a particular challenge for aligning incentives (Che and Hausch 1999). "Hold-up" refers to the constraint that renegotiation puts on the problem of designing a contract to motivate the seller to invest.

Many authors have analyzed the severity of hold-up in this general setting. (Public messages are usually included in the time line but add nothing in the cases analyzed in this paper.) Prominent models yield different predictions, but their disparate assumptions make comparisons difficult. Watson (2007) shows an essential difference between various models in the literature lies in the modeling of the actions that consummate trade. "Individual-action" models explicitly account for how the contracting parties take individual actions to consummate trade (such as the buyer's action of whether to install the good produced by the seller). "Public-action" models abstract a bit by considering trade actions to be taken by the external enforcer as a function of messages from the contracting parties. Watson demonstrates that public-action models essentially constrain attention to the class of "forcing" contracts, which do not consider how individual trade actions can serve as options.[2] Thus, to examine the full range of feasible contracts, it is important to explicitly model the players' individual trade actions — in other words, to provide a noncooperative game-theoretic account of trade.

Buzard and Watson (2009) take the analysis a step further by investigating how the ability to motivate ex ante investments depends critically on which of the parties has the action that consummates trade. Fix the setting described above, where the seller makes the investment choice. In Buzard and Watson's case of *divided* investment and trade actions, trade is determined by the buyer's choice of whether to install the intermediate good. In the *unified* case, it is the seller's act of delivery that consummates trade. Buzard and Watson show that efficient investment can be supported in the divided case but not generally in the unified case.[3]

In this paper, we examine another dimension of the trade technology: the extent to which the trading opportunity is durable. In relationships with a durable trading opportunity, if the parties do not complete the trade in a given period then they will have another opportunity to do so in the future. Most of the literature on investment and hold-up focuses on models with a nondurable trading opportunity, in which trade can only take place at a single point in time. However, the motivations for these analyses often involve stories of a durable trading opportunity. For example, Nöldeke and Schmidt (1998) argue that suitably defined option contracts function to induce efficient investment when parties can consummate trade at any time. Edlin and Hermalin (2000), on the other hand, argue that a party

[2] Lyon and Rasmusen (2004) also argue that models in the previous literature make limiting assumptions about how option contracts may function. They note that options can be exercised even following impasse in renegotiation.

[3] Buzard and Watson (2009) concentrate on the setting in which a single player has the trade action. They also show how it becomes easier to encourage investment if both parties have verifiable trade actions, which is a characteristic of Evans' (2008) model that we discuss below.

can effectively let an option expire and then renegotiate from scratch. Che and Sákovics (2007), in summarizing the literature, point to durability as a contributor to the hold-up problem.

Our objective here is to provide a precise analysis of durability and its effect on the hold-up problem. We construct and analyze a model that explicitly accounts for the technology of trade, including the durability of the trading opportunity. As Buzard and Watson (2009) do, we limit attention here to settings in which just one of the players has a trade action. The model has cross-investment (as in Che and Hausch 1999), whereby the seller makes an investment that influences the buyer's benefit of trade later.

By comparing the durable trade environment with a benchmark model that has only one trading opportunity, we develop our main theme: that durability of a trading opportunity does not complicate the hold-up problem per se. Specifically, if an outcome is supported in the nondurability environment then it is also supported in the environment in which the trading opportunity is durable. Furthermore, we show that the nature of the trade action (whether it is the buyer's or seller's action) plays an important role in the durability model just as it does in the nondurability model. To be precise, in the simple class of contractual relationships studied here, the outcomes supported in the durability and nondurability environments are identical. Therefore, the efficient outcome can always be obtained in the divided case (where the buyer has the trade action) but not in the unified case (where the seller has the trade action).[4]

We show that simple open-ended option contracts suffice to deliver our results in the durability environment. These are stationary contracts that trigger a transfer from one party to the other in the period in which trade occurs, whenever it may be. The specified transfer is independent of the time period. We also show that, in some cases, these stationary contracts gives rise to multiple equilibria in the post-investment continuation. Interestingly, the two equilibria that emerge represent the different outcomes described by Nöldeke and Schmidt (1998) and Edlin and Hermalin (2000). However, we prove that the desired outcome (that which gives the seller the greatest incentive to invest ex ante) can be uniquely implemented by using a nonstationary contract. Furthermore, our notion of unique implementation requires this property from every continuation under the original contract. As an aside, we discuss how our results relate to the "outside option principle" from bargaining theory.[5]

This paper adds to the substantial literature on contracting with unverifiable investments. Many of the papers in this literature examine public-action mechanism-design models.[6] Examples of individual-action models in the literature, among others, are the articles

[4]Our analysis is particularly straightforward because, for the class of relationships we analyze, there is no benefit of introducing messages so we can leave them out of the model.

[5]Some recent papers, including Edlin and Hermalin (2000) and Wickelgren (2007), have motivated the link between durability and hold-up by appealing to the outside option principle, which basically states that a bargainer's ability to opt out of negotiation does not affect the outcome of bargaining if the option gives a sufficiently low payoff to this player. This result does not hold if the outside option is close to efficient, which is the case in the contractual setting studied here.

[6]These include Aghion, Dewatripont, and Rey (1994), Che and Hausch (1999), Hart and Moore (1999),

of Hart and Moore (1988), MacLeod and Malcomson (1993), Nöldeke and Schmidt (1995), Lyon and Rasmussen (2004), Watson (2007), and Buzard and Watson (2009).

Most closely related to our work is that of Evans (2008), who also studies an individual-action model with durable trading opportunities. Evans' model is very general in terms of the investment technology and the available times at which the players can trade and renegotiate. He examines a single trade technology in which trade requires an individual action of the seller (verifiable production and delivery) followed by an individual action of the buyer (verifiable acceptance of the good). Evans shows that contracts that generate multiple equilibria in the post-investment continuation can be usefully employed to give incentives to invest, by having the contracting parties condition their equilibrium selection on the unverifiable investments. He proves that the efficient outcome can be achieved in a broad range of settings, most generally if the players can commit to a joint financial hostage (money deposited with a third party until trade occurs, if ever).

Our work complements Evans (2008) by showing that, for the setting we study: (a) holding the trade technology fixed, the outcomes supported in the cases of nondurable and durable trading opportunities are equivalent; and (b) with or without durability, vastly different outcomes arise depending on the technology of trade (whether the buyer or seller has the action that consummates trade). Also, the analysis of unique implementation in the durability model is unique to our modeling exercise.

In the next section we describe and analyze the model of a nondurable trading opportunity, for both the cases of divided and unified investment and trade actions. In Section 3, we describe the model of a durable trading opportunity and we study its relation to the nondurability model. Section 4 presents our analysis of nonstationary option contracts and unique implementation. In Section 5 we provide some general analysis of bargaining with outside options that serves to clarify the outside option principle and its implications for hold-up. The Conclusion contains some additional comments about the literature and extensions.

