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Samurai Accountant: A Theory of Auditing and Plunder

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A risk neutral principal wishes to exact a payment from a risk neutral agent whose wealth he does not know, but may verify through a costly auditing procedure. We characterize efficient schemes for the principal when he is allowed to choose schedules for preaudit and postaudit payments and audit probabilities, subject to the constraint that only monetary incentives can be used and that the principal may never make a net payment to the agent. The main results are that efficient schemes involve preaudit payments which are increasing in the agent's wealth, audit probabilities are decreasing in the agent's wealth and also satisfy certain constraints as equalities. In general, such schemes involve stochastic auditing and rebates after an audit.

In this paper we analyse the problem of a principal extracting wealth from an agent who is reluctant to part with it and is also better informed than the principal as to the amount he owns. We assume that the principal has a costly means of discovering the agent's wealth. This problem occurs in a variety of contexts. A prominent example is the way income taxes are collected. In this case the taxpayer knows his own income, but the taxing authority does not. Typically though, the tax collector can expend resources to audit the taxpayer. Another example is that of a manager who asks his subordinates to report on the profitability of their divisions. If the output of a division has value to the subordinate, then he will have an incentive to hold back some of the output. Again, the manager can audit the subordinate, but at a cost. Both of these examples can occur in a context where the tax collector or manager has a legitimate claim to the wealth that is being extracted, but the same problems arise for illegitimate appropriation. Consider the situation described by Kurosawa (1970) where a band of brigands assails a peasant village and demands tribute. The brigands can plunder the village, but this can be costly, particularly if the village harbours masterless samurai.

We adopt the following model of this general problem. The principal knows the probability distribution of wealth for the population from which the agent is drawn, but the principal does not know the wealth of the agent. The principal may audit the agent in order to verify his wealth, but this is costly. The principal may choose schedules detailing the amount of wealth to be surrendered according to messages sent by the agent, the probability that each message will trigger an audit, and the amount of wealth to be surrendered in the event that an audit takes place. The agent, treating the policy of the principal parametrically, acts to maximize his expected net income. The principal chooses his policy in order to maximize some function of his revenue and auditing activity.

We will not specify the objective function for the principal. For instance, a band of brigands or a manager may wish to maximize expected revenue net of audit costs. A tax collector may wish to minimize audit costs subject to meeting some net revenue goal. Or a taxing authority may wish to maximize the sum of taxpayers' utilities subject to a net revenue constraint. For risk neutral taxpayers, the sum of utilities is just total expected wealth less gross revenue collected. For each of these objective functions, if auditing is costly, then an optimal scheme will have the property that it is not possible to raise the same gross revenue with less auditing. Schemes with this property we call audit efficient.

Our major results are the following. Net revenue maximizing schemes may not exist unless payments are bounded exogenously. We therefore restrict attention to schemes with bounded payments. Without loss of generality we can restrict attention to incentive compatible direct revelation schemes, i.e. those in which the agent truthfully announces his wealth and makes a payment (which for convenience we will call a tax) based on his announcement to the principal and the principal chooses the probability of auditing based on reported wealth. Optimal schemes typically involve stochastic auditing. In an efficient direct scheme, in which the principal induces truthful reporting, taxes are monotonically increasing and audit probabilities are monotonically decreasing in reported wealth. An agent who is audited and found to be telling the truth receives a rebate; thus, honest agents prefer to be audited. Finally, efficient schemes have the property that for each wealth level reported, the principal demands all the agent's wealth as a tax, or promises to take nothing after an audit, or does not audit the report at all.

Several assumptions limit our analysis. We assume that the distribution of wealth is fixed and unaffected by the principal's policies. Thus, we abstract from the distortions that an income tax creates on the labour-leisure decisions made by wage earners (see e.g. Mirrlees (1971)). There are two reasons for this. One is to be able to attack the problem of compliance per se, the other is that the solution of this problem provides insight for solving the more general case in which policies affect the distribution of wealth. Kanodia (1985) analyses a similar model with moral hazard, but under the restriction that preaudit transfers be independent of type. An analogous assumption in our model would be to require that the tax be independent of reported wealth.

The principal/agent approach adopted here does matter. We assume that the principal can commit to an audit policy and that the agent will respond optimally to this policy. Since the principal can induce truthful reporting of wealth, he knows what an audit will reveal. Furthermore, we show that the result of auditing a truthful report must be to give a rebate, so there are two reasons for the principal not to want to conduct the audit. We do not address the issue of how the principal can make commitments, but analyze the case of partial commitment in Proposition 4 below. Reinganum and Wilde (1986) address the case of not being able to make binding commitments in a tax compliance model.

Another major assumption of our analysis is that the agent seeks to maximize his expected net wealth, that is, the agent is risk neutral. We discuss the effect of risk aversion on our results following Theorem 1. Mookherjee and P'ng (1986) treat a model in which agents are risk averse and the principal selects taxes and audit probabilities to maximize a welfare function.

We also make the restrictive assumption that the agent cannot surrender more than his wealth. This limits the sorts of schedules that a principal can impose upon the agent, and on the kinds of reports the agent can issue. This means that a modified version of the revelation principle must be used. It also rules out imposing harsh penalties on the agent. If perfect, ex post observations are feasible, then it is typically true that enforcement costs can be made arbitrarily small by forcing the agent to pay large penalties with arbitrarily small probabilities if the agent fails to do what the principal prefers. This result is often associated with Becker (1968) and Stigler's (1970) work on the economics of crime prevention. In general, Mirrlees (1975) shows that even when observations are not perfect, it may be possible to approximate (but not attain) full-information optima with incentive schemes that require an agent to pay large penalties with small probabilities We restrict attention to payments that do not exceed wealth. Consequently, approximating full-information optima with large penalties is not feasible in our model. There are several reasons for making this restriction. One is that legal constraints outside the model may limit what the principal can do to the agent. Managers would be barred by the courts from torturing their subordinates. In the U.S., the eighth amendment to the constitution prohibits the government from torturing tax evaders. It will follow from our results that the principal may want to use this punishment even if he doesn't audit the agent, and not reserve it solely for punishing misreporting. We will also be forced to rule out the converse sort of incentives, that is, offering the agent a large reward with a small probability This too can be used to induce any sort of behaviour. This restriction is justified if the principal's resources are limited to what he can extract from the agent.

