Testing and the Composition of the Labor Pool

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We consider a continuous-time labor matching model with endogenous separation. Firms initially lack information about the quality of workers with whom they are matched. They acquire the necessary information from one of two sources: screening and on-the-job performance. Prospective employees are screened through a costly testing process. The test provides a valuable, but imperfect, signal. The accuracy of the signal can be improved by devoting more resources to testing. Firms can also learn about their employees by observing their performance after they have been hired. Workers who perform poorly on the test are not offered employment; those who perform poorly on the job are eventually fired (after some delay). Worker quality is not match-specific; low quality workers are less productive with all firms. We show that, in equilibrium, there is an inverse and complementary relation between the level of testing that firms optimally perform and the overall quality of the workers in the matching pool. If the matching pool is predominantly comprised of poor quality workers, then firms have the incentive to test workers more rigorously and consequently will be more selective in their hiring decisions. Since workers who test poorly are quickly returned to the matching pool, more rigorous testing implies a lower average quality of workers seeking employment. We consider the properties of a steady-state, stable equilibrium in such an environment. The complementarity between testing and the composition of the unemployed pool introduces the possibility of multiple equilibria.

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I. Introduction

Firms spend considerable resources meeting their labor requirements. Placing advertisements, reviewing applications, conducting interviews, training, and so on, all consume scarce resources in advance of the firm receiving any productive benefit from a new employee.[1] The need to expend these resources is partly due to frictions in the labor market that make it difficult to find available workers. These "search frictions" have received much attention in the economics literature (Diamond 1982, and subsequent work).

This paper considers an exacerbating difficulty: that, in settings where workers are of heterogeneous quality, firms are compelled to gather information about prospective employees before hiring them. Because qualified employees are more productive than are under-qualified ones, a bad hire may be very costly to the firm.[2] Thus, it is important for managers to determine whether or not a given applicant is qualified for a vacant job. The more resources devoted to screening and testing applicants, the better information the manager will have about potential hires. Examples of the sort of "testing" we have in mind would include subjecting applicants to background checks, multiple interviews, drug testing and/or formal, written examinations.

Naturally, the decision of how much to spend on this sort of testing will depend on the

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[1]Many of these costs may not be explicitly recognized. This does not imply that such costs are trivial. For example, Payne (2003) finds that the average cost to hire a single new employee for a medical practice was $1,580.

[2]For example, Mornell (1998) estimates that — as a rule of thumb — even relatively quickly rectified bad hires can cost a company two and a half times the employee's annual salary. Regardless of the precise cost of a bad hire, it seems that this cost will greatly outweigh the simple cost of hiring a replacement in many environments. In contrast to the $1,580 cost of a new hire reported above, Payne (2003) finds that the average cost to terminate and replace a bad hire cost a medical practice $68,112.

expected quality of the applicant pool. Furthermore, a consequence of a successful screening effort is that those applicants who are not offered positions will return to the ranks of the job-searching unemployment pool. This feedback mechanism has consequences for the quality composition of those remaining unemployed, and thus future testing decisions.

We investigate the properties of the feedback between firms' testing expenditures and the quality of the pool of unemployed workers. In what follows, we introduce a matching model in which workers differ in their abilities (skills) and firms endogenously determine how much effort and cost to devote to screening (testing) job applicants. We depart from some of the literature on matching labor markets in that we are interested in the quality of the *worker* rather than the quality of the firm-specific *match*.[3] While clearly both firm-specific compatibility and innate ability are important in evaluating a worker's potential productivity, we focus on the latter because of the implications for the compositions of the labor pool.[4]

The output of the test administered to a job applicant is an imperfect signal about the applicant's abilities. The accuracy of this signal depends on the amount the firm spends on the test. For example, a bad hire might be avoided by a second interview or a drug test, even if the applicant made a good impression in a primary interview. Under-qualified workers are sent back to the unemployment pool after poor test results, which introduces a complementarity between the average quality of the unemployed workers and the firm's testing decision: those not offered employment lower the quality of those remaining unemployed, which, in turn, raises the benefit to firms from additional testing. Additional testing more effectively weeds out the under-

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[3]For examples of search models which address match quality, see Jovanovic (1979) or Diamond (1982).

[4]Note that when firms only care about match quality, a bad match with one firm has no bearing on that worker's match quality with another firm, i.e., future matches.

qualified and further lowers the quality of those remaining unemployed.

To explore the relation between pre-employment testing and the composition of the matching pool of workers, we analyze a simple model of matching and employment in which firms engage in costly testing of newly matched workers before deciding whether to make job offers. We characterize the steady-state equilibria of the model and provide comparative statics for stable equilibria. This permits us to investigate the impact of environmental changes such as a reduction in the cost of testing applicants due to, for example, the emergence of Internet-based testing services.[5] The complementarity of testing and the composition of the matching pool leads to the possibility of multiple stable, steady-state equilibria. That is, for some fundamentals, there can be both an equilibrium with high testing and a low average skill of workers in the matching pool, as well as an equilibrium with low testing and a high average skill in the matching pool.

The model therefore provides a theoretical explanation of significant variations in testing across nearby geographical regions that may have similar fundamentals.[6]

Related Literature

Several components of our modeling exercise have appeared elsewhere in the economics

[5]For example, in a recent survey, the Aberdeen Group reports that "[o]n line assessment tools are becoming integral to successful senior management recruitment...." ("Web-Based Pre-Employment Assessments Help Firms Hire Better Candidates, Retain Stronger Workforce, Says AberdeenGroup," July 8, 2005 Press Release, available at http://www.aberdeen.com/press/releases/07-07-05_PreHire.asp.)