Maskin and Moore (1999), Segal (1999), and Segal and Whinston (2002). The more recent entries by Roider (2004) and Guriev (2003) have the same basic public-action structure. Demski and Sappington (1991), Nöldeke and Schmidt (1998), and Edlin and Hermalin (2000) examine models with sequential investments in a tradeable asset; in these models, transferring the asset is essentially a public action. In some papers, such as with Edlin and Reichelstein (1996) and Stremitzer (2009), trade actions are modeled as public but simple contracts (or breach remedies) are sufficient to achieve an efficient outcome anyway. Related as well is the work of Boeckem and Schiller (2008) and Ellman (2006).

2 The Benchmark Model of a Nondurable Trading Opportunity

We start with an individual-action model with unverifiable investment and a verifiable individual trade action, as in Watson (2007). A buyer and a seller interact as described in the introduction. The order of actions is:

- **Period 0**
  - The parties write a contract $\mathcal{C}$ specifying an externally enforced monetary transfer m from the buyer to the seller to be compelled if trade occurs in period 1. We normalize to zero the payment specified for the contingency in which trade does not occur. The contract may also specify an up-front transfer, which will not affect the subsequent analysis.[7]
  - The seller makes an investment choice, selecting between H at personal cost c and L at zero cost. Let θ denote the seller's choice, which we call the *state*. This investment action is observed by the buyer but is unverifiable to the external enforcer.
- **Period 1**
  - The parties have an opportunity to renegotiate the contract, altering the specification of $\mathcal{C}$ and potentially including an up-front transfer.
  - One of the parties takes the *trade action* $a \in \{0, 1\}$, where $a = 1$ means that trade occurs and $a = 0$ means that trade does not occur. This choice is verifiable.
  - The external enforcer compels the contractually specified monetary transfer, which is m if the trade occurred (that is, if $a = 1$) and zero otherwise.

We will examine two versions of the model which differ in the technology of trade. In the first version, the buyer has the trade action, which one can regard as a choice of whether to install ($a = 1$) or not install ($a = 0$) the intermediate good supplied by the seller. Using the terminology of Buzard and Watson (2009), we call this the case of *divided* investment and trade actions (because investment is chosen by one party, whereas the other chooses whether to trade). In the second version, the seller has the trade action, which one can interpret as whether to deliver the intermediate good ($a = 1$) or not ($a = 0$). This is called the *unified* case. The trade technology is exogenously given.

Here is an example of the divided case: Suppose that the seller is a public-relations firm that designs logos and other graphics for its clients; the buyer is a health-care provider that wants to "re-brand" with a new high-quality logo to put on its web site and letterhead. The seller invests to create a logo. High investment implies that the logo is valuable to the buyer

[7] The assumption of zero payment conditional on no trade is therefore without loss of generality.

(it would attract new customers). The buyer observes the logo and recognizes the value it would generate. Then the buyer takes the trade action, which is either to place the new logo on its web site or to continue using its old logo. This choice is verifiable. That is, the external enforcer can easily access the Internet to see whether the new logo was installed, but does not observe its benefit to the buyer.

For an example of the unified case, suppose that the seller is an advertisement agency that owns billboards, while the buyer is a company that would gain from advertising on a billboard. The seller invests to create the layout of the advertisement. The buyer observes the layout and recognizes what the benefit would be of posting it on a billboard. Subsequently, the seller chooses whether or not to mount it on one of its billboards; this is the trade action.

We suppose that if the seller invests in period 0 (so that $\theta = \mathrm{H}$), then trade gives benefit $x > c$ to the buyer. If the seller does not invest ($\theta = \mathrm{L}$), then the intermediate good is worthless. To keep the model simple, we assume that the trade action has no direct affect on the seller's payoff; in the unified case, for instance, there is no delivery cost. The joint value of high investment and trade is $x - c$. Note that the specification of the trade action as an individual action is the key to having a precise account of the technology of trade. We have not included in the specification of the game a phase in which the parties can make announcements, because messages will not affect our results; we discuss this further in the Conclusion.

In the divided case, the contract $\mathcal{C}$ is called a *forcing contract* if it specifies either $m < 0$ (giving the buyer the incentive to install regardless of the seller's investment level) or $m > x$ (giving the buyer the incentive to never install). We call $\mathcal{C}$ an *option contract* if it specifies $m \in [0, x]$, since the buyer has the incentive to install in state H and not in state L. In the unified case, since the seller faces no direct cost of delivery, the contract forces trade if $m > 0$, it forces no trade if $m < 0$, and it functions as an option when $m = 0$.

The seller's choice of investment action and the behavior of the player with the trade action are assumed to be consistent with sequential rationality. Thus, each player selects his individual actions to maximize his/her expected payoff, and the players anticipate rational behavior in the future.

If the contract would induce an inefficient trade action (in particular, $a = 0$ in state H), then the parties will renegotiate just before the time that the trade action must be selected. The outcome of renegotiation is assumed to be consistent with a "black-box" cooperative bargaining solution in which the players divide surplus according to fixed bargaining weights π_{B} and π_{S} for the buyer and seller. Surplus is defined relative to the continuation value of proceeding under the original contract. This characterization of renegotiation along with the sequential rationality conditions identify a *contractual equilibrium* (see Watson 2004).[8]

[8] Because the players are risk neutral in money, the cooperative solution yields the same expected payoffs as does the following non-cooperative specification of negotiation: Nature selects one of the players to make an ultimatum offer to the other player, who either accepts or rejects it; Nature selects player i with probability π_i. We assume that (i) on the self-enforced component of contract, players behave as agreed whenever this is

A *value function* gives the continuation payoffs from the start of period 1, gross of investment cost, as a function of the state.[9] Without loss of generality we can focus on contracts that yield a payoff vector of $(0,0)$ to the players in state L. Thus, the value function is characterized by the continuation payoff vector in state H, which we denote $v = (v_{\mathrm{B}}, v_{\mathrm{S}}) \in \mathbb{R}^2$.

The normalization of payoffs in state L implies a restriction on the domain of m. In the divided case, we can assume that $m \geq 0$. This is because $m < 0$ is a forcing contract that ensures the seller's payoff in state L is strictly negative. We can increase m to bring the seller's payoff in state L up to zero, without changing incentives in either state (so the seller's payoff in state H rises by the same amount). Likewise, in the unified case we can assume that $m \leq 0$. A strictly positive m would be a forcing contract that ensures the seller a strictly positive payoff in state L; resetting m to zero would make the seller's payoff zero in state L and it shifts her payoff in state H by the same amount (without changing incentives in either state).

The value vector v is said to be *implemented by contract* $\mathcal{C}$ if there is an equilibrium (combining sequential rationality and the bargaining solution) of the game from period 1 that achieves this payoff vector in state H and gives each player zero in state L. Our interest is in finding a contract that gives the seller the incentive to invest efficiently. From the time of her investment action in period 0, the seller will obtain $-c + v_{\mathrm{S}}$ by investing at level H and will obtain zero (no cost and zero value from period 1) by investing at level L. Thus, our objective is to determine whether some value $v = (v_{\mathrm{B}}, v_{\mathrm{S}})$ that satisfies $v_{\mathrm{S}} \geq c$ can be implemented.

For our simple model of nondurability, it is easy to determine whether a contract can induce efficient investment and trade. The result differs significantly depending on whether investment and trade actions are divided or unified, as Watson (2007) and Buzard and Watson (2007) emphasize.