The last restrictive assumption that we discuss relates to the auditing technology. We assume that an audit discovers true wealth without error. Baron and Besanko (1984), Laffont and Tirole (1986), and others present models in which auditing is possible, but cannot be done perfectly.

This work generalizes the recent work of Reinganum and Wilde (1985), which deals with a net revenue maximizing tax collector who must use a lump sum tax scheme. They show that optimal schemes are deterministic. Scotchmer (1986) analyses a model in which the tax collector can only choose the audit function to maximize revenue, given an increasing tax function and fines which are proportional to the amount underreported.

MODEL

We assume that the wealth of the agent is a random variable taking values in the finite set $X=\{x_1, \ldots, x_n\}$. We will sometimes say that an agent is of *type i* if his wealth is x_i The probability that wealth takes on the value x_i is h_i ($h_i > 0$). We label the wealth levels so that $0 \leqq x_1 < \cdots < x_n$. A *mechanism*[1] for the principal consists of a set M of messages, a function $t: M \to \boldsymbol{R}$, a function $p: M \to [0, 1]$, and a function $f: X \times M \to \boldsymbol{R}$. The function t is called the *tax function.* If the agent reports the message m to the principal, he must send an amount $t(m)$ to the principal. The function p is called the *audit function.* If the principal receives the message m, he commits to auditing the agent with probability $p(m)$ The function f is called the *penalty function.* In the event of an audit, the principal returns the payment $t(m)$ and instead collects $f(x, m)$ where x is the agent's wealth. Since we assume that the agent can pay no more than his wealth we limit the principal's choice of mechanism by requiring that there be at least one message m with $t(m) \leqq x_1$ and by requiring that $f(x, m) \leqq x$ for all x and m.

Call a mechanism a *direct revelation mechanism* if its message space is X, the set of types, and say that a direct revelation mechanism is *incentive compatible* if truthfully stating his wealth is an optimal message for the agent. The next fact simplifies our analysis.

Proposition 0 (The Revelation Principle). *Given any mechanism and any set of agent's optimal responses, there is an incentive compatible direct revelation mechanism which* *is equivalent from the point of view of both the principal and the agent, when the agent tells the truth.*

This version of the revelation principle is not immediate from standard statements of the revelation principle because the agent can never pay more than his wealth. The revelation principle need not apply to situations in which the set of reports available to the agent depends on his type.[2]

An incentive compatible direct revelation mechanism in our framework can thus be described by a triple $(t_i, p_i, f_{ij})_{i=1,\dots,n, j=1,\dots,n}$ where t_i is interpreted as $t(x_i)$, p_i as $p(x_i)$, and f_{ij} as $f(x_i, x_j)$. That truthfully revealing his wealth is an optimal response for the agent is captured by the inequalities

$$(1-p_i)(x_i - t_i) + p_i(x_i - f_{ii}) \geqq (1-p_j)(x_i - t_j) + p_j(x_i - f_{ij}) \quad \text{for all } i \text{ and all } j \text{ with } t_j \leqq x_i. \tag{1}$$

These simply say that reporting type i when his wealth is x_i gives the agent an expected utility at least as large as reporting type j. The agent has no incentive to lie, and if he truthfully reports his wealth the principal's expected gross revenue (exclusive of the costs of auditing) is

$$\textstyle\sum_{i=1}^{n} [(1-p_i)t_i + p_i f_{ii}]h_i. \tag{2}$$

The inequalities in (1) are as weak as possible when f_{ij} is made as large as possible for $j \neq i$, which given the restriction that the agent can pay no more than his wealth, is accomplished by setting $f_{ij} = x_i$. This allows us to replace (1) with

$$(1-p_i)(x_i - t_i) + p_i(x_i - f_{ii}) \geqq (1-p_j)(x_i - t_j) \quad \text{for all } j \neq i \text{ with } t_j \leqq x_i. \tag{1'}$$

But since $t_i \leqq x_i$ and $f_{ii} \leqq x_i$, (1′) will hold for all j. Letting f_i stand for f_{ii}, we can describe any incentive compatible direct revelation mechanism by means of three n-vectors t, p, and f which satisfy the conditions

$$t_i \leqq x_i \qquad i = 1, \dots, n \tag{3.1}$$

$$f_i \leqq x_i \qquad i = 1, \dots, n \tag{3.2}$$

$$0 \leqq p_i \leqq 1 \qquad i = 1, \dots, n \tag{3.3}$$

and the incentive constraints

$$x_i - [(1-p_i)t_i + p_i f_i] \geqq (1-p_j)(x_i - t_j), \qquad i = 1, \dots, n, x_i \geqq t_j, j \neq i. \tag{3.4}$$

Note that while the set of such triples (t, p, f) is closed, it is not compact, as t_i and f_i are not bounded below. This may not seem important if the principal's objective function is increasing in t and f, but it does matter.

The intuition is this. Any solution to the above problem must force agent to want to tell the truth. There are two ways to do this. One is to punish him for lying by setting $f_{ij} = x_i$ for $i \neq j$, the other is to reward him for telling the truth by making $f_i < 0$. It may be that a large reward for telling the truth can be offset by a tiny audit probability and thus economize on audit costs. Thus if the principal wants to maximize expected revenue net of audit costs, it might pay to offer large rewards and audit with small probabilities.

Example

The example given in Table I demonstrates this intuition. The example uses three wealth levels and presents a family of mechanisms indexed by $\varepsilon > 0$.