[6]An example may be the dire economic conditions in many U.S. central cities relative to those in nearby suburbs. Holzer (1996) finds that screening levels are higher for inner city job applicants relative to other areas. Holzer partially attributes the discrepancy between the inner city and suburban labor markets to racial discrimination. While surely racism in the labor and housing market exacerbates these difficulties, our model offers a complimentary explanation: that inner city labor markets have more rigorous screening and, as a result, a lower quality matching pool and higher unemployment.

literature. Jovanovic (1979) offers a seminal treatment of matching labor markets. Other examples of matching models include Pissarides (1994) and Mortensen and Pissarides (1994). In this literature, a poor match with one firm has no bearing on whether a worker will enjoy a better fit at another firm. This independence eliminates the need to consider any feedback mechanism.

Malueg and Xu (1997) introduce a model where firms test employees to learn about their abilities and the test quality is endogenous. However, the firm's optimal test quality does not generally depend on the proportion of skilled workers among the unemployed. There is also no feedback mechanism. Workers whom the firm believes to be unskilled are offered unskilled positions rather than being separated from the firm.

MacDonald (1982) considers a model in which firms sequentially accumulate information (signals) about their workers' types. Firms ultimately assign workers to one of two jobs based on the available information. Those whom the firm knows more about are more easily assigned and more highly paid. Workers with little experience or conflicting signals are paid less.

The work that is most closely related to our model is that of Acemoglu (1999). In Acemoglu's model, firms react to the composition of the labor pool by adjusting the type of job offered. The firm irreversibly selects what *type* of job opening it will offer before meeting with any potential employees. The goal of Acemoglu's paper is to explain the growing wage differential between more educated and presumably more skilled workers and those who are less so. There is the possibility of a "pooling" equilibrium, where all jobs are suitable for any worker. Under different circumstances -- e.g., a skills biased technological change -- a "separating" equilibrium can emerge, where the firm will only accept certain types of workers. The idea is that as firms offer more demanding positions, workers whose skill level had previously been sufficient find themselves ill-equipped to compete for the available work. An important distinction between Acemoglu's model and ours is that Acemoglu assumes that once a firm is matched with a worker, the firm is instantly able to determine whether or not that worker is suitable for the type of job offered. Therefore, there is no need to test employees in Acemoglu's model. We build on Acemoglu's work by constructing a model with a similar framework that focuses specifically on the firm's testing decision, rather than the firm's "investment" choice.[7] This allows us to derive conclusions about the distinction between testing and observing "on-the-job" performance, which is our focus.

II. The Model

In this section we introduce a continuous-time model in which workers differ in their innate skills. The development of the model here is interspersed with analysis, culminating in the conditions for equilibrium.

Workers, Firms, and Matching

Firms and workers interact in continuous time, and we assume that all agents are risk neutral and discount the future at rate *r*. There is an infinite mass of infinitely-lived firms and a unit mass of workers. Each firm has a single job, which can be vacant or filled in a given instant of time. If the firm has a vacant job and wishes to locate a worker, the firm must pay a flow cost

[7] In effect, in our model all firms open the same type of job which is directed towards higher skilled workers.

of $a > 0$ to be in the matching market. Matching occurs with frictions, in that firms and workers are not matched instantaneously; rather matching takes place via a Poisson process in which λ^f is the flow rate at which firms are matched with workers, and λ^w is the rate at which workers are matched with firms. These rates depend on the masses of firms and workers in the matching pool (which are endogenously determined in equilibrium). Specifically, to keep the model simple, we assume that the flow of new matches is given by

$$m(V, U) = \theta[\min\{V, U\}],$$

where V is the mass of firms in the matching market (vacancies), U is the mass of workers in the matching market (unemployment in the modeled sector), and θ is an exogenous parameter. Then the flow rates λ^f and λ^w are given by:

$$\lambda^f = \frac{m(V,U)}{V} \tag{1}$$

and

$$\lambda^w = \frac{m(V,U)}{U} \quad . \tag{2}$$

Once a match has occurred, neither the worker nor the firm can consider other employment options until they separate. Workers, regardless of employment status, retire at flow rate q and new workers enter the market at the same rate, so the mass of workers remains at one over time.

Suppose that there are two types of workers: high and low. The inflow of new workers consists of a fraction ρ^0 who are high type and a fraction $1-\rho^0$ who are low type, so the fraction of high-type workers in the economy is always ρ^0 (although the fraction of high types in the matching pool may be different). High-type workers are more productive than are low types. We assume a production technology whereby the firm would always prefer not to hire a worker known to be of low type. Specifically, high-type workers produce a flow value of $z_H > 0$ to the firm, whereas low types produce a flow value of $z_L < 0$.[8] If firms could perfectly determine the quality of a potential hire, then low types would never be employed. Thus, firms are interested in testing to avoid hiring low types.

Testing and Employment

When a firm matches with a worker, the firm has an opportunity to conduct a test instantaneously, before deciding whether to offer the worker a job. The firm decides a testing level $\alpha \in [0,1)$ at cost $c(\alpha)$, which is strictly increasing. The test delivers a signal $\sigma \in \{H, L\}$ with the obvious interpretation that $\sigma = H$ is a favorable indication that the worker is a high type. A test of level α will correctly identify a low-type worker with probability α in the sense that

$$\text{Prob}\,[\sigma = L \mid \text{low type}] = \alpha.$$

We assume that high types always "pass" the test, meaning that

$$\text{Prob}\,[\sigma = H \mid \text{high type}] = 1.$$

Note that it is possible for a low-type worker to generate the signal $\sigma = H$ (this happens with conditional probability $1 - \alpha$), but this becomes less likely as the firm spends more on testing. Based on the test result, the firm decides whether or not to offer employment to the worker.