Proposition 1: *In the model with a nondurable trading opportunity and divided investment and trade actions, v can be implemented if and only if $v_{\mathrm{S}} \in [0, x]$ and $v_{\mathrm{B}} = x - v_{\mathrm{S}}$. There is an option contract that induces efficient investment and trade.*

Proof: Take any vector v that satisfies the conditions of the proposition. Let the contract $\mathcal{C}$ specify $m = v_{\mathrm{S}}$. Then under contract $\mathcal{C}$, the buyer has the incentive to select $a = 1$ in state H and he has the incentive to choose $a = 0$ and in state L. No renegotiation will take place in either state, because the buyer's trade action yields an efficient outcome. So the buyer's payoff from the beginning of period 1 is $x - m$, the seller's payoff is m, and therefore v is implemented.

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consistent with individual rationality; and (ii) if an offer is rejected, then the equilibrium in the continuation of the game does not depend on the identity of the offerer or on the nature of the offer. These are the Agreement and Disagreement Conditions described in Watson (2004).

[9] That is, the value function does not incorporate any cost of investment that the seller may incur in period 0.

It is easy to verify that no other value vectors can be implemented. For instance, specifying $m > x$ will force the buyer to choose $a = 0$, and renegotiation then implies that $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$ is implemented. Alternatively, if $m = 0$ were specified and the buyer would select $a = 1$ in both states, then there is no renegotiation gain. This contract implements $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (x, 0)$ in state H. $\|$

Note that setting $v_{\mathrm{S}} \in [c, x]$ induces efficient investment and trade, so we conclude that the efficient outcome will be obtained in the divided case. As the next result shows, the unified case gives a less encouraging result.

Proposition 2: *In the model with a nondurable trading opportunity and unified investment and trade actions, v is implementable if and only if $v_{\mathrm{S}} \in [0, \pi_{\mathrm{S}} x]$ and $v_{\mathrm{B}} = x - v_{\mathrm{S}}$. Efficient investment cannot be achieved if $c > \pi_{\mathrm{S}} x$.*

Proof: Consider a contract specifying $m = 0$. This contract will implement every v in the set described in the proposition. To see this, observe that under this contract the seller would be indifferent between selecting $a = 0$ and $a = 1$ in both states. Suppose that in state L the seller would select $a = 0$ and in state H she would select $a = 1$ with probability α. Then in state H the renegotiation surplus is $(1 - \alpha)x$, which the parties split according to their bargaining weights. Therefore

$$(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\alpha x + (1 - \alpha)\pi_{\mathrm{B}} x, (1 - \alpha)\pi_{\mathrm{S}} x)$$

is implemented. Varying α from zero to one achieves exactly the set of implementable values described in the proposition. No other values can be implemented. In particular, specifying $m < 0$ forces the seller to choose $a = 0$ in both states, renegotiation occurs in state H, and the normalized implemented value is then $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$. Further, if $c > \pi_{\mathrm{S}} x$ then we necessarily have $v_{\mathrm{S}} < c$ and so the seller cannot be given the incentive to invest efficiently. $\|$

Proposition 2 shows that the conclusion of Che and Hausch (1999) applies to our simple model in the case of unified investment and trade actions. That is, the best contract is a "null" contract and the only way for the seller to extract part of the benefit of her investment is through ex post renegotiation. Proposition 1 shows that this negative conclusion *does not* apply in the case of divided investment and trade actions. In fact, the efficient outcome can be attained in this case. This point — that the technology of trade has implications for hold-up and efficiency — was first made by Watson (2007), with general analysis provided by Buzard and Watson (2009).[10]

[10]Recall that public-action models, such as that of Che and Hausch (1999), do not make the distinction between the divided and unified cases.

3 The Model of a Durable Trading Opportunity

We next consider a durable trading opportunity in which the opportunity to trade persists indefinitely. Time is discrete and the parties discount the future using discount factor δ. Interaction in each period $t = 1, 2, \ldots$ is identical to period 1 of the model without durable trade except that, from a given period t, the game continues into period $t + 1$ if and only if $a = 0$ was selected in period t. The game ends at the end of the period in which $a = 1$ is chosen. Here is the time line:

- **Period 0**
  - The parties write a contract $\mathcal{C} = \{m^t\}_{t=1}^{\infty}$ which, for each period t, specifies an externally enforced monetary transfer m^t to be compelled if $a = 1$ in period t.
  - The seller makes an investment choice, selecting between H at personal cost c and L at zero cost. Let θ denote the seller's choice, which we call the *state*. This investment action is observed by the buyer but is unverifiable to the external enforcer.
- **Period** $t = 1, 2, \ldots$
  - The parties have an opportunity to renegotiate the contract.
  - The player who has the trade action selects $a \in \{0, 1\}$, where $a = 1$ means that trade occurs and $a = 0$ means that trade does not occur. This choice is verifiable.
  - The external enforcer compels the contractually specified monetary transfer, which is m^t if $a = 1$ was selected and zero otherwise. If $a = 1$, then the buyer obtains the benefit of the intermediate good and the relationship ends. Otherwise, the relationship continues into the next period.

We model the players' behavior as in Section 2. In each period, the player with the trade action selects a to maximize his/her payoffs. Furthermore, the resolution of renegotiation in each period is consistent with the bargaining solution that divides surplus according to the fixed bargaining weights. A value vector v is *implemented by contract* $\mathcal{C}$ if there is an equilibrium (combining sequential rationality and the bargaining solution) of the game from period 1 that achieves this payoff vector in state H and gives each player zero in state L.

The equilibrium construction is more complicated here in the setting of a durable trading opportunity than it was in the setting of nondurability, because there is an infinite horizon. However, note that delaying trade in state H is an off-equilibrium-path phenomenon because, conditional on arriving at any period t, renegotiation will ensure that trade occurs at t. Still, we have to analyze what would happen if any given period t were reached with the original contract $\mathcal{C}$ in force.

Because we now have an infinite number of periods, there will be a sequence of state-contingent continuation values. As before, we can normalize so that the continuation values in state L are always zero. Correspondingly, without loss of generality we can assume that $m^t \geq 0$ in the divided case and $m^t \leq 0$ in the unified case. We then express the relevant values in terms of a sequence $\{v^t\}_{t=1}^{\infty}$ of continuation values in state H. For each t, $v^t = (v^t_{\mathrm{B}}, v^t_{\mathrm{S}}) \in \mathbb{R}^2$ is the continuation payoff vector from period t in state H under the original contract $\mathcal{C}$.

Also, we shall need to describe the behavior over time of the player who has the trade action. For most of our analysis, this player can be assumed to always select $a = 0$ in state L. We describe the behavior in state H as a sequence $\{a^t\}_{t=1}^{\infty}$, with the interpretation that if the game reaches period t in state H then a^t is the trade action that is chosen in this period. Where we need to look at random choices, we will use α^t to denote the probability that $a = 1$.

We separate the analysis of the two trade technologies into the following two subsections, starting with the case of divided investment and trade actions.