TABLE I

$h_1 = 1/2$	$x_1 = 1$	$p_1 = 1 - \frac{(1-\varepsilon)}{2}$	$t_1 = 1$	$f_1 = 1$
$h_2 = 1/4$	$x_2 = 2$	$p_2 = \varepsilon$	$t_2 = 2$	$f_2 = 2 - \frac{1-\varepsilon}{2\varepsilon}$
$h_3 = 1/4$	$x_3 = 3$	$p_3 = 0$	$t_3 = 2 + \varepsilon$	$f_3 = 3$

Simple calculations show that this mechanism is incentive compatible. Suppose the cost of conducting an audit is 1 unit. Then the expected revenue net of audit costs is $9/8 - \varepsilon/8$. Note that $f_2 \to -\infty$ as $\varepsilon \to 0$. Tedious computation shows that 9/8 is the supremum of expected net revenue and is not achievable by any mechanism.

We could stop here and claim that the "solution" to the principal's problem involves offering infinitely large rewards with infinitesimal probabilities, but this is not a satisfactory answer. The entire principal/agent approach is predicated on the principal's ability to make commitments to carry out actions. The offer to pay arbitrarily large rewards must at some point strain the credulity of the agent. The principal's resources (or, at least, the resources of the entire population) place an upper bound on the size of the reward that could be offered. If we do place an upper bound on rewards, then it will in general be a binding constraint. We will require $t_i \geqq 0$ and $f_i \geqq 0$. The choice of zero as a lower bound is somewhat arbitrary. However, provided that taxes and penalties are bounded below the qualitative results do not change.

We thus define a *feasible auditing scheme* to be a triple of n-vectors (t, p, f) satisfying

$$0 \leqq p_i \leqq 1, \qquad i = 1, \ldots, n \tag{4.1}$$

$$0 \leqq t_i \leqq x_i, \qquad i = 1, \ldots, n \tag{4.2}$$

$$0 \leqq f_i \leqq x_i, \qquad i = 1, \ldots, n \tag{4.3}$$

$$x_i - [(1-p_i)t_i + p_i f_i] \geqq (1-p_j)(x_i - t_j), \qquad i = 1, \ldots, n \tag{4.4}$$

$$\text{If } p_i = 0, \text{ then } f_i = t_i \text{ and if } p_i = 1, \text{ then } t_i = x_i. \tag{4.5}$$

In order to facilitate the discussion, the following way of describing auditing schemes is convenient. Let $r_i = (1-p_i)t_i + p_i f_i$, the expected revenue extracted from the agent when his wealth is x_i and he reports truthfully, and let $u_i = x_i - r_i$, the agent's expected utility from truthful reporting. The expected gross revenue when the distribution of wealth is h, is then $h \cdot r$. When $p_i = 0$, then f_i is irrelevant and when $p_i = 1$, then t_i is irrelevant. Condition 4.5 embodies the following conventions: if $p_i = 0$, then $f_i = t_i$ and when $p_i = 1$ then $t_i = x_i$. These conventions make the statement of the results tidier and clearly have no real force. Denote the inequality $x_i - r_i \geqq (1-p_j)(x_i - t_j)$ by $IC\ (i, j)$.

EFFICIENCY

Other things being equal, smaller values of p_i are better because they reduce audit costs. Call a scheme (t, p, f) *audit efficient with respect to h* if it is feasible and there is no other feasible scheme (t', p', f') satisfying $h \cdot r' \geqq h \cdot r$, $p' \leqq p$, and $p' \neq p$. That is, it is not feasible to raise at least as much expected gross revenue and decrease an audit probability without increasing some other audit probability. Call a scheme *audit efficient* if it is audit efficient with respect to h for every $h > 0$. We show below that these notions coincide.

Theorem 1 characterizes schemes which are audit efficient with respect to some h. In order to simplify the statement of the theorem, say that j *attracts* i, denoted $i \to j$, if $i \neq j$, $x_i \geqq t_j$, and $IC(i, j)$ holds as an equality, i.e., $x_i - r_i = (1-p_j)(x_i - t_j)$. Following Corollary 1 we provide a description of our results.

Theorem 1. *The scheme (t, p, f) is audit efficient with respect to $h > 0$, if and only if*

If $i > j$, then

$r_i \geqq r_j$, *with equality if and only if* $p_i = p_j = 0$. (5.1a)

$u_i \geqq u_j$, *with equality if and only if* $u_i = 0$. (5.1b)

For each i, $t_i = x_i$ *or* $f_i = 0$ *or* $p_i = 0$. (5.2a)

For each i, $t_i \geqq r_i \geqq f_i$. (5.2b)

If $i > j$, then

$p_i \leqq p_j$, *with equality if and only if* $p_i = 1$ *or* $p_j = 0$. (5.3a)

$t_i \geqq t_j$, *with equality if and only if* $p_j = 0$. (5.3b)

For each $i > 1$, *there is some* j *for which* $i \to j$. (5.4a)

If $p_j > 0$, *then there is some* $i > j$ *for which* $i \to j$. (5.4b)

If $i \to j$ *and* $0 < p_j < 1$, *then* $i > j$. (5.4c)

If $i > j$, $i \to k$, $j \to m$, *and* $0 < p_m < 1$, *then* $k \geqq m$. (5.4d)

$p_n = 0$. (5.4e)

$n \to n-1$ (5.4f)

If $\bar{\imath} = \max\{j: p_j > 0\}$, *then* $\bar{\imath} + 1 \to \bar{\imath}$. (5.4g)

Let $\underline{i} = \min\{i: p_i < 1\}$. *If* $p_1 > 0$, *then* $u_j = 0$ *if and only if* $j \leqq \underline{i}$. (5.5)

The proof of the theorem is tedious and is reserved for the appendix. The first thing to note about the theorem is that the results (5.1)-(5.5) do not depend on h, the probability distribution of agent types. This is not surprising since we deal only with incentive compatible mechanisms. Thus we have the following corollary.

Corollary 1. *If the scheme (t, p, f) is audit efficient for some $h > 0$, then it is audit efficient.*

There is another simple consequence of our arguments.

Corollary 2. *If (t, p, f) and (t', p, f') are audit efficient and $p_1 > 0$, then $t = t'$ and $f = f'$.*

This corollary is proved in the appendix.