The outcome of the test is privately observed by the worker and firm and is not observed

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[8] It is thus costly for a firm to make a bad hire. In addition to the opportunity cost of not hiring a more capable worker, there are costs associated with training, supervising and terminating that raise the cost of hiring workers that are ill-suited for the duties they were hired to perform. Our model would operate the same way if z_L were positive; however, it is convenient to assume that this parameter is negative because it rules out an equilibrium in which the firms are happy to hire either type of worker and do no testing.

by other outside firms.[9] In fact, if the worker is not hired then other firms do not even observe that the worker was matched at all. Also, we assume that firms either do not observe, or at least do not take into consideration, the worker's age when deciding the testing level.[10] We assume that the testing cost function c is differentiable, with $c'(0) = 0$, $c'' > 0$, and

$$\lim_{\alpha \to 1} c(\alpha) = \infty.$$

This will guarantee that firms want to test at a positive level contingent on matching with workers.

At the conclusion of the test, the firm must decide whether to hire the worker. Rationality dictates that upon obtaining the signal $\sigma = L$, the firm immediately terminates the relationship. A worker who is not offered employment return to the matching pool, where he is indistinguishable from the other workers there (because other firms do not learn that he was matched and they do not see the outcome of the test). Conversely, if the signal is $\sigma = H$, then the worker is offered a job. To see that this hiring rule will be optimal, note that otherwise it would not make sense to devote any resources to testing.

Once the worker is hired, the worker and firm proceed to negotiate a wage to divide the expected surplus arising from their employment relationship. That is, the firm and the worker share a quasi-rent resulting from the advantage of continuing their relationship and producing,

[9]This assumption can be justified by noting that neither the firm nor the worker has the incentive to divulge a poor test result: the firm will not want its competitors free riding on its costly information acquisition; similarly, workers will not want to diminish their prospects with another potential employer.

[10]One way to motivate this assumption is to posit that each firm has to choose its screening technology before matching with a worker, because it takes more time to develop the test than is available between when the firm matches with a worker and when the test must be administered. Furthermore, a fixed cost of the test makes it impractical for a firm to design several versions of the test to be given to workers of different ages. Another motivation for our assumption would be that legal rules constrain firms from age discrimination. In the Conclusion, we comment on the alternative setting in which the firms can discriminate on the basis of age.

relative to their respective alternatives. Let w denote the flow wage resulting from this negotiation. The wage determination process is discussed below.

Production takes place immediately after wage negotiation. Note that, despite the firm's effort to screen the applicant, with positive probability a low-type worker will be hired and will eventually be revealed as a low type. However, we assume that it is impossible to instantaneously rectify this situation because the firm cannot immediately release a worker. Specifically, we assume that the firm must retain a new hire for a time period of at least $T > 0$. To build intuition, suppose that the job entails participation in an important task or project which will require a time period of at least T to complete. Once an employee is hired, he must be retained for the duration of the project.[11]

We assume that the entire market — both the hiring firm and all other firms — learn the type of the worker who is employed, by observing this worker's performance on the project. Thus after the period T of employment, the worker's type becomes common knowledge. At this point, the firm will fire the low-type worker. After being fired, the low-type worker has no recourse but to leave the labor market we are analyzing and seek employment in some other (un-modeled) sector, the value of which we take to be zero. The worker does not return to the matching market because there is no demand for his services. In effect, he is "branded" as low type by his poor performance. Conversely, the high-type worker who remains on the job for time period T will benefit from other firms observing the outcome of production. We assume that a worker who is commonly known to be a high type does not have to return to the matching

[11] Note that some delay is essential for any testing to occur, because with $T = 0$ there would be no penalty for hiring low-type workers. The firm could simply hire any workers with whom it came into contact; after instantaneously learning of the worker's ability, the firm would be free to terminate the employment if the worker is found to be unproductive.

market to find employment. Competition for his services will permit him to earn his full value z_H in production.[12]

To summarize, information based on production is common knowledge in the market, whereas the information content of a test result is private information. This information structure is motivated by the observation that it is difficult to determine if an applicant has been tested (interviewed) by a competitor and subsequently not offered a job, but employment experience is more readily observable.

Firms looking to match with, and subsequently test, workers will be primarily concerned with the quality of the workers in the matching pool. Let ρ denote the proportion of workers in the matching pool who are of the high type. It will generally be the case that $\rho \neq \rho^0$. For intuition, note that workers enter the matching pool from two distinct sources: new entrants to the labor market, ρ^0 of which are high-type (all "births" are initially unmatched), and those workers whom are returned to the unmatched pool following an inadequate test result. The latter will be comprised entirely of low-types, as high-types always perform well on the test. Some low-types will leave the unmatched pool, but how many will depend on a firm's choice of testing level. Of course, the firm's testing decision will likewise depend on the expected quality of the available workforce.