Divided Case

Consider the divided case, where the buyer has the trade action in each period. We examine contracts satisfying $m^t \geq 0$ for all t, due to our normalization that equilibrium payoffs in state L are zero. In state H it is rational for the buyer to install ($a^t = 1$) in period t if and only if $x - m^t \geq \delta v^{t+1}_{\mathrm{B}}$, because when the buyer does not trade he gets zero in the current period and then waits for the continuation value from the start of the next period. Thus, in equilibrium, the buyer's individual behavior and continuation payoffs in state H must satisfy

$$\begin{array}{lll} a^t = 1 & \text{only if} & x - m^t \geq \delta v^{t+1}_{\mathrm{B}}, \ \text{and} \\ a^t = 0 & \text{only if} & x - m^t \leq \delta v^{t+1}_{\mathrm{B}}. \end{array} \tag{1}$$

The opportunity for renegotiation in period t implies that, in state H,

$$v^t = \underline{w}^t + \pi[x - \underline{w}^t_{\mathrm{B}} - \underline{w}^t_{\mathrm{S}}], \tag{2}$$

where

$$\underline{w}^t = \begin{cases} (x - m^t, m^t) & \text{if } a^t = 1 \\ \delta v^{t+1} & \text{if } a^t = 0. \end{cases}$$

In these expressions, $\underline{w}^t$ denotes the disagreement point of negotiation, which is the value of continuing in the current period under the original contract. The parties can achieve the maximum joint surplus by renegotiating to a contract that forces trade, so the surplus of renegotiation in state H is $x - \underline{w}^t_{\mathrm{B}} - \underline{w}^t_{\mathrm{S}}$.

In summary, a pure-strategy equilibrium is characterized by a sequence $\{a^t, v^t\}$ such that Conditions 1 and 2 hold for all t. Thus, the H-state value v is implemented by contract $\mathcal{C}$ if there is an equilibrium (combining sequential rationality and the bargaining solution) of the game from period 1 such that $v = v^1$. Consideration of mixed actions add nothing to the implementable set in the divided case. We call $\mathcal{C}$ a *simple open-ended contract* if it specifies a single price m that the buyer must pay when he installs the good, so $m^t = m$ for all t. That is, the price m is constant across periods and the contract does not expire.

We next calculate the set of implementable values and compare it to the implementable set in the benchmark model with a nondurable trading opportunity.

Proposition 3: *In the model with a durable trading opportunity and divided investment and trade actions, v can be implemented if and only if $v_{\mathrm{S}} \in [0, x]$ and $v_{\mathrm{B}} = x - v_{\mathrm{S}}$. Thus, the implementable set is the same as in the benchmark (nondurability) model. There is a simple open-ended option contract that induces efficient investment and trade.*

Proof: Consider a contract that, for some $m \in [0, x]$, specifies $m^t = m$ for all t. This is a simple open-ended contract. Note that the contract specifying m implements $v = (x - m, m)$ in the setting of a nondurable trading opportunity. We shall demonstrate that the open-ended version of the contract implements the same H-state payoff vector in the setting of a durable trading opportunity. Specify $\{a^t, v^t\}$ such that $a^t = 1$ and $v^t = (x - m, m)$ for all t. We show that $\{a^t, v^t\}$ is an equilibrium by checking that Conditions 1 and 2 hold for all t. The first condition reduces to $x - m \geq \delta(x - m)$ and so clearly holds. Note that $\underline{w}^t = (x - m, m)$ so the surplus of renegotiation is zero and the second condition holds.

To see that no other values can be implemented, let β be the maximum implementable continuation value for the seller in state H, with the value in state L normalized to zero. That is, β is the maximal implementable v_{S}. Note that, because the trading environment is stationary, β characterizes the maximum v_{S}^t for every period t (over all contracts). We can use a recursive formulation to determine β. Consider some period t and let us find an upper bound on v_{S}^t using Conditions 1 and 2, with the constraint that $v_{\mathrm{S}}^{t+1} \leq \beta$.

Note that if m^t is set to induce $a^t = 0$ in state H (for instance, if $m^t \geq x$), then the seller's payoff in state H would be $v_{\mathrm{S}}^t = \delta v_{\mathrm{S}}^{t+1} + \pi_{\mathrm{S}} x(1-\delta) < \delta\beta + \pi_{\mathrm{S}} x(1-\delta)$. If we instead wanted to induce $a^t = 1$ in state H (by specifying $m^t \leq x$), we would have $v_{\mathrm{S}}^t = m^t$. These expressions imply that v_{S}^t is maximized at $\max\{\delta\beta + \pi_{\mathrm{S}} x(1-\delta), x\}$. This maximum must equal β. If $\delta\beta + \pi_{\mathrm{S}} x(1-\delta) \geq x$ were the case then the implication is $\delta\beta + \pi_{\mathrm{S}} x(1-\delta) = \beta$, which simplifies to $\beta = \pi_{\mathrm{S}} x$. Thus we know that β must equal x, which coincides with the upper limit of values implemented by the simple open-ended contract. ‖

Unified Case

Next consider the unified case, where the seller has the trade action in each period. We examine contracts satisfying $m^t \leq 0$ for all t, due to our normalization that equilibrium payoffs in state L are zero. In fact, we can fix attention to $m^t = 0$ for all t, because the seller would not trade in either state if $m^t < 0$. Then in state H it is rational for the seller to choose $a = 1$ in period t if and only if $\delta v_{\mathrm{S}}^{t+1} \leq 0$. Thus, in equilibrium, the seller's individual behavior and continuation payoffs in state H must satisfy

$$\begin{array}{lll} a^t = 1 & \text{only if} & \delta v_{\mathrm{S}}^{t+1} \leq 0 \text{ and} \\ a^t = 0 & \text{only if} & \delta v_{\mathrm{S}}^{t+1} \geq 0. \end{array} \tag{3}$$

The opportunity for renegotiation in period t implies Condition 2 described in the previous subsection.

A pure-strategy equilibrium is therefore characterized by a sequence $\{a^t, v^t\}$ such that Conditions 3 and 2 hold for all t. We will have to examine mixed actions as well, but there is no need to complicate the exposition with the mixed-action versions of Conditions 3 and 2. A value v is implemented by contract $\mathcal{C}$ if there is an equilibrium (combining sequential rationality and the bargaining solution) of the game from period 1 such that $v = v^1$. The following result shows that we have the same relation between the settings of nondurable and durable trading opportunities in the unified case as we found in the divided case.

Proposition 4: *In the model with a durable trading opportunity and unified investment and trade actions, v is implementable if and only if $v_{\mathrm{S}} \in [0, \pi_{\mathrm{S}} x]$ and $v_{\mathrm{B}} = x - v_{\mathrm{S}}$. Thus, the implementable set is the same as in the benchmark (nondurability) model. Efficient investment cannot be achieved if $c > \pi_{\mathrm{S}} x$.*

Proof: Because we focus on contracts that specify a transfer of zero if trade does not occur in a period (and the corresponding normalization that continuation payoffs are zero in the L state), the proof here is a bit involved. In footnote 11 (at the end of this proof), we sketch a simpler method using a contract that specifies $m^t > 0$ with an up-front transfer (or transfer to the buyer if trade does not occur in a given period).