Some of the consequences of audit efficiency are quite striking, others are expected. Result 5.1 says that the higher the realized wealth level of the agent the more he expects to pay to the principal and also the more he expects to keep.

Result 5.2 is quite striking. It implies that for any wealth level, the tax due is equal to the agent's entire wealth ($x_i = t_i$), or the principal promises to take nothing after an audit($f_i = 0$) or else the principal never audits that report ($p_i = 0$). (Both $x_i = t_i$ and $f_i = 0$

are possible simultaneously.) As a result, an agent is always at least as well off after an audit ($t_i \geqq f_i$), so that efficient schemes require in effect the giving of rebates (perhaps of size zero) to the agent after an audit reveals that he reported truthfully. The reason for the result in our model is straightforward. When p_i is fixed, the principal has two instruments to raise a given amount of revenue, r_i, from an agent with wealth x_i: the tax, t_i, and the penalty, f_i. Increasing t_i while reducing f_i in a way that holds r_i constant is beneficial to the principal because it weakens the incentive constraints $IC(k, i)$. Therefore, the principal can improve on a tax scheme in which $f_i > t_i$ by increasing t_i and reducing f_i. We can construct other models in which an agent would prefer not to be audited. Specifically, if some types of agent always tell the truth regardless of the incentive scheme (i.e. are honest), but the principal cannot directly observe honesty, some taxpayers could find it optimal to be dishonest in spite of grave consequences if discovered.[3] Alternatively, audits could be imperfect and the agents could have different information regarding the probability that the principal will discover a lie. Even so, rebates for truthful reporting cannot be ruled out.

Result (5.3) says that audit probabilities decline with reported wealth and that taxes increase with wealth. That taxes are increasing in reported wealth follows for rather subtle reasons. It is easy to show that the incentive constraints require audit probabilities to increase whenever taxes decrease. This result is true even with risk aversion. It is also easy to see that for the risk neutral case, expected payments r_i are increasing in x_i (5.1a): for if $j > i$ and $r_j < r_i$, then $IC(i, k)$ implies that $IC(j, k)$ is not binding for any k, which by (5.4a) is inefficient. Note that this last part of the argument requires the assumption of risk neutrality. Now suppose that there are income levels i and j with $i > j$, but $t_i < t_j$. In particular then, $t_i < x_i$, so by result (5.2), either $f_i = 0$ or $p_i = 0$. Assume for simplicity that both p_i and p_j are positive, the general case being covered in the appendix. Then the only revenue raised from type i is through taxes, since $f_i = 0$. This expected revenue is $(1 - p_i)t_i$ which is less than $(1 - p_j)t_j$, since $t_i < t_j$ and since audit probabilities and taxes move in opposite directions, $p_i \geqq p_j$. But the revenue raised from type j is at least $(1 - p_j)t_j$ which violates the monotonicity of revenue, and so is inefficient. Since the taxes are increasing in wealth, the incentive constraints require that low reported wealth be more likely to be audited in order to keep the agent reporting honestly. A simple consequence of these two results is that a stochastically dominating shift in the distribution of wealth will result in a larger expected net revenue for the principal. The result that the low wealth types are audited more frequently may seem paradoxical, particularly in a tax auditing context. There are two factors that make this more palatable. First it is low reports that trigger audits, not low wealth. This is reasonable because the gains from misrepresentation come from underreporting when taxes are increasing. The other factor is that the probability distribution h can be interpreted as being conditional on observable traits of the agent which are not part of the model. Thus tax collectors can condition on occupation or residential location and then audit low reports for that category of taxpayer.

Part (5.4) of the theorem lists technical results regarding constraints which bind in efficient schemes. Part (5.4b) states that there is no reason to audit reports that are unattractive to other wealth levels; the principal audits to encourage compliance. Part (5.4c) implies that we need only include the incentive constraints that are downward constraints, that is, we need only include $IC(i, j)$ for $i > j$.[4] Part (5.4d) states that the set of reports that attracts a wealth class increases with wealth. Part (5.4e) states that the principal need not audit those agents who report the highest wealth level.

The second corollary states that the vector of audit probabilities completely characterizes an audit efficient mechanism that involves a positive probability of auditing. If the probability of an audit is zero for all reports and the minimum of the distribution of wealth (x_1) is positive, then there is a continuum of efficient schemes, in which all reports pay a tax t which everyone can afford. This is similar to the result (Myerson (1981)) that for an optimal auction, the vector of probabilities of winning the prize completely determines the transfers between the players.

Part (5.5) has the unpleasant consequence that if any auditing at all has positive probability, then an agent with the lowest wealth level will lose all his wealth. If this were not so, since only the downward incentive constraints bind, the principal could increase the revenue raised from a type 1 agent. This increased revenue could then be traded off to reduce auditing probabilities. The details of this argument can be found in Lemma 6 of the Appendix. The condition $p_1 > 0$ is to ensure that some auditing is possible. If $p_1 = 0$, then by (5.3a), $p_i = 0$ for all i and no report is ever audited. This can be audit efficient if $t_i = t < x_1$ for all i. In this case $u_i > 0$ for each i. Such a scheme, while audit efficient, does not raise the maximal expected gross revenue possible for the given vector of audit probabilities. If $p_1 > 0$, then every audit efficient scheme actually maximises expected gross revenue given the vector of audit probabilities.

It follows from (5.3) that if $t_i = t_j$, then $p_i = p_j$. This allows an alternate method of implementing a scheme. Instead of asking the agent to report his wealth, the principal could just ask for a payment of whatever size the agent wishes. The principal could announce an audit schedule as a function of the offered payment and a penalty function as a function the offer and the amount of wealth discovered. Formally, given a scheme (t, p, f) define the schedule

$$\hat{p}(t) = \begin{cases} p_i & \text{if } t = t_i \text{ for some } i \\ 1 & \text{if } t \neq t_i \text{ for any } i \end{cases}$$

This is well defined by (5.3b). Define

$$\hat{f}(x_i, t) = \begin{cases} x_i & \text{if } t \neq t_i \\ f_i & \text{if } t = t_i \end{cases}.$$

If the principal announces that given an amount of tribute t that he will audit with probability $\hat{p}(t)$ and that after an audit he will take $\hat{f}(x_i, t)$ when he was offered t and finds x_i, then a best response of an agent of type i is to offer t_i in tribute. This mechanism is equivalent to (t, p, f) from the point of view of both the agent and principal. This approach seems to be more the norm with brigands than with tax collectors.