We are now in a position to precisely specify the firm's problem of testing. Let v be the ex ante value which the firm derives from searching for a worker. In order to attract the attention of potential workers in the matching market, firms seeking an employee must pay the flow posting cost a while searching. Let $\hat{v}$ be the expected value to the firm from a match with a

[12]Specifically, if we define $\tilde{w}$ as the wage for a known, high-type worker, then we have $\tilde{w} = z_H$ in equilibrium.

worker, conditional on that match having taken place. Considering the rate of matches and the discount rate, we thus have

$$v = \frac{\lambda^f \hat{v} - a}{r + \lambda^f} \quad . \tag{3}$$

Once a match has occurred, the firm will select a testing level α to maximize $\hat{v}$, where

$$\begin{aligned} \hat{v} = \rho\left[\left(1 - e^{-(r+q)T}\right)\left(\frac{z_H - w + qv}{r+q}\right) + e^{-(r+q)T} v\right] \\ + (1-\rho)\,(1-\alpha)\left[\left(1 - e^{-(r+q)T}\right)\left(\frac{z_L - w + qv}{r+q}\right) + e^{-(r+q)T} v\right] + (1-\rho)\alpha\, v - c(\alpha). \end{aligned} \tag{4}$$

The validity of this expression relies on the firm wanting to hire the worker if the test result is $\sigma = H$, which will be the case in the equilibria we analyze. The first term of expression (4) is the probability of matching with a high-type worker multiplied by the continuation payoff to the firm from such a match. The continuation payoff comprises two terms. The first term is the firm's expected payoff over the initial T time period following a match and a successful test result. Here, while the worker remains employed, the firm pays wage w and receives output z_H. The worker quits during this period at probability rate q, in which case the firm returns to the matching market and earns v. The final term in the firm's continuation payoff resulting from a match with a high-type worker is the firm's expected return from time period T forward. We assume that while firms do not have to advertise to find known workers, they may return to the matching market to find new workers because they cannot earn a positive return on those "known" high-type workers with time period T experience.[13]

The second term of expression (4) is the (ex ante) expected consequence of an inaccurate test result. The probability of this event is the probability of the joint event of a worker being of low-type and passing the test.[14] The continuation payoff is the same as the high-type case, except that low-types yield output z_L. The third term is the analogous payoff resulting from an accurate test administered to a low-type worker.[15]

We assume that there is no barrier to entry for firms, so new firms will enter until the expected value of finding a worker is exactly offset by the advertising cost. This implies that in equilibrium we must have

$$v = 0. \tag{5}$$

To bolster intuition for (5), note that if it were profitable ($v > 0$), then new firms would surely enter the market, leading to a reduction in λ^f. Clearly, a functioning market relies on $\hat{v} > 0$. In effect, firms require a strictly positive value over the first T of employment time in order to offset the cost of advertising and its inadvertent employment of low-type workers. The firm tests potential new hires to reduce the latter cost and protect its quasi-rent.

That $v = 0$ simplifies the analysis of the firm's choice of α to maximize $\hat{v}$. It will be useful to characterize how the optimal testing level α^{**} depends on the high-type proportion ρ of

[13] In equilibrium, firms will be indifferent between offering these workers $\tilde{w} = z_H$ and returning to the matching market. In either case, their expected return will be zero.

[14] Note that Prob[low-type $\cap$ $\sigma = H$] = Prob[$\sigma = H$ | low-type)Prob[low-type] = $(1-\alpha)(1-\rho)$.

[15] Note that an individual firm will take the relevant labor market quality, ρ, as given in its selection of the optimal testing level. Of course, in equilibrium, the firms' aggregate testing decisions will be instrumental in the determination of ρ.

the matching pool. To do this, we begin by characterizing the equilibrium wage level, w. We assume that the wage is determined by a bargaining process which takes place directly after the conclusion of testing. As noted above, workers who test poorly are not offered employment. Those with $\sigma = H$ are entitled to a fraction β of the flow surplus generated by their *employment*. Let $\tilde{\rho}$ be the probability that a worker is of high type given that he has received $\sigma = H$. We have:

$$\tilde{\rho} = \frac{\rho}{1-(1-\rho)\alpha}. \tag{6}$$

Then w is set so that the worker retains a fraction β of the post testing surplus flow. That is, the worker gets

$$w = \beta\left[\tilde{\rho} z_H + (1-\tilde{\rho}) z_L\right]. \tag{7}$$

Implicit in this expression is that the worker's outside option while negotiating a wage with a particular firm is zero, as we assume that he does not have other offers to consider at the time.[16] We may now substitute for w, using (6) and (7), and for v, using (5), into the expression for the expected, pre-test surplus generated by the match, (4). The firm's problem then becomes one of choosing α to maximize

$$\hat{v} = (1-\beta)\rho\left[\frac{z_H(1-e^{(r+q)T})}{r+q}\right] + (1-\beta)(1-\rho)(1-\alpha)\left[\frac{z_L(1-e^{(r+q)T})}{r+q}\right] - c(\alpha).$$

[16]Technically, we are assuming that the worker must choose to disengage with the matched firm and reenter the matching market in order to meet with another firm. This is the idea of the "outside option principle," where the outside option serves as an inequality constraint on the bargaining outcome and is not binding in our setting.

The firm's optimal testing level α^{**} as a function of ρ is therefore described by the first-order condition to this problem:

$$\frac{\partial \hat{v}}{\partial \alpha} = -\frac{(1-\rho)(1-\beta)(1-e^{-(r+q)T})z_L}{r+q} - c'(\alpha) = 0.$$

That is, the optimal testing level solves

$$c'(\alpha) = \Gamma\,(1-\rho), \qquad \textbf{(8)}$$

where

$$\Gamma \equiv -\frac{(1-\beta)(1-e^{-(r+q)T})z_L}{r+q}.$$

Our assumptions on c guarantee a unique solution to this equation for every ρ; in addition, we know that α^{**} is strictly decreasing in ρ. The precise shape of the relation between α^{**} and ρ depends, of course, on the nature of the cost function c.