Consider a contract that specifies $m^t = 0$ for all t. We first show that this contract implements both $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (x, 0)$ and $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$. The first value arises if we prescribe $\{a^t, v^t\}$ such that $a^t = 1$ and $v^t = (x, 0)$ for all t. The second value is achieved by prescribing $a^t = 0$ and $v^t = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$ for all t. It is easy to check that Conditions 3 and 2 hold for all t in both of these specifications.

To obtain intermediate values we can construct an equilibrium in which the seller would sometimes randomize between $a = 1$ and $a = 0$. The simplest way to proceed is to assume that the players have access to a public randomization device, which they could use to obtain arbitrary mixtures of the two values just described. But we haven't assumed such a randomization device and so our construction will be a bit more involved.

Define the contract so that $m^t = 0$ for all t. Let T be a positive integer and let us prescribe that $a^t = 1$ and $v^t = (x, 0)$ for all $t > T$, so that the equilibrium continuation from period $T + 1$ gives the seller a payoff of zero. Suppose that in period T the seller would pick $a^T = 1$ with probability α, which is rational. The renegotiation surplus in period T is $(1 - \alpha)(1 - \delta)x$, implying that

$$v_{\mathrm{S}}^T = \pi_{\mathrm{S}}(1 - \alpha)(1 - \delta)x.$$

Backward induction then determines behavior and continuation values in earlier periods. In particular, for $t = 1, 2, \ldots, T - 1$ we have $a^t = 0$ and (omitting algebra)

$$v_{\mathrm{S}}^t = \pi_{\mathrm{S}} x[1 - \delta^{T-t}(\delta + \alpha(1 - \delta))].$$

The implemented value for the seller from period 1 is

$$v_{\mathrm{S}}^1 = \pi_{\mathrm{S}} x[1 - \delta^{T-1}(\delta + \alpha(1-\delta))].$$

It is not difficult to verify that for any $y \in [0, \pi_{\mathrm{S}} x]$, we can find numbers T and α so that $v_{\mathrm{S}}^1 = y$. Also, using the same logic employed in the proofs of Propositions 2 and 3, one can show that no other values can be implemented.[11] $\|$

4 Multiple Equilibrium and Unique Implementation

In this section we explore the issues of multiple equilibrium and unique implementation. We focus on the case of divided investment and trade actions, and we add footnotes to discuss the unified case.

As Proposition 3 shows, with a simple open-ended option contract, there is an equilibrium of the game with a durable trading opportunity in which the parties invest and trade efficiently. That is, the hold-up problem is completely avoided. This would seem to verify Nöldeke and Schmidt's (1998) intuition regarding open-ended option contracts. Interestingly, in addition to demonstrating that durability does not necessarily exacerbate the hold-up problem, our modeling exercise gives some support to Edlin and Hermalin's (2000) intuition about the buyer being able to credibly commit to refrain from installing the good until the contract is renegotiated. More precisely, a simple open-ended option contract may give rise to *multiple equilibria*, including one that is consistent with Edlin and Hermalin's story.

Proposition 5: *Consider a durable trading opportunity and divided investment and trade actions. An open-ended option contract specifying a price of* $m \geq (1 - \delta\pi_{\mathrm{B}})x$ *implements* both *the value* $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (x - m, m)$ *and the value* $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$*, in both cases with payoff vector* $(0, 0)$ *in state L.*

Proof: That $v = (x - m, m)$ is implemented follows from Proposition 3. To see that $v = \pi x$ is also implemented, specify $a^t = 0$ and $v^t = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$ for all t. Under the assumption $m \geq (1 - \delta\pi_{\mathrm{B}})x$, we have $x - m \leq \delta\pi_{\mathrm{B}} x$ and so Condition 1 holds for all t. Since $a^t = 0$ for all t, $\underline{w}^t = \delta v^{t+1}$ and so Condition 2 clearly holds as well. $\|$

[11] Here is a sketch of a simpler proof that utilizes an up-front transfer. Let $m^t = m \in [0, \pi_{\mathrm{S}} x]$ for all t. Clearly, this gives the seller the incentive to deliver in state L (strictly if $m > 0$). Suppose that, under the contract, the seller would never deliver in state H, so we have $a^t = 0$ for all t. Continuation values are $v^t = (\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$ for all t. It is easy to see that these specifications constitute an equilibrium for δ close enough to 1. In particular, the seller is happy to wait for renegotiation in period $t+1$ rather than trade in period t. Thus, the contract implements $(\pi_{\mathrm{B}} x, \pi_{\mathrm{S}} x)$ in state H and $(-m, m)$ in state L. An up-front transfer of m from the seller to the buyer then makes the implemented values conform to our normalization. Another way to implement the same values is to specify $m^t = 0$ but have the seller make a transfer to the buyer in the event that he does not deliver in a given period. With either method, for low values of δ we can specify randomization appropriately to achieve the desired value in state H.

If the players are patient (δ is close to one) and the buyer has substantial bargaining power relative to the seller's cost of high investment, then an open ended option with m near x supports an equilibrium in which the players divide the trade value according to their bargaining weights. In this equilibrium, unless $c < \pi_s x$, the seller will not have the incentive to invest efficiently.

In summary, stationary (constant price) contracts induce multiple equilibria for relatively high prices.[12] One of the equilibria leads to efficient outcomes, whereas another exhibits hold-up. It is important to note that the parties have an *ex ante* joint interest in selecting an equilibrium that induces an efficient outcome. Thus, if hold-up persists in environments with durable trading opportunities, it is because of adverse equilibrium selection.

Consideration of non-stationary contracts allows us to reach an even stronger conclusion. Ex ante, the buyer and the seller may wish to ensure selection of an equilibrium that gives the seller the incentive to invest efficiently. We shall demonstrate that a nonstationary option contract can be structured so that, in each state, there is a *unique* equilibrium from period 1 and, furthermore, that the seller invests efficiently. We say a contract $\mathcal{C}$ *uniquely implements* a value v if there is one and only one sequence $\{a^t, v^t\}_{t=1}^{\infty}$ that satisfies Conditions 1 and 2 for all t, and such that $v^1 = v$. This is a stronger notion of unique implementation than is typically studied, because we are insisting that the entire sequence $\{a^t, v^t\}_{t=1}^{\infty}$ be uniquely determined, rather than just v^1. We want a unique equilibrium value in state L as well, but it will trivially be the case here given the range of m^t considered and that trade yields zero in state L.

Proposition 6: *In the setting with a durable trading opportunity and divided investment and trade actions, any $v = (v_{\mathrm{B}}, v_{\mathrm{S}})$ satisfying $v_{\mathrm{S}} \in [0, x)$ and $v_{\mathrm{B}} + v_{\mathrm{S}} = x$ can be uniquely implemented (by a nonstationary contract).*

Proof: Consider any $v = (v_{\mathrm{B}}, v_{\mathrm{S}})$ such that $v_{\mathrm{S}} \in [0, x)$ and $v_{\mathrm{B}} + v_{\mathrm{S}} = x$. Select an integer T. For all $t \geq T$, let $\mathcal{C}$ specify $m^t = 0$. In state H, therefore, the buyer will exercise the option in period T (choosing $a^t = 1$) and will also do so in every subsequent period (Condition 1 is met). Clearly, continuation values in all periods $t \geq T$ are uniquely determined.