We summarize the qualitative features of efficient schemes as follows. There is a (possibly empty) group of wealth levels who report low wealths and are always audited. Agents with these wealth levels who make these reports pay all of their income to the principal. There is a nonempty group of high reports that are never audited. The principal audits intermediate reports with a probability strictly between zero and one. We have no results on the behaviour of marginal tax rates for agents who are audited with a probability strictly between 0 and 1 aside from the immediate consequence of (5.1): the marginal tax rate for these agents is strictly between 0 and 1. While we do not have a simple, general condition that guarantees that a revenue-maximizing scheme requires selecting $0 < p_i < 1$ for some i, these schemes are necessary in general.[5] A routine verification, aided by the results in the theorem, shows that the net revenue maximizing scheme for the example is the element in the family that we described in which $\varepsilon = 1/5$. Thus, for the example, $p_1 = 3/5$ and $p_2 = 1/5$ in the optimal scheme.

While the results of theorem are for the most part intuitively appealing and quite consistent with results in related types of mechanism-design problems, we should remark that the results do no go as far as results in similar models and that our proofs, while elementary, are delicate. In particular, unlike the auction design problem (see Maskin and Riley (1984a, b) and Myerson (1981)), there is no guarantee that the local downward incentive constraints (constraints of the form $(IC(i, i-1))$ bind at an optimum.[6] The fact that only the downward incentive constraints may bind at the optimum allows us to simplify our problem along the lines of Moore (1984). However, this type of analysis provides a qualitative description of efficient schemes rather than an explicit characterization of optima.

The assumption that agents are risk neutral is a particularly strong one. Let us describe which of the qualitative properties of audit efficient programs would hold even if agents were risk averse. The results that we describe follow directly from our proofs for the risk neutral case. We refer the reader to Mookherjee and P'ng (1986) for additional results.[7] The result that honest taxpayers prefer to be audited (5.2b) continues to hold and for the same reasons as in the risk neutral case. In general, when the agents are risk averse it is not efficient to push either t_i to its upper bound or f_i to its lower bound. When agents are risk averse spreading out their possible payments is costly. Indeed, Mookherjee and P'ng show that if the agents' common utility function $u(\cdot)$ satisfies

$$\lim_{c\to\infty} u(c)/c = 0,$$

then there exists a solution to the auditing problem even if penalties are not exogenously bounded below. Part (5.4b) continues to hold when agents are risk averse. From (5.4b) it follows that audit probabilities and taxes move in opposite directions: $p_i \geqq p_j$ if and only if $t_i \leqq t_j$. However, our proof that taxes are monotonically increasing with income uses risk neutrality in an essential way. It is not known if the monotonicity results of (5.1) and (5.3) extend to the risk averse case.[8] Finally, it is not difficult to show that every agent that is not audited pays the same tax, which is the maximum of the t_i. Conversely, precisely those agents paying the highest tax are never audited (5.5).

SPECIAL SCHEMES

In this section we discuss the relationship between special properties of auditing schemes. A scheme (t, p, f) is *lump sum* if there is some t^* such that $f_i = t_i = \min\{x_i, t^*\}$ for each i. That is, the principal has a target revenue t^*; if t^* is unaffordable for the agent, then the principal takes everything. A scheme is *deterministic* if $p_i \varepsilon \{0, 1\}$ for each i. Reinganum and Wilde (1985) showed that with a continuum of income levels and restricting attention to lump sum schemes, then net revenue maximizing schemes are deterministic. This result requires only slight modification in our framework.

Proposition 1. *If (t, p, f) is audit efficient and lump sum with $x_j \leqq t^* < x_{j+1}$, and if $p_1 > 0$, then $p_i = 1$ for $i < j$, $p_i = 0$ for $i > j$ and $p_j = (t^* - x_j)/(x_{j+1} - x_j)$. In particular, if $t^* = x_j$, then (t, p, f) is deterministic.*

Proof. If $i \leqq j$, then $u_i = 0$, so by (5.5), $p_i = 1$ for $i < j$. If $i > j$, then (5.3b) implies $p_i = 0$. If $x_j = t^*$, then (5.3b) implies $p_j = 0$. If $x_j < t^*$, then (5.3b) implies $p_j > 0$, so (5.4g) implies $x_{j+1} - t^* = (1 - p_j)(x_{j+1} - x_j)$, from which the lemma follows. ‖

Proposition 2. *If (t, p, f) is audit efficient and deterministic, then it is lump sum.*

Proof. Since (t, p, f) is deterministic, then

$$\underline{i} = \min\{i \colon p_i < 1\} = \min\{i \colon p_i = 0\}.$$

If $p_1 = 0$, then (5.3b) and (6.5) imply that (t, p, f) is lump sum with $t^* \leqq x_1$. If $p_1 > 0$, then by (5.5), $u_i = 0$ for $i \leqq \underline{i}$, so $p_i = 1$ and $f_i = x_i (= t_i$ by (6.5)). By (5.3) for $i > \underline{i}$, $t_i = x_{\underline{i}}$ ($= f_i$ by (6.5)). Thus (t, p, f) is lump sum with $t^* = x_{\underline{i}}$. ||

It follows from Proposition 2 that $f_i > 0$ whenever $x_i > 0$ for deterministic schemes. This property in fact characterizes deterministic schemes in which some auditing takes places.