Steady-State Conditions

In order to investigate the long-run properties of the model, we must describe the conditions necessary for the existence of a steady-state equilibrium. In a steady state, the measure of workers in the matching pool is constant over time. That is, the net flow of workers in and out of the matching pool must be zero. Therefore, we must have

$$q = Uq + U\lambda^w[1-(1-\rho)\alpha].$$

The left hand side is just the new workers entering the labor market for the first time. All new entrants are initially unmatched. The right hand side comprises two terms. The first is simply those workers who retire and leave the labor market while unemployed. The second term is the net flow out of unemployment into employment. The negative term inside the square brackets represents those workers who return to the unmatched pool after being matched with a firm. These workers are identified as low-type during testing. Workers identified as high type during testing ($\sigma = H$) never return to the unmatched pool. They are immediately hired based on their test result, and their type becomes known by all firms at the end of their employment tenure.[17] After T, these workers enter a market with complete information regarding their type. The high-types can secure all of the surplus that their ability generates from a future employer. The low-types must seek employment in another (un-modeled) sector of the economy.

The second requirement for a steady-state equilibrium is that there must not be any change in the composition of the matching pool. This requires that the net flow of *high-type* workers in and out of the matching pool must be zero. Therefore we have

$$\rho^0 q = \rho U[q + \lambda^w].$$

The left hand side is the high-type new entrants to the labor market. The right hand side represents the measure of high-types leaving the unmatched pool. High types exit unemployment for one of two reasons: they retire with probability q, or they get matched with an employer with probability λ^w. The latter never return to the unmatched pool, because they will certainly have $\sigma = H$. This test result will ensure their employment status until they retire, as firms never terminate high-types.

Rearranging the equality regarding the composition of the matching pool, we get an expression for the steady-state size (measure) of the unemployment pool:

[17] Of course, other firms do not learn the worker's type if he retires prior to T. Retired workers, by definition, do not return to the labor pool in any event.

$$U = \frac{\rho^0 q}{\rho(q+\lambda^w)}. \qquad \textbf{(9)}$$

Using this and the other steady-state condition, we have:

$$\rho = \frac{\rho^0(q+\lambda^w) - \rho^0\lambda^w\alpha}{(q+\lambda^w) - \rho^0\lambda^w\alpha}. \qquad \textbf{(10)}$$

This equation describes, for a given matching rate for workers, the relation between the firms' chosen testing level α and the implied proportion of high types in the matching pool ρ in steady-state.

Summary of Equilibrium Conditions and Existence

Note that there are ten endogenous variables in our model economy: U, V, λ^w, λ^f, v, $\hat{v}$, w, $\tilde{\rho}$, ρ, and α. As long as equation (4) is valid, a steady-state equilibrium (U^*, V^*, λ^{w*}, λ^{f*}, v^*, $\hat{v}^*$, w^*, $\tilde{\rho}^*$, ρ^*, α^*) is a solution to equations (1)–(10). We claim without proof that an equilibrium exists generally and involves employment if $\rho^0 z_H + (1-\rho^0)z_L > a$. But rather than work at the most general level, we will keep things simple by focusing on equilibria in which $V > U$ (that is, where there are more firms that workers in the matching market). This inequality is ensured if a is sufficiently close to zero. In this case, we have

$$\lambda^f = \frac{m(V,U)}{V} = \frac{U}{V}\theta$$

and

$$\lambda^w = \frac{m(V,U)}{U} = \theta \ .$$

Substituting for λ^w in equation (10), we then have that equations (8) and (10) form a system that determines the equilibrium testing level α^* and high-type matching proportion ρ^*, which are the two variables we are most interested in relating.

Using $\lambda^w = \theta$ and reproducing equations (8) and (10) give us

$$c'(\alpha) = \Gamma\,(1-\rho)$$

and

$$\rho = \frac{\rho^0(q+\theta) - \rho^0\theta\alpha}{q+\theta-\rho^0\theta\alpha}.$$

The points (α, ρ) that satisfy the first equation we call the *testing line*. The points that satisfy the second equation we call the *flow line*. These lines are shown in the Figure below (see the left graph).