Next, define $\{m^t\}_{t=1}^{T-1}$ inductively so that, for each $t < T$,

$$m^t = \min\left\{\frac{m^{t+1}}{2} + \frac{\delta m^{t+1} + x(1-\delta)}{2}, v_{\mathrm{S}}\right\}.$$

By construction, we have

$$x - m^t > \delta(x - m^{t+1})$$

[12] Multiple equilibria with open-ended options also exist in the unified case. The construction in footnote 11 relies on selecting an equilibrium in state H where the parties renegotiate because they anticipate that the seller will not deliver otherwise. There is another equilibrium in which the seller delivers without renegotiation and πx is the implemented value in the state H.

for every t, which means that the buyer strictly prefers to install in period t rather than wait to do so in period $t+1$. Because continuation values from period T are uniquely determined, this implies (via backward induction) that continuation values in all previous period are also uniquely determined. In each period, contingent on trade not occurring earlier, the buyer will install. Letting $a^t \equiv 1$ for all t, we therefore know that $\{a^t, v^t\}_{t=1}^{\infty}$ uniquely satisfies Conditions 1 and 2.

By choosing T sufficiently large, we have $m^1 = v_{\mathrm{S}}$ and the desired value v^1 is uniquely implemented. It is clear also that $(0,0)$ is the sole equilibrium value in state L. Note that, since it would leave the buyer with a payoff of zero (and be indifferent between actions), a value with $v_{\mathrm{S}} = x$ cannot be uniquely implemented. $\|$

The proof constructs a contract that yields a unique equilibrium from every period and in which renegotiation never occurs.[13] There are simpler contracts that achieve the same objective in a less elegant way, but may have some realistic features. Consider a contract that gives the buyer the option of trading at a fixed price $m \in [0,x)$ in any period $t < T$, but then would force the buyer to trade at price m in any period $t \geq T$. From period T all continuations have a unique equilibrium in both states. One can easily use backward induction to check that the equilibrium is unique from all previous periods as well. From period 1, the payoff vector in state H is $(x-m, m)$ and the payoff vector in state L is $(\pi_{\mathrm{B}}\delta^{T-1}m, \pi_{\mathrm{S}}\delta^{T-1}m)$. Normalizing the latter to $(0,0)$ yields the same set of values described in the proposition.

Our point in this section has not been to pick sides in the debate between Edlin and Hermalin (2000) and Nöldeke and Schmidt (1998). It may be interesting that their separate intuition regarding open-ended option contracts is played out in the multiple equilibria supported by a stationary contract in the case of divided investment and trade actions. But rather, we emphasize the significant difference in implementable outcomes between the divided and unified cases. Note that the models of Edlin and Hermalin (2000) and Nöldeke and Schmidt (1998) abstract from consideration of the technology of trade; that is, they do not specify whether the divided or unified case is being perceived. We stress that it is illuminating to model the technology of trade — by treating trade actions as individual actions and, for the setting of a durable trading opportunity, by modeling the time periods.

[13]Unique implementation in the case of unified investment and trade actions is more challenging because, in our simple model, the seller's delivery cost does not vary with the state. Thus, the seller's preferences regarding the trade action are exactly the same in both states. However, it is still possible to uniquely and approximately (within a small error) implement the set of values delineated in Proposition 4, for δ close to 1. For instance, the value $(v_{\mathrm{B}}, v_{\mathrm{S}}) = (\pi_{\mathrm{B}}x, \pi_{\mathrm{S}}x)$ can be uniquely implemented using a contract that forces no trade in each period. More complicated contracts (for example, ones that force no trade up to some period T and then force trade afterward) can implement values that give the seller between zero and $\pi_{\mathrm{S}}x$ in state H.

5 Comments on the Outside Option Principle

Many authors have noted the relationship between contracting models with renegotiation and models of bargaining with outside options. For example, Edlin and Hermalin (2000) appeal to the *outside option principle*, which states that, with some bargaining protocols, if a player has an outside option then it affects the outcome of negotiation only if this player's value of taking the outside option exceeds his equilibrium value in the setting without the outside option. In other words, the option serves as a constraint on the player's payoff, in that the player with the option must get at least his outside option payoff. If this payoff is less than the subgame perfect equilibrium payoff when there is no outside option, then any threat to opt out is non-credible and it does not affect the bargaining.

The outside option principle is motivated by a non-cooperative bargaining game, but in fact the set of equilibrium outcomes is very sensitive to assumptions about the timing of the option.[14] Also, it is typically assumed that a single player has an outside option which, if taken, gives a positive payoff only to this player. In contrast, in the contract renegotiation under investigation here, opting out means exercising the contractual option (by installing the good) which generally gives positive payoffs to *both* players. In fact, exercising the option yields a payoff vector that is on the efficient frontier.

We next show that the outside option principle is generally invalid in settings where the outside option yields close to an efficient outcome. More precisely, when the outside option is close to efficient but gives little to the player with the option, there are multiple equilibria. One equilibrium yields the payoff vector predicted by the outside option principle, but another equilibrium yields a payoff vector that is close to that of the outside option. This helps to explain why the outside option principle offers incomplete intuition regarding the properties of equilibrium in contractual settings with renegotiation and durable trading opportunities.[15]

Consider a simple model of bargaining in which player 1 and player 2 are bargaining over a surplus of size one. In each period one of the players is randomly selected to propose a division of the surplus. If the other party accepts the proposal, the game ends and the parties get the proposed payoffs. If the proposal is rejected, then player 1 has an opportunity to take an outside option and end the game. If the proposal is rejected and the outside option is not taken, the interaction repeats in the next period. Players discount future periods using discount factor δ. The probability that player 1 gets to make the offer in any given period is π_1, whereas π_2 is the probability that player 2 gets to make the offer. If player 1 takes the outside option, he gets a payoff of w_1 and player 2 gets a payoff of w_2.

The bargaining model described here assumes that the offers made by the parties are efficient (the proposed payoffs sum to one). In addition, we focus on equilibria in which the outcome is efficient from the start of any given period. More generally, there may exist

[14] For an analysis of the different cases, see Osborne and Rubinstein (1990).

[15] The point we are making here is distinct from the message of Avery and Zemsky (1994). They show that in various examples in the bargaining literature, multiple equilibria arise because there is an outside option that yields an inefficient outcome. See also Evans (2008) on this idea.

equilibria with delay (and therefore equilibrium payoffs that sum to less than one).[16] The following two results characterize the equilibrium payoffs; the first result, which illustrates the basic idea of the outside option principle, is a special case of the second. See Figure 1 for a graphical depiction. Define $p^* \equiv \delta\pi_1/(1-\delta\pi_2)$.

Result 1: *Suppose $w_2 = 0$. Then:*

1. *For $w_1 < \delta\pi_1$, the unique equilibrium payoff vector is $\pi = (\pi_1, \pi_2)$.*
2. *If $\delta\pi_1 \leq w_1 \leq p^*$ then, for all*

$$z \in \left[w_1 + (1-\delta)\pi_1, \frac{p^*}{\delta}\right],$$

   there exists an equilibrium with payoffs $(z, 1-z)$.