Proposition 3. *If (t, p, f) is audit efficient and $p_1 > 0$, if $x_i > 0$ and $f_i = 0$, then $0 < p_i < 1$.*

Proof. If $p_i = 0$, then by (6.5), $t_i = f_i = 0$. But then $IC(j, i)$ implies $r_j = 0$ for all j, so efficiency implies $p_j = 0$ for all j, contradicting $p_1 > 0$. If $p_i = 1$, then $r_i = p_i f_i = 0$. Then (5.1a) implies $i = 1$. By (5.3e) and (5.5), then $u_i = 0$, contradicting $x_i > 0$ and $r_i = 0$. ||

The next result characterizes efficient schemes when the principal's ability to make commitments is imperfect. Specifically we consider the case where the only credible commitment is to take everything after an audit, regardless of whether the truth was told.

Proposition 4. *If the principal is restricted to schemes in which he appropriates the agent's entire wealth after an audit, that is, if $f_i = x_i$ for all i, then there exists a revenue maximizing auditing scheme that is deterministic.*

When the condition of Proposition 4 is met our problem reduces to a standard mechanism-design problem. In particular, local downward incentive constraints bind; we can use this fact, and familiar arguments (see, for example, Myerson (1981) and Maskin and Riley (1984*a*)) to show that we can take the auditing scheme to be deterministic. We point out that having the local incentive constraints bind is not of itself sufficient to obtain a deterministic scheme, as our previous example demonstrates.

This result underscores the importance of the ability of the principal to make commitments. Since in general, $f_i = 0$ for some i, the agent must believe that the principal after conducting an audit will be satisfied with taking nothing even after having gone to the expense of an audit. Peasant villagers might never find this a credible promise by a band of brigands because it is ex post optimal for the brigands to carry off everything. If that is so, then the brigands may be forced to use a scheme with $f_i = x_i$ for all i, which is in general not a net revenue maximizing scheme. The brigands then have an incentive to create a commitment mechanism to make the schedules credible. One provocative speculation about how the schedules might be made believable is for it to be common knowledge that there is a strong and vindictive god by which the brigands could swear oaths.[9] Another possibility for the brigands might be to create a bureaucracy for the enforcement of the schedules and incentives for bureaucrats which would keep them from making ex post optimal decisions, to the ex ante benefit of the brigands.

APPENDIX: PROOFS

Proof of Necessity in Theorem 1

In order to prove necessity in Theorem 1 it is convenient to reformulate the problem. Set $q_i = (1 - p_i)$, the probability that the report x_i is not audited (and hence the payment is t_i). If $q_i < 1 (p_i > 0)$, then $f_i = (r_i - q_i t_i)/(1 - q_i)$. When $q_i = 1$ $(p_i = 0)$ then our conventions set $f_i = t_i$. Thus given (r, t, q) we can recover (t, p, f). In this framework $IC(i, j)$ becomes

$$x_i - r_i \geqq q_j (x_i - t_j) \tag{4.4'}$$

and (4.3) becomes

$$r_i \geqq q_i t_i. \tag{4.3'}$$

The advantage of this formulation is that each r_i only enters the left-hand side of the constraints and q_i and t_i enter only the right-hand sides. This makes the effect of changes easier to keep track of. Also note that 4.4′ holds for all i and j.

Also define $A(i)=\{j: i\to j\}$ and $A^{-1}(j)=\{i: i\to j\}$.

The proof of necessity is divided into several lemmas. The following table indicates which lemmas are used directly in the proof of necessity of any part of the theorems. There are some statements in the theorem which do not follow immediately from any lemma, but the details are easily filled in.

Part of Theorem	Lemmas that Apply
5.1	7
5.2	2
5.3	12, 13, 14
5.4a	18
5.4b	1
5.4c	15
5.4d	19
5.4e	17a
5.4f	17c, 14
5.4g	17c
5.5	21

Throughout this section the statements of all results are understood to be preceded by the phrase, "If (r, t, q) is audit efficient with respect to $h>0$ then. . . ."

Lemma 1. *If $q_j<1$, then there exists i such that $i\to j$.*

Proof. If not, increase q_j adjusting t_j so that $r_j \geqq q_j t_j$ continues to hold. This violates no constraints, contradicting efficiency. ‖

Lemma 2. *For each j, $t_j=x_j$ or $r_j=q_j t_j$ (or both).*

Proof. If $q_j=1$, then $r_j=q_j t_j=t_j$. If $q_j<1$ and $t_j<x_j$ and $r_j>q_j t_j$, increase t_j. This weakens $IC(i,j)$ for all i and makes $A^{-1}(j)$ empty, contradicting Lemma 1. ‖

Lemma 3. *Suppose $r_k=q_k t_k$, $x_k>0$, $q_i<1$, and $j\to k\to i$. Then $q_k>q_i$ and $j>k$.*

Proof. $IC(j,i)$ and $j\to k$ imply

$$q_k(x_j-t_k)=u_j \geqq q_i(x_j-t_i). \tag{1}$$

Combining $r_k=q_k t_k$ and $k\to i$ yields

$$q_i(x_k-t_i)=x_k-r_k \geqq q_k x_k - q_k t_k. \tag{2}$$

Rewrite $k\to i$ as $(1-q_i)x_k=r_k-q_i t_i$ and use $q_i<1$, $x_k>0$, and $r_k=q_k t_k$ to conclude $q_k t_k-q_i t_i>0$. Rearrange (1) and (2) to get $x_j(q_k-q_i)\geqq q_k t_k-q_i t_i$ and $q_k t_k-q_i t_i \geqq x_k(q_k-q_i)$, respectively. The first inequality implies $q_k>q_i$ and so together they imply $x_j>x_k$. ‖

Lemma 4. *If $q_i\geqq q_j$ and $q_j>0$, then $t_i\geqq t_j$.*

Proof. If $q_j=1$, then $IC(j,i)$ becomes $x_j-t_j\geqq x_j-t_i$, so $t_i\geqq t_j$. If $0<q_j<1$, then by Lemma 1 there is some k with $k\to j$. Then $IC(k,j)$ and $IC(k,i)$ imply $u_k=q_j(x_k-t_j)\geqq q_i(x_k-t_i)$, so $t_i\geqq t_j$. ‖