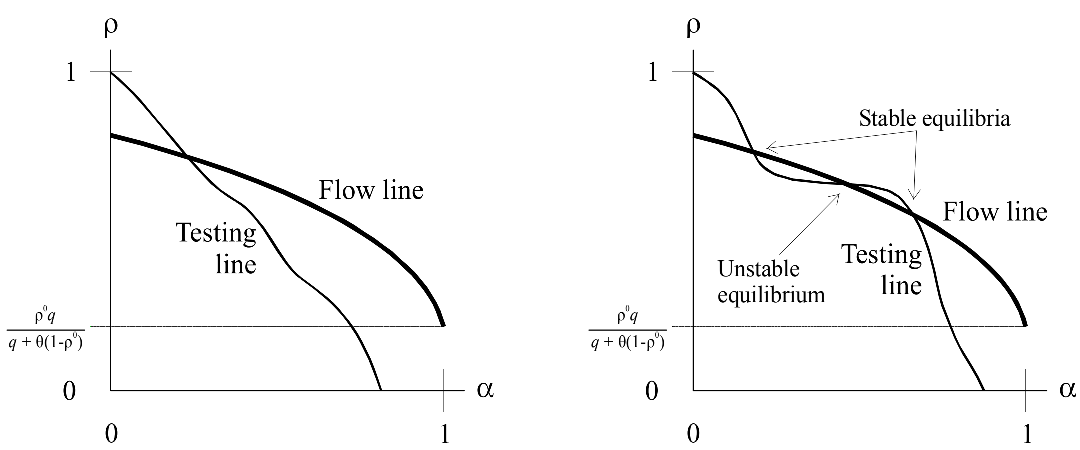

Figure: Illustration of Steady-State Equilibria

The flow line is strictly concave and decreasing, intersects the ρ-axis at $\rho = \rho^0$, and takes the value $\rho^0 q/[q + \theta(1-\rho^0)]$ at $\alpha = 1$. Given our assumptions on c, the testing line is continuous, strictly decreasing, intersects the ρ-axis at $\rho = 1$, and intersects the α-axis at some $\alpha < 1$. Thus, these lines intersect at least once; furthermore, there is an intersection where the testing line is steeper, which will imply stability.[18] Recall that equation (4) is valid if a firm obtains non-negative expected value from hiring a worker whose test result is $\sigma = H$. This is equivalent to the condition that $\hat{v}$ be non-negative. Straightforward examination of the firm's testing problem (recall the analysis leading up to equation (8)) shows that the value $\hat{v}$ is continuous and strictly increasing in ρ, strictly positive at $\rho = 1$, and strictly negative at $\rho = 0$. Thus, fixing the other parameter values, there is a number $\underline{\rho} \in (0, 1)$ such that (i) $\hat{v} = 0$ at $\rho = \underline{\rho}$ and (ii) $\hat{v} > 0$ if and only if $\rho > \underline{\rho}$. We therefore have $\rho > \underline{\rho}$ as a necessary condition for equilibrium. In summary, the only question for existence is whether $\rho > \underline{\rho}$ and whether the implied steady-state matching pools have $V > U$.

THEOREM 1: For z_H sufficiently large and a sufficiently close to zero, there exists a stable steady-state equilibrium with $\lambda^w = \theta$ (that is, $V > U$).

Proof: It is easy to see that, for any $\rho > 0$, the value of the firm's α-choice maximization problem (to which equation (8) gives the solution) is strictly positive for sufficiently large z_H. (The value is positive for the choice of $\alpha = 0$, in particular.) This implies that $\underline{\rho}$ converges to zero as z_H approaches infinity. Thus, for large enough z_H, the first intersection of the testing and

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[18] If the slope of the testing line is lower (more negative) than that of the flow line, slight deviations from α^* imply deviations from ρ^* in the direction that will push α back toward α^* (when firms act on the testing line).

flow lines (from the left) occurs at a value $\rho > \underline{\rho}$. Next note that, from equations (1), (3), and (5), a can be taken small enough to ensure that $V > U$. Thus, the flow and testing lines characterize stable, steady-state equilibrium values α^* and ρ^*, and the rest of the variables are easily determined from the other equilibrium equations. ■

The Figure illustrates one striking feature of the model: There is a complementary relation between the firm's optimal testing level and the composition of the matching labor pool. For example, an exogenous change in the environment which changes the ease of matching workers and firms will produce a direct effect on the composition of the labor pool. Further, the change in the composition of the labor pool will have an indirect effect on the firm's testing decision, which also would impact the composition of the labor pool. A similar chain of consequences will follow some exogenous change in the firm's problem. This spillover effect is cataloged more precisely in the propositions presented in the next section.

The feedback between testing and the composition of the labor matching pool creates the possibility of multiple equilibria, as illustrated in the right graph of the Figure. The next result establishes that multiple stable steady-state equilibria exist on a positive measure of the parameter space.

THEOREM 2: For a large enough z_H and a close to zero, there exists a cost function $\underline{c}$ such that there are multiple stable, steady-state equilibria. Furthermore, any cost function that is sufficiently close to $\underline{c}$ (in the sense of c' being uniformly close to $\underline{c}'$) will have the same property.

Proof: Because the flow line is strictly decreasing, we can find a function $\underline{c}$ that yields a testing line that intersects the flow line in three places, two from above, as shown in the right graph of the Figure. Clearly, any function sufficiently close to $\underline{c}$ will have the same property. Note that z_H does not directly influence the testing line. The argument employed in the proof of Theorem 1 for z_H and a can then be used to establish that the intersection occurs where $\rho > \underline{\rho}$ and $V > U$. ■

The key to multiple equilibria is large variations in c'' over the range of α. That is, there have to be separate regions in the unit interval where c'' is relatively large (so that α^{**} does not respond much to changes in ρ) and between which c'' is relatively small. For example, the test technology may feature roughly two different testing styles, one which is much more informative but also costs much more. Firms may be able to vary the testing style at the margin, but there is a relatively dramatic shift when wholly adopting the more informative style.

The potential for multiple equilibria allows our model to explain differences in firms' efforts to screen applicants between geographic areas, between industries, or over time. For example, firms in suburban areas which attract a highly skilled labor pool will not be compelled to test their applicants very strenuously prior to hiring them, while inner city firms facing a low skilled labor pool will test applicants more. Holzer (1996) finds evidence of this phenomenon in a study of four major U.S. metropolitan areas. He found that central cities exhibit higher vacancies despite higher unemployment levels (p.37). In addition, there was some evidence that central-city applicants faced more rigorous screening than applicants elsewhere (pp.54-55). Such phenomenon are consistent with the multiple equilibria model presented here.

Remember that where we have multiple equilibria, not every one is stable. Starting from an arbitrary initial point in the neighborhood of an equilibrium, the model's out-of-equilibrium dynamics will move the firm's testing decision and the proportion of high-type workers in the unemployed pool towards a stable steady-state equilibria, but not towards an unstable equilibrium. Therefore, unstable steady state equilibria can only result where the model's initial conditions coincide precisely with that equilibrium. For this reason, we will concentrate on stable, steady-state equilibria in what follows.

III. Comparative Statics Results

In this section we provide some comparative static results for *stable* steady-state equilibria. The proofs rest on the fact that a stable steady-state equilibrium is completely described by the intersection of the flow and testing lines, where the testing line has a lower (more negative) slope, as described above.