3. *For $p^* < w_1$, the unique equilibrium payoff vector is $(\pi_1 + \pi_2 w_1, \pi_2(1-w_1))$.*

Result 1 demonstrates the basic idea behind the outside option principle: the ability to opt out only affects the outcome of the bargaining when the value of the option is greater than the bargaining outcome in the game without an outside option (that is, when $w_1 > \delta\pi_1$). Settings of contract renegotiation, however, generally do not have the property that $w_2 = 0$. The following result describes the equilibria with general outside option payoff vectors.

Result 2: *Consider any $w = (w_1, w_2)$.*

1. *(Region A) If $w_1 \leq \delta\pi_1$ and $w_1 + p^* w_2 \geq p^*$, then for all*

$$z \in \left[\pi_1(1-w_2) + \pi_2 w_1, \pi_1\right],$$

   there exists an equilibrium with payoff vector $(z, 1-z)$.

2. *(Region B) If $w_1 > \delta\pi_1$ and $w_1 + p^* w_2 > p^*$, then the unique equilibrium payoff vector is $(\pi_1(1-w_2) + \pi_2 w_1, \pi_1 w_2 + \pi_2(1-w_1))$.*
3. *(Region C) If $w_1 < \delta\pi_1$ and $w_1 + p^* w_2 < p^*$, then the unique equilibrium payoff vector is π.*
4. *(Region D) If $w_1 \geq \delta\pi_1$ and $w_1 + p^* w_2 \leq p^*$, then for all*

$$z \in \left[w_1 + (1-\delta)\pi_1, \frac{p^*(1-w_2)}{\delta}\right],$$

   there exists an equilibrium with payoff vector $(z, 1-z)$.

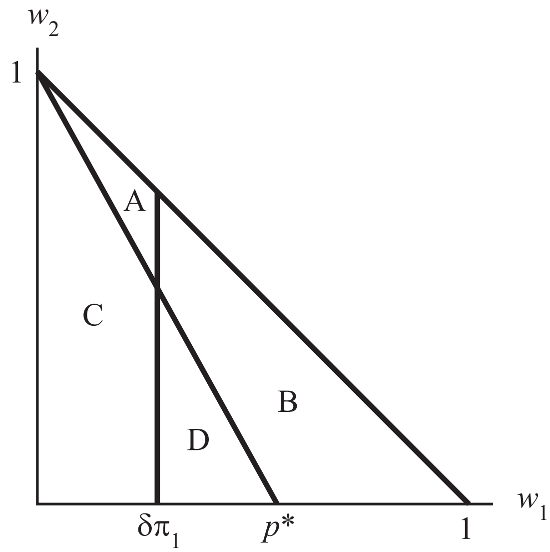

Figure 1: Regions of the space of outside-option payoff vectors.

To understand the implications of this result, consider a setting in which an efficient outcome occurs if player 1 takes the outside option; that is, $w_1 + w_2 = 1$. Recognize that this is essentially the contractual situation evaluated in the divided case of Section 3 with the contract specifying $m \in [0, x]$. Refining further, consider the case in which $w_1 < \delta\pi_1$ (which corresponds to having m close to x in the contracting model). Then Region A is the relevant region of Result 2 and we have a continuum of equilibrium payoff vectors. There is an equilibrium in which player 1's payoff z is equal to w_1, another equilibrium in which player 1's payoff is $z = \pi_1$, and a continuum of equilibria between (that is, each $z \in [w_1, \pi_1]$ can be supported in an equilibrium).

This analysis demonstrates that arguments about the outside option principle, for example by Edlin and Hermalin (2000), are implicitly arguments about equilibrium selection. Since player 2 gets a positive payoff if player 1 takes the option, and because the sum of the players' outside option payoffs equals the whole surplus, Result 2 applies and there exists an equilibrium of the bargaining game (specifically, with $z = w_1$) that gives the seller a sufficient ex ante incentive to invest. Edlin and Hermalin's story possibly relates to a different equilibrium of the bargaining game, in which $z = \pi_1$.[17]

[16] In particular, equilibria with delay will arise in the cases with multiple equilibria described below.

[17] Wickelgren (2007) also appeals to the outside option principle in his analysis of buyer-option contracts. He asserts that the seller's valuation of the outside option does not affect the set of equilibrium payoffs, which is contrary to what we find in Result 2. In addition, his discounting rule is equivalent to an assumption that the outside option yields a payoff vector on the line $w_1 + w_2 = \delta$, meaning that by renegotiating the players somehow can trade sooner. Following Watson (2007), we view it more realistic to consider that the trade action takes place at a fixed point in time, whether or not renegotiation occurs before.

6 Conclusion

We have shown for a model of unverifiable investment that durability of the trading opportunity does not contribute to the hold-up problem. In fact, simple open-ended option contracts achieve in the durability environment what their static counterparts achieve in the nondurability environment. Furthermore, nonstationary option contracts can be utilized to uniquely implement the desired outcome. In the end, the major determinant of whether efficient investment and trade can be achieved is the nature of the trade action. Efficiency is obtained in the case of divided investment and trade actions, whereas it is not generally obtained in the unified case.

We demonstrated that, under some stationary contracts (for example, with m close to x in the divided case) multiple equilibria exist in the continuation from period 1 in state H. This multiplicity was not useful in expanding the set of implementable value functions in our setting, but, as Evans (2008) shows, it may be useful more generally. In fact, this can be demonstrated in a simple variant of our model in which both the buyer and the seller have investment choices. Suppose that the parties simultaneously choose between low and high investment, and let c_{B} and c_{S} be the buyer's and seller's costs of high investment. Assume that he investment choices are commonly observed but not verifiable. Let θ be either H or L as before, which indicates whether the buyer's benefit of trade is x or 0; this as well is unverifiable. Assume that if both players invest high then $\theta = \mathrm{H}$ for sure, whereas $\theta = \mathrm{L}$ for sure if they both invest low. If one player invests high and the other invests low, then $\theta = \mathrm{H}$ with probability q. Assume that $x > c_{\mathrm{B}} + c_{\mathrm{S}}$ and $(1-q)x > c_{\mathrm{B}}, c_{\mathrm{S}}$, which means that it is efficient for both players to invest high and then for trade to occur.

In the example just described, the state of the relationship is given by the players' investment choices and θ. Clearly, to motivate the players to invest high, we want to differentiate between cases in which $\theta = \mathrm{H}$ but one of the player cheated by investing low. We can use the equilibrium multiplicity to do this. For example, take a contract that specifies m very close to x. In the event that $\theta = \mathrm{H}$ we know there are two equilibria from period 1. In one of the equilibria, the buyer obtains a payoff of $x - m \approx 0$, whereas in the other the buyer gets $\pi_{\mathrm{B}} x$. It enhances the buyer's incentive to invest when the players plan to coordinate on the latter equilibrium if the buyer invests high and on the former equilibrium if the buyer invests low.

So, in a more general version of our model, if one does not insist on unique implementation, we expect that a larger set of value functions can be implemented in the environment of a durable trading opportunity than can be implemented in the nondurability environment. An interesting topic for future research would be to characterize the set of uniquely implementable value functions in a broader class of contractual relationships (with messages) and compare it to the implementable set in the nondurability model.[18]

[18] Evans' (2008) use of a financial hostage may raise the question of whether such arrangements can operate well in reality. On this subject, Baliga and Sjöström (2009) have suggested conditions under which third-party budget-breaking schemes can withstand collusive side deals. Bull (2009) shows that these schemes fall apart if side contracts can condition on messages sent in the original contract.