Lemma 5. *If $q_j=1$, $i\to j$ and $r_i=q_i t_i$, then $q_i=1$ and $t_i=t_j$.*

Proof. Since $i\to j$ and $q_j=1$, $t_j=r_i=q_i t_i$. If $t_j>0$, it follows that $q_i>0$. Then Lemma 4 implies $q_i=1$ and $t_j=t_i$. If $t_j=0$, then $IC(k,j)$ implies $r_k=0$ for all k, and so efficiency implies $q_k=1$ for all k. ‖

Lemma 6. *For all i,, $u_i=0$ or $q_i=1$ or for some j, $i\to j$.*

Proof. Suppose $u_i>0$, $q_i<1$, but $i\to j$ for no j. Then r_i can be increased. This allows us to reduce the revenue raised from other types in such a way as to allow us to increase q_i, contradicting efficiency.

The obstacles to increasing q_i are the constraints $IC(k, i)$. These will be removed if we can reduce r_k for $k \varepsilon A^{-1}(i)$. If $r_k > q_k t_k$ just reduce it. If $x_k = q_k t_k$, there are two cases, $r_k = 0$ and $r_k > 0$. If $r_k = 0$, then $k \to i$ implies $x_k = q_i(x_k - t_i)$. If $x_k = 0$, then $t_i = 0$ (since $x_k \geqq t_i$) and we can increase q_i without violating $IC(k, i)$. If $x_k > 0$, then $t_i = 0$ and $q_i = 1$, contrary to hypothesis. Thus we need only deal with the case $r_k = q_k t_k > 0$. In order to reduce r_k, we need to reduce t_k. The obstacles to this are the constraints $IC(j, k)$.

Thus we have reduced the problem of weakening $IC(k, i)$ for all $k \in A^{-1}(i)$ to the problem of weakening $IC(j, k)$ for all $j \in A^{-1}(k)$ where $x_k > 0$ and $r_k = q_k t_k > 0$. Proceeding recursively, Lemmas 1 and 3, tells us that we need only show how to reduce $IC(j, k)$ for all $j \in A^{-1}(k)$ where $q_k = 1$ and $r_j = q_j t_j$. By Lemma 5, $q_j = 1$ and $t_j = t_k$ and furthermore $j' \to j$ implies $j' \to k$. Thus we can weaken $IC(j, k)$ for all such j by decreasing t_j and t_k the same amount.

Thus we have shown that we can increase q_i keeping all other q_j's the same by reducing the amount of revenue extracted from types other than i. If we keep these changes small relative to the increase in r_i, the same overall revenue can be extracted. This contradiction to efficiency proves the lemma. ||

Lemma 7. *If $i > j$, then*

(a) $r_i \geqq r_j$.

(b) $u_i \geqq u_j$ *and if* $u_i > 0$, *then* $u_i > u_j$.

Proof. It follows from Lemma 6 that $r_i = \min_k \{(1 - q_k)x_i + q_k t_k\}$, which is increasing in x_i. Similarly, $u_i = \max_k q_k(x_i - t_k)$, from which 7(b) follows. ||

Lemma 8. *If for some $x_i > 0$, $t_i = 0$, then for all j, $q_j = 1$ and $r_j = t_j = 0$.*

Proof. If $q_i = 1$, then $IC(k, i)$ implies $r_k = 0$ for all $k \neq i$ and efficiency implies then that $q_k = 1$ for all k. If $q_i < 1$, then Lemma 2 implies $r_i = q_i t_i = 0$, so $u_i = x_i > 0$. Thus Lemma 6 implies that for some k, $i \to k$, i.e., $x_i = q_k(x_i - t_k)$ so $q_k = 1$ and $t_k = 0$. The first case now applies to k. ||

Lemma 9. *If $i > j$ and $t_i \leqq t_j$, then $q_i \geqq q_j$.*

Proof. It follows from Lemmas 2 and 7(a) that $q_i t_i = r_i \geqq r_j \geqq q_j t_j$. Thus if $t_i > 0$, $q_i \geqq q_j$. If $t_i = 0$, it follows from Lemma 8 that $q_i = q_j = 1$. ||

Lemma 10. *If $t_i = t_j$, then $q_i = q_j$.*

Proof. Let $t = t_i = t_j$. If $t = 0$, Lemma 8 applies. Suppose $t > 0$ and $q_i > q_j$. Then by Lemma 1, there is some k with $k \to j$. Combining $k \to j$ and $IC(k, i)$ yields $u_k = q_j(x_k - t) \geqq q_i(x_k - t)$. Thus $x_k = t = r_k = t_k$ and $u_k = 0$. Now $x_j \geqq t = x_k$ implies $x_j > x_k = t$, so by Lemmas 2 and 7(a) we have $q_j t = r_j \geqq r_k (= t)$, which implies $q_j = 1$ or $t = 0$, both contradictions. Thus $q_i \leqq q_j$ and a symmetric argument proves $q_i = q_j$. ||

Lemma 11. *If $i > j$ and $q_i = 0$, then $q_j = 0$.*

Proof. If $q_i = 0$, then by (6.5) $x_i = t_i$. Lemma 1 implies the existence of $k \in A^{-1}(i)$. $IC(k, j)$ and $k \to i$ imply $0 = q_i(x_k - x_i) = u_k \geqq q_j(x_k - t_j)$, but $x_k \geqq t_i = x_i > x_j \geqq t_j$, so $q_j = 0$. ||

Lemma 12. *If $i > j$, then $t_i \geqq t_j$ and $q_i \geqq q_j$.*

Proof. We first show that $q_i \geqq q_j$. If $t_i < t_j$, then Lemma 9 implies $q_i \geqq q_j$. If $t_i = t_j$, then Lemma 10 gives the result. If $t_i > t_j$, suppose that $q_i \geqq q_j$ is false, i.e. that $q_i < q_j$. Then the contrapositive of Lemma 4 (exchanging the roles of i and j) implies $q_i = 0$, so $q_j = 0$ by Lemma 11. Thus $q_i \geqq q_j$.

