Cheap Talk

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SIMPLE COMMUNICATION MODEL

- Two agents.
- One (Sender) has private information, \( t \). The other (Receiver) takes action, \( a \).
- Nature picks \( t \in [0, 1] \) from prior, \( F(\cdot) \).
- Sender learns \( t \). Receiver does not.
- Sender sends message \( m \in M \) to Receiver.
- Receiver takes action \( a \in \mathbb{R} \).
Preferences

\[ U^i(t, a), \ i = R, S. \]

Note (important): \( U^i \) does not depend on \( m \).

Talk is cheap.
ASIDE: OTHER POSSIBLE ASSUMPTIONS

- Standard “Spence” signaling: $U^i(\cdot)$ depends on $m$. Normally assume single crossing.
- Verifiable information. $M(t)$ set of messages available to $t$. ($M(t) = t$, truth required. $M(t) = M$, cheap talk.)
WHAT IS THIS MODEL ABOUT?

1. Communication in everyday settings.
2. Avoiding inefficiency caused by incomplete information.
3. Advertising
4. Expert Advise
5. Legislative Decision Making
$a^i(t)$ solves: $\max U^i(t, a)$.

$\bar{a}(t', t'')$ be the unique solution to $\max_a \int_{t'}^{t''} U^R(a, t) dF(t)$.

Leading example:
$U^R(t, a) = -(a - t)^2$ and $U^S(a, t) = -(a - t - b)^2$, $b > 0$.

Uniform prior.

More generally: $U^i$ concave in $a$ and with positive mixed partial so that $a^i$ is increasing.

Assume: $a^S(t) > a^R(t)$.

So there is $\varepsilon > 0$ such that $a^S(t) - a^R(t) \geq \varepsilon$.

(In quadratic example, $a^S(t) = t + b$ and $a^R(t) = t$.)
CONFLICT OF INTEREST

- S and R have similar interests: \( a^i(t) \) increases in \( t \).
- S and R have different interests: \( a^S(t) > a^R(t) \).
- Sometimes add parameter \( b \), intuitively decreasing \( b \) decreases conflict.
STRATEGIES

Three elements:

1. Message for each type: \( \mu : [0, 1] \rightarrow M \) for \( S \).
2. Action for each message: \( \alpha : M \rightarrow \mathbb{R} \) for \( R \).
3. Interpretation of message: \( \beta(t \mid m) \).
EQUILIBRIUM CONDITIONS

1. for each $t \in [0, 1]$, $\mu(t)$ solves $\max_m U^S(\alpha(m), t)$, \hspace{1cm} (1)

2. for each $m \in M$, $\alpha(m)$ solves $\max_a \int_0^1 U^R(a, t) \beta(t \mid m) dt$, \hspace{1cm} (2)

3. $\beta(t \mid m)$ is derived from $\mu$ and $F$ from Bayes’s Rule.

An equilibrium with strategies $(\mu^*, \alpha^*)$ induces action $a$ if \{ $t : \alpha^*(\mu^*(t)) = a$ \} has positive prior probability.
Simplifications

- $R$ uses a pure strategy by concavity.
- $M$ is finite and $S$’s strategy pure.

Finiteness is a conclusion.
Most $S$ types will have unique best response.
ON FINITENESS

Assume that $a < a'$ induced in equilibrium. Then

1. There exists $t$ such that $U^S(t, a) = U^S(t, a')$.
2. $a^S(t) \in (a, a')$.
3. No $t' > t$ induces $a$.
4. $a \leq y^R(t)$
5. No $t' < t$ induces $a'$.
6. $a' \geq y^R(t)$
7. $a^R(t), a^S(t) \in [a, a']$.
8. $a' - a \geq \varepsilon$. 

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NECESSARY CONDITION FOR MEANINGFUL CHEAP-TALK

Senders must differ in preferences over Receiver actions. Why?
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This rules out:

1. Cheap Talk in Labor Market
2. Cheap Talk about Quality
ANOTHER NECESSARY CONDITION

Receivers and Senders have some common interest.
That is, $R$ can learn $t$ only if there exists $a$, such that
$U^i(a^R(t), t, m) \leq U^i(a, t, m)$ for $i = R, S$ ($a^R(t)$ is $R$’s best
response to $t$).
Naively this rules out: Communication with Enemies
Proposition