The first of our comparative static results concerns the ramifications of an improvement in testing technology. To build intuition, consider a relatively recent advance in screening applicants: testing using the Internet. The testing service providers that offer this service often allow firms to customize tests for applicants based on their needs. Applicants are given a log-in name and a password which allows them to take a pre-employment screening test from anywhere at anytime. The results can be instantaneously e-mailed to human resource personnel. This service greatly reduces the travel costs and time required to administer tests to job applicants. However, these services are quite new. For example, two industry leaders, Personnel Services Corporation and Baden Employee Selection and Development Services, only began offering "e-testing" in the first quarter of 2000. The following result may offer some insight on how the firm's screening efforts and the overall quality of the unemployed workforce will be impacted as this technology becomes more prevalent.

PROPOSITION 1: Consider a small,[19] uniform downward shift in $c'(\cdot)$, the marginal cost of testing.[20] This change will affect the steady-state values of a stable equilibrium in the following way: (a) α^* will increase, (b) ρ^* will decrease, and (c) U^* will increase.

Proof: Rather than provide a calculus-based proof, which would require us to parameterize the shift in c', we content ourselves with a graphical proof here. An indication of the more formal method of proof can be gleaned from the proof of Proposition 2 below, where we provide the differential conditions as well. Lower c' implies that the testing line shifts to the right and leaves the flow line unchanged. Because the testing line has a lower (more negative) slope, the intersection is at a higher value of α^* and a lower value of ρ^*. As a consequence, more low types will be discovered during testing and be returned to the unmatched pool. By equation (9), the effect is to increase the size of the matching pool (unemployment) U^*. ■

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[19]In this result, as well as those that follow, we will need to restrict attention to parameter shifts that are "small" in the sense that the change is not large enough to shift the relevant curve (i.e., the testing or flow line) enough to eliminate the initial equilibria under study. Without more structure on $c(\alpha)$, it is not possible to deliver the results algebraically, and graphical proofs offered here are restricted to changes that do not upset the underlying equilibria.

[20]For example, suppose that $c(\alpha) = \bar{c}\,\xi(\alpha)$, where $\bar{c} \in \Re$ and $\xi(\alpha)$ is a function with $\xi'(0) = 0$ and $\lim_{\alpha \to 1} \xi(\alpha) = \infty$. In this case, a small decrease in $\bar{c}$ would correspond to uniform downward shift in c'.

The intuition for this result is straightforward. Part (a) merely indicates that testing is a normal good: If its price goes down, then firms will buy (i.e., test) more.[21] The heightened testing will mean that more low-type applicants are returned to the unemployment pool following a poor test result. These returned low types swell the ranks of the unemployed.

An unexpected feature of our model merits some digression here. We might expect that increased testing, and the corresponding improved screening of job applicants, would result in improved productivity of those employed. Higher expected productivity would, in turn, translate into higher wages. However, this is not necessarily the case. The cause of the ambiguity lies in the dynamic interaction between the firm's testing decision and the quality of the workforce. Continuing with the example of Proposition 1, as more workers are found to be low-type and returned to the unemployed pool, the quality (and productivity) of those who are employed will increase (on average). If we ignore the effect on the quality of the labor pool, this would unambiguously raise wages. However, from (b), we know that the quality of the labor pool will deteriorate as a result of better screening as well. This will lower the firm's expectation of a worker's ability *even though the worker passed a more rigorous screening process*. The net effect depends on the slope of the testing line. Resolving the ambiguity — resulting from a change in *any* exogenous variable — requires more specific assumptions about the cost of testing function, $c(\alpha)$.

While the model as currently formulated is not able to offer predictions about wage levels, it may offer some insight into the complexity of the wage determination process. In

[21]Note that (a) only applies to the "level," or accuracy, of testing in equilibrium, not *total* expenditure on testing.

particular, the model indicates that wages are a complicated reflection of firms' expectations about the quality of the workers they manage to hire, and how these expectations interact with the firms' effort to protect themselves from hiring low quality workers. The focus on the quality of the labor force also offers a perspective on labor productivity that is distinct from any notion of capital intensity.

The next result investigates a set of parameter changes which collectively increase the firm's "stake" in its hiring decisions.

PROPOSITION 2: Consider (i) a small decrease in β, the worker's share of the post-match surplus; (ii) a small decrease in r, the discount rate; (iii) a small increase in T, the time required to complete the important task; or (iv) a small decrease in z_L, the damage done by low-types while employed. Such a change will affect the values of a stable steady-state equilibrium in the following way: (a) α^* will increase, (b) ρ^* will decrease, and (c) U^* will increase.

Proof: Note that each of these changes causes Γ to rise, so we need only analyze the effect on the equilibrium values α^* and ρ^* of a small change in Γ. A graphical argument starts with the observation that, from equation (8), we have $\partial\alpha^{**}/\partial\Gamma > 0$. That is, the optimal testing level is strictly increasing in Γ. Thus, the testing line shifts to the right and we obtain the same conclusions as in Proposition 1: More low types will be discovered during testing and be returned to the unmatched pool, lowering the average quality of those left unmatched and increasing unemployment.