If $q_j > 0$, then Lemma 4 implies $t_i \geqq t_j$. If $q_j = 0$, then Lemma 1 implies that there is some $k \in A^{-1}(j)$ and so $k \to j$ and $IC(k, i)$ imply $0 = u_k \geqq q_i(x_k - t_i)$. It follows that either $x_k = t_i$ or $q_i = 0$. If $t_i = x_k$, then $x_k \geqq t_j$ implies $t_i \geqq t_j$. If $q_i = 0$, then $t_i = x_i$ by convention, so $i > j$ implies $t_i \geqq t_j$. ||

Lemma 13. *If $0 < q_i = q_j < 1$, then $i = j$.*

Proof. It follows from Lemma 4 that $t_i = t_j$. We now assume $i > j$ and argue to a contradiction. We have $x_i > x_j \geqq t_j = t_i$; therefore $u_i > 0$ and

$$r_j \geqq q_j t_j = q_i t_i = r_i \tag{1}$$

where the last equality follows from Lemma 2. Since $i>j$, Lemma 7a and (1) imply that

$$r_i = r_j. \tag{2}$$

Lemma 6 implies that for some k,

$$x_i - r_i = q_k(x_i - t_k) = q_k(x_i - x_j + x_j - t_k) \leqq q_k(x_i - x_j) + x_j - r_j \tag{3}$$

where the inequality follows from $IC(j, k)$. Since $x_i - x_j > 0$, (2) implies that (3) holds only if $q_k = 1$ and therefore

$$t_k = r_i = r_j \tag{4}$$

and thus

$$q_i t_i = r_i = t_k \geqq t_i \tag{5}$$

where, in (5), the first equality follows from (1), the second equality from (4) and the inequality from Lemma 12 since $q_k = 1 > q_i$. Thus (5) implies $t_i = 0$, so by Lemma 8, $q_i = q_j = 1$, a contradiction. ‖

Lemma 14. *If $i \neq j$, then $t_i = t_j$ if and only if $q_i = q_j = 1$.*

Proof. From Lemma 10, $t_i = t_j$ implies $q_i = q_j$. Thus Lemma 13 implies that if $i \neq j$ and $t_i = t_j$, then either $q_i = q_j = 0$ or $q_i = q_j = 1$. However, if $i \neq j$ and $q_i = q_j = 0$, then $t_i = x_i \neq x_j = t_j$. This proves that if $i \neq j$ and $t_i = t_j$, then $q_i = q_j = 1$.

Conversely if $q_i = q_j = 1$, then $r_i = t_i$ and $r_j = t_j$. Thus $IC(i, j)$ and $IC(j, i)$ combine to imply that $t_i = t_j$. ‖

Lemma 15. *If $0 < q_j < 1$ and $i \to j$, then $i > j$.*

Proof. Suppose $j > i$. Then $i \to j$ and Lemmas 12 and 14 imply $x_j > x_i \geqq t_j > t_i$. Thus Lemmas 2 and 12 imply $r_j = q_j t_j > q_i t_i = r_i$. But $i \to j$ and $r_j = q_j t_j$ yield $r_i - r_j = (1 - q_j)x_i \geqq 0$, a contradiction. ‖

Lemmas 1 and 15 combined have the following immediate consequence.

Lemma 16. *If $q_j < 1$, then for some $i > j$, $i \to j$.*

Lemma 17. *If $\bar{\imath} = \max\{j: q_j < 1\}$, then*

(a) $\bar{\imath} < n$.
(b) *For all $i \neq j$, if $i, j > \bar{\imath}$, then $i \to j$.*
(c) $\bar{\imath} + 1 \to \bar{\imath}$.

Proof. Part a follows directly from Lemma 16. To prove the remainder of the lemma, recall that Lemma 14 implies that for all $i, j > \bar{\imath}$

$$t_i = t_j. \tag{1}$$

Part b follows directly from $q_i = q_j = 1$ and (1). Finally, Lemma 16 guarantees that there exists an $i > \bar{\imath}$ such that $i \to \bar{\imath}$. Therefore, by IC, we have

$$x_i - r_i \geqq q_{\bar{\imath}}(x_i - t_i)$$
$$\text{or} \quad x_i \geqq (r_i - q_i t_i)/(1 - q_{\bar{\imath}}), \tag{2}$$

with equality for some $i > \bar{\imath}$. However, the right-hand side is independent of i for $i > \bar{\imath}$ because of (1) and the definition of $\bar{\imath}$. Therefore, (2) holds as a strict inequality if $i > \bar{\imath} + 1$. ‖

Lemma 18. *If $i > 1$, then there is some j for which $i \to j$.*

Proof. If $q_i = 1$, Lemma 17 applies. If $u_i > 0$ and $q_i < 1$, then Lemma 6 applies. If $u_i = 0$ and $i > 1$, then $IC(i, 1)$ implies $i \to 1$. ‖

Lemma 19. *If $\bar{\imath} \geqq j$ and $i > j$, where $\bar{\imath}$ is as defined in Lemma 17, then $r_i > r_j$.*

Proof. Lemma 6 implies

$$r_j = \min_k \{(1 - q_k)x_j + q_k t_k\}. \tag{1}$$

Lemma 14 implies that if $k > \bar{\imath}$, then $t_i = t_n$. Lemma 17c implies that

$$t_n = (1 - q_{\bar{\imath}})x_{\bar{\imath}+1} + q_{\bar{\imath}} t_{\bar{\imath}}. \tag{2}$$

Thus for $j \leqq \bar{\imath}$,

$$r_j = \min_{k \leqq \bar{\imath}} \{(1 - q_k)x_j + q_k t_k\}. \tag{3}$$

Then if $i > j$ and $j \leqq \bar{\imath}$, since $x_i > x_j$ and $q_k < 1$