In our model of trade, installation by the buyer ends the interaction. This action, therefore, is not reversible. That is, once trade has occurred, the parties cannot revert to the no-trade outcome. It would be interesting to examine a more general model with reversible trade actions and analyze how differential reversibility affects the scope of contracting and the hold-up problem. A small step in this direction is taken in Watson (2005). On a related note, one might wonder about variations of durability. For example, instead of the buyer's benefit of trade being constant across time (so discounting is the only cost of waiting), one could imagine that the benefit x declines each period by a fixed amount, until it reaches zero. Consider the divided case, where the buyer has the trade action. It should be clear that a stationary open-ended option contract will not perform well in this setting, because eventually the buyer will essentially be forced to not trade and will extract a share of the benefit in renegotiation. On the other hand, our results extend using nonstationary contracts in which the option price tracks the buyer's trade benefit over time.

A Appendix - Proof of Outside Option Results

Result 1 follows immediately from Result 2 (setting $w_2 = 0$), so we proceed to prove Result 2. The proof uses the standard technique of constructing relations between bounds on equilibrium payoffs in various subgames. We focus on equilibria in which the outcome is efficient in the continuation from the start of any given period.

Let $\overline{\eta}$ and $\underline{\eta}$ be the maximum and minimum of player 1's payoff from the beginning of a period, over all efficient subgame perfect equilibria in the game from this point. (The construction will establish that the bounds are met, so the maximum and minimum exist.) Let $\overline{\gamma}$ and $\underline{\gamma}$ be the maximum and minimum of player 1's continuation values from the stage at which he has the opportunity to take the outside option. Similarly, let $\overline{\mu}$ and $\underline{\mu}$ be the maximum and minimum of player 2's continuation values from the option stage. Then we have

$$\overline{\eta} = \pi_1(1 - \underline{\mu}) + \pi_2\overline{\gamma} \quad \text{and} \quad \underline{\eta} = \pi_1(1 - \overline{\mu}) + \pi_2\underline{\gamma}. \tag{4}$$

For intuition, consider the first of these and note that when player 1 is selected to make the offer, the best equilibrium outcome for him is to hold player 2 down to her worst equilibrium outcome, which gives player 1 the payoff $1 - \underline{\mu}$.

Our analysis continues by separately examining three cases having to do with whether player 1 has the incentive to take the outside option in a given period:

Case 1: $w_1 > \delta\overline{\eta}$, so player 1 always takes the outside option.

Case 2: $w_1 < \delta\underline{\eta}$, so player 1 never takes the outside option.

Case 3: $w_1 \in [\delta\underline{\eta}, \delta\overline{\eta}]$, so player 1 may take the outside option following some histories and forego the outside option following others.

In Case 1, the equilibrium continuation values are nailed down uniquely: $\overline{\gamma} = \underline{\gamma} = w_1$, $\overline{\mu} = \underline{\mu} = w_2$, and

$$\overline{\eta} = \underline{\eta} = \pi_1(1 - w_2) + \pi_2 w_1.$$

Define $p^* = \delta\pi_1/(1-\delta\pi_2)$. Note that, by substituting for $\overline{\eta}$, the presumption for this case ($w_1 > \delta\overline{\eta}$) becomes the necessary condition $w_1 + p^* w_2 > p^*$.

The continuation values are also uniquely determined in Case 2. We obtain $\overline{\gamma} = \delta\overline{\eta}$, $\underline{\gamma} = \delta\underline{\eta}$, $\overline{\mu} = \delta(1-\underline{\eta})$, and $\underline{\mu} = \delta(1-\overline{\eta})$. The presumption for this case becomes the necessary condition $w_1 < \delta\pi_1$.

Case 3 is more complicated. Considering player 1's incentive in the option stage of a period, we see that the best equilibrium continuation for player 1 from the option stage is to forego the outside option and get $\overline{\eta}$ from the start of the next period. Thus, we have $\overline{\gamma} = \delta\overline{\eta}$. Likewise, the worst continuation has player 1 receiving $\underline{\eta}$ from the next period, which would motivate player 1 to take the outside option, implying $\underline{\gamma} = w_1$. Player 2's worst and best continuation values from the option stage will depend on the relative magnitude of w_2:

$$\overline{\mu} = \max\left\{w_2, \delta\left(1 - \frac{w_1}{\delta}\right)\right\}$$
$$\underline{\mu} = \min\{w_2, \delta(1-\overline{\eta})\}$$

Here, attaining the bounds requires a selection over equilibria in the continuation from the next period.

We continue the analysis of Case 3 by exploring the possibilities for w_2. The inequality $w_1 \leq \delta\overline{\eta}$ implies that $\delta(1-\overline{\eta}) \leq \delta(1 - w_1/\delta)$, and therefore there are three subcases to consider:

Subcase 3a: $w_2 \leq \delta(1-\overline{\eta})$.

Subcase 3b: $\delta(1-\overline{\eta}) < w_2 < \delta\left(1 - \frac{w_1}{\delta}\right)$.

Subcase 3c: $\delta(1 - \frac{w_1}{\delta}) \leq w_2$.

Working through the straightforward implications of these inequalities, we obtain the following. In Subcase 3a, we have $\overline{\mu} = \delta - w_1$ and $\underline{\mu} = w_2$. Substituting for $\overline{\mu}$, $\underline{\mu}$, $\overline{\gamma}$, and $\underline{\gamma}$ in equations 4 and solving for $\overline{\eta}$ and $\underline{\eta}$ then yields

$$\overline{\eta} = \frac{p^*(1-w_2)}{\delta} \quad \text{and} \quad \underline{\eta} = w_1 + (1-\delta)\pi_1.$$

The presumptions for Case 3 and Subcase 3a translate into the necessary conditions $w_1 \geq \delta\pi_1$ and $w_1 + p^* w_2 \leq p^*$. (The inequality defining Subcase 3a is implied by these two inequalities.)

Subcases 3b and 3c are handled the same way. In Subcase 3c, we obtain $\overline{\eta} = \pi_1$ and $\underline{\eta} = \pi_1(1-w_2) + \pi_2 w_1$ with necessary conditions $w_1 \leq \delta\pi_1$ and $w_1 + p^* w_2 \geq p^*$. (The inequality defining Subcase 3c is implied by these two inequalities.) Subcase 3b turns out to be vacuous because solving for $\overline{\eta}$ and $\underline{\eta}$ and substituting in the presumptions for this subcase yields a contradiction.

Consider the four regions described in Result 2. Note that Region A satisfies the necessary conditions for Case 3c. In addition, the interior of Region A satisfies the necessary conditions for Cases 1 and 2 which identify equilibrium payoffs that are contained in the set identified by Case 3c. Thus, Case 3c gives the set of equilibrium payoffs for Region A. Region D is consistent with the necessary conditions of Case 3a and only this case. Region B is consistent with the necessary conditions of only Case 1, and Region C satisfies the necessary conditions of only Case 2. These facts imply the results.

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