Here are the calculus steps that provide the conclusion more formally. Consider the testing-line and flow-line equations as defining an identity relating α, ρ, and Γ. This identiy implicitly defines (α*, ρ*) as a function of Γ; that is, we can write $g(\Gamma) = (\alpha^*, \rho^*)'$. Using the Implicit Function Theorem (implicit differentiation of g) and a few lines of algebra, we obtain

$$\frac{d\alpha^*}{d\Gamma} = \frac{1-\rho}{c''(g_1(\Gamma)) - \Gamma\Omega}$$

and

$$\frac{d\rho^*}{d\Gamma} = \frac{-\Omega(1-\rho)}{c''(g_1(\Gamma)) - \Gamma\Omega},$$

where

$$\Omega = \frac{\rho^0\theta(1-\rho^0)(q+\theta)}{(q+\theta-\rho^0\theta\alpha)^2}.$$

Clearly we have $\Omega > 0$. Furthermore, because the slope of the testing line is more negative than is the slope of the flow line, we have $c''(g_1(\Gamma)) - \Gamma\Omega > 0$. This is implies that $d\alpha^*/d\Gamma > 0$ and $d\rho^*/d\Gamma < 0$. The conclusion for U^* follows from equation (9), as before. ■

The intuition for this result comes from the idea that all four parameter changes effectively increase the firm's stake in the outcome of production. For (i), this is clear. The motivation is that an exogenous change, say a decline in unionism, could lower workers' bargaining power. Obviously, the firm's share of the post match surplus, $1-\beta$, is the complement of the worker's share. Because the workers' bargaining power is reduced, the firm is entitled to more of the quasi-rent. This means that there is more to protect via testing.

To see how a change in the discount rate (ii) effects the firm's decision making, note that if the firm becomes more patient, then the cost of testing a worker (paid immediately) will decrease relative to the discounted value of the post-match quasi-rent. The result will be more stringent testing to better protect future returns. The intuition for (iii) is similar. The amount that can be appropriated by the firm before the worker is discovered by the market increases, so firm will test more. As for (iv), recall that z_L is negative. Thus a decrease in z_L (or, equivalently, an increase in $|z_L|$) corresponds to hired low-types being more destructive. Where production is more sensitive to the presence of low-types, the firm will want to test more to guard against hiring them.[22]

The next result considers the impact of an improvement in the quality of new, initially unemployed workers.

PROPOSITION 3: Consider a small increase in ρ^0, the proportion of high-types among new entrants to the unemployed labor pool. This change will affect the steady-state values of a stable equilibrium in the following way: (a) α^* will decrease and (b) ρ^* will increase.

Proof: An increase in ρ^0 will shift the flow line up, with $\partial\rho/\partial\rho^0 > 0$. There is no change in the

[22]Note that the complimentary result for z_H does not hold. An increase in z_H has no effect on the firm's testing choice (other than to guarantee that the firm earns a positive return following a match). The cause of this asymmetry is the structure of the testing technology: since high-types always test well, optimal testing levels are aimed solely at avoiding hiring low-types.

testing line. Therefore, the intersection of the testing and flow lines occurs at a higher value of ρ^* and a lower value of α^*. A calculus argument is straightforward along the lines shown in the proof of Proposition 2. ■

The intuition for Proposition 3 is straightforward. Better quality new workers directly improve the quality of the labor pool. As a result of the better quality pool of unemployed workers, there is less need for costly testing by firms. The lower testing level means that more low types will become employed, which further improves the quality of the unemployed workforce.

The last of our comparative statics results concerns small changes in the manner in which firms and workers are able to find one another, i.e., the matching technology.

PROPOSITION 4: Consider a small increase in θ, the (exogenous) matching function parameter. This change will affect the steady-state values of a stable equilibrium in the following way: (a) α^* will increase and (b) ρ^* will decrease.

Proof: An increase in θ will shift the flow line down, with $\partial\rho/\partial\theta < 0$. There is no change in the testing line. Therefore, effect on the stable steady-state equilibrium is just the opposite of the case of Proposition 3. ■

The intuition for (b) is clear: better matching technology, e.g., an Internet jobs board, leads to more matches. Due to the testing technology, of the workers who are matched with a firm, those hired will be disproportionately high-type. This reduces the quality of the remaining workforce. It follows that this will force firms to test its applicants more in order to protect its quasi-rent (a).

Note that in this case, the impact on unemployment is ambiguous. Better matching technology means that it is easier for both workers to find firms (equation (9)) and for firms to find workers (equation (8)). This will lead to more hires and thus lower unemployment. However, in equilibrium, easier matching will result in more low-type workers being matched, discovered in the testing phase and returned to the unemployment pool (b). As a consequence, the lower quality workforce will force firms to test more stringently (a). This will increase the percentage of applicants being denied employment. The latter effect will tend to raise unemployment. Specific conditions on the testing function are required to pin down which effect is dominant.

IV. Conclusion

We have presented a simple model that addresses the feedback mechanism between the labor pool and the hiring decisions of firms. This introduces a potentially important complementarity between firms' screening decision and the quality of the workforce. However, several extensions to the model would improve its applicability to real world labor markets. Useful extensions might include a richer treatment of the workers' types and the testing technology, as well as a more nuanced treatment of employees' work experience, whereby, for example, a firm can get some information about whether an applicant had an unfavorable experience with a previous employer. Another useful extension might be to endogenize the quality of the entering workforce. For example, suppose that workers could invest in human capital to make themselves more productive or to assist them in successfully navigating the screening process. We believe that further empirical work on testing technology and employment variables would be informative.

We have assumed that firms either do not observe how long workers have been in the matching pool or that they simply do not condition the testing level on the ages of workers with whom they are matched. In fact, if firms observe workers' ages then they may want to discriminate between older and younger workers. This is another extension worth studying. In fact, one of the authors has conducted preliminary analysis elsewhere on how age might signal quality in the matching market, but we have not thoroughly considered how to incorporate this feature in the present model. We can offer some intuition, however: Because older workers in the matching pool are more likely that are younger workers to have already been matched with firms and failed tests, age is a negative signal of quality. Thus, in a model in which the firms consider age when selecting a testing level, we expect that older workers would be more stringently tested and that possibly an age cutoff would exist beyond which workers would transfer to another sector.

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