There exists $N^*$ such that for every $N$ with $1 \leq N \leq N^*$, there exists an equilibrium in which the set of induced actions has cardinality $N$ and there is no equilibrium which induces more than $N^*$ actions. Equilibria are described by a partition $t(N) = (t_0(N), \ldots, t_N(N))$ with $0 = t_0(N) < t_1(N) < \ldots < t_N(N) = 1$, and signals $m_i$, $i = 1, \ldots, N$, such that for all $i = 1, \ldots, N - 1$

$$U^S(\bar{a}(t_i, t_{i+1}), t_i)) - U^S(\bar{a}(t_{i-1}, t_i), t_i)) = 0,$$  

(3)

$$\mu(t) = m_i \text{ for } t \in (t_{i-1}, t_i],$$  

(4)

and

$$\alpha(m_i) = \bar{a}(t_{i-1}, t_i).$$  

(5)
PROPERTIES OF EQUILIBRIA

1. Unit interval partitioned.
2. Types in each partition element send the same message.
4. Incentive constraints determine edges of partition.
5. “Babbling” equilibrium always exists.
6. Typically multiple equilibria.
REGULARITY CONDITION

Definition
The cheap-talk game satisfies the Monotonicity (M) Condition if for any two solutions to (3), \( \hat{t} \) and \( \tilde{t} \) with \( \hat{t}_0 = \tilde{t}_0 \) and \( \hat{t}_1 > \tilde{t}_1 \), then \( \hat{t}_i > \tilde{t}_i \) for all \( i \geq 2 \).

- Exactly one equilibrium partition for each \( N = 1, \ldots, N^* \).
- Ex-ante equilibrium expected utility for both \( S \) and \( R \) is increasing in \( N \).
- \( N^* \) decreasing in \( b \).
- Ex-ante equilibrium expected utility for both \( S \) and \( R \) is decreasing in \( b \) for fixed \( N \).
NITS

Definition
An equilibrium \((\mu^*, \alpha^*)\) satisfies the No Incentive to Separate (NITS) Condition if 
\[
U^S(\alpha^*(\mu^*(0)), 0) \geq U^S(a^R(0), 0).
\]

NITS states that the lowest type of Sender prefers her equilibrium payoff to the payoff she would receive if the Receiver knew her type (and responded optimally).
Proposition

If an $N$-step equilibrium fails to satisfy NITS, then there exists an $(N + 1)$-step equilibrium. Moreover, if an equilibrium satisfies NITS, then so will any equilibrium with a shorter first segment.
Proposition

If there is only one equilibrium partition with \( N \) induced actions for any \( N \in \{1, \ldots, N^*\} \), then there exists \( \hat{N} \in \{1, \ldots, N^*\} \) such that an equilibrium with \( N \) actions satisfies NITS if and only if \( N \geq \hat{N} \).
NITS AND Condition (M)

Proposition

*If a cheap-talk game satisfies *(M)*, then only the equilibrium partition with the maximum number of induced actions satisfies NITS.*
NITS in Single-Crossing Models

1. $S$’s preferences are monotonic in $R$’s actions.
2. $R$’s action is monotonically increasing in $S$’s type.
3. Costly signals.

$t = 0$ satisfies NITS in any perfect bayesian equilibrium.
Restriction on Belief

Proposition

If there exists a message $m^*$ such that R’s beliefs given $m^*$ are supported within $[0, t^*]$ where $t^* = 1$ if $U^S(\bar{a}(0, t), 0) > U^S(a^R(0), 0)$ for all $t \in (0, 1)$ and $t^*$ is the unique positive solution to $U^S(\bar{a}(0, t^*), 0) = U^S(a^R(0), 0)$, then NITS must hold because type 0 can always send $m^*$.

The restriction is a weak relative to Farrell’s Credible Neologism.
Definition
A credible neologism relative to a fixed equilibrium exists if there exists a set of types, $T$, such that precisely types in $T$ prefer $R$’s optimal response to $T$ than the equilibrium payoff.

*If NITS fails, then there is a credible neologism containing $t = 0$. The problem with Credible Neologisms is that they typically destroy all equilibria.*
Assume: Each type of $S$ can prove that her type is no greater than her true type.

Conclude:
1. No equilibria created.
2. NITS holds (so equilibria typically destroyed).

The ability to avoid being pooled with higher types is typically unattractive.
Assume: Each type of $S$ can prove that her type is no less than her true type.

Conclude:

1. New equilibria created.
2. Unique outcome: separation due to unraveling at the top.
CS with perturbations (some $S$ and $R$ follow fixed strategies):

- $M = [0, 1]$
- Some $S$ must send $m = t$.
- Some $R$ must set $a = a^R(m)$.

*NITS holds if there is positive probability of non-strategic types and strategic Senders use non-decreasing strategies. So limiting equilibria make selection in CS.*
A proof by contradiction: Assume NITS fails.

1. Dishonest low Senders pool at $m = 0$.  
   (Otherwise attractive deviation to 0.)

2. There exists a “small” positive message that is attractive to $t = 0$. 
   (Any low message induces a nice response from non-strategic $R$. A simple argument also shows that one such message must also induce an attractive response from strategic $R$.)
Kartik perturbs CS:
Sender payoffs: $U^S(a, t) - kC(m, t)$, for $k > 0$.
Assume:

- $C$ is twice continuously differentiable.
- $C_1(t, t) = 0$.
- $C_{11}(m, t) > 0 > C_{12}(m, t)$.
- $C(t, t) = 0$.

Example: $C(m, t) = -(m - t)^2$.
Model approaches CS as $k$ goes to zero.
Properties

- Look at pure-strategy equilibria in which $S$ and $R$ use weakly increasing strategies.

- Result: Limiting equilibria must satisfy NITS.
Intuition

*A proof by contradiction: Assume NITS fails.*

1. Low Senders pools at $m = 0$.
   (Otherwise attractive deviation to 0 - better action and less cost.)

2. There exists a “small” positive message that is attractive to $t = 0$.
   (Any low message induces a nice response from non-strategic $R$. A simple argument also shows that one such message must also induce an attractive response from strategic $R$.)

3. High types not in the pool.
   ($t = 1$ can deviate to $m = 1$ and receive a better action at less cost.)

4. The existence of another “on-path” action (step 3) and lying costs implies that pool must be small.
In Chen and Kartik, $m = 1$ must be used in equilibrium. (Otherwise, $t = 1$ deviates.)

- Pooling Equilibria Must Pool at $m = 1$.
- $m = 0$ is a profitable deviation if NITS fails.
Veto Threats: Model

- Players: Chooser (C) and a Proposer (P).
- Quadratic Preferences.
  - C’s ideal point $t$.
  - P’s ideal point 0.
- $t$ is private information.
Veto Threats: Game

- Chooser learns her type
- Chooser sends (cheap) message to Proposer.
- Proposer proposes $a$.
- Chooser rejects (final outcome 1) or accepts final outcome $a$.
- $t$ is supported on $[t, \bar{t}]$ with $t < 1$ and $\bar{t} > \frac{1}{2}$.
Veto Threats: Equilibria

- Always Babbling.
- Sometimes size 2: $C$ induces $P$’s ideal or a compromise.
- Never more than two serious messages in equilibrium.
**Veto Threats: NITS**

*NITS: t does at least as well in equilibrium as by revealing.*

- 2 step satisfies NITS.
- If 1 step fails NITS, then 2 step exists.
- Both 1 and 2 step may satisfy NITS.
Preferences: $U^S(a, t) = (1 - a)(1 + ky) + a(t + r)$
$U^R(a, t) = a(1 + kt) + (1 - a)(y + r)$.

- $k$ degree of relationship.
- $y$ fitness of “mother.”
- Like CS, but not smooth and Sender likes lower value of $a$.
- Apply NITS at $t = 1$.
- Conflict of interest: self interest dominates.
- Common interest: if $k$ is large and $t$ is large, then both sides want $a$ to be low.
1. At most two actions induced in equilibrium.
2. Babbling Equilibrium Exists
3. 
   \[ y^* := \frac{y}{k} + 1 - \frac{1}{k}. \]  
   \[(6)\]

The Receiver finds it uniquely optimal to set \( a = 0 \) if \( \mathbb{E}[t|m] < y^* \), uniquely optimal to set \( a = 1 \) if \( \mathbb{E}[t|m] > y^* \), and is indifferent over all \( a \) otherwise.
Results

- The babbling equilibrium satisfies NITS if and only if $E[t] \geq y^*$. 
- A two-step equilibrium exists if and only if 
  \[ E[t \mid t < 1 - k(1 - y)] \leq y^*. \] (7)
- If a two-step equilibrium exists, it satisfies NITS.
- If the one-step equilibrium fails NITS, then a two-step equilibrium exists.
- If the one-step equilibrium satisfies NITS, a two-step equilibrium may or may not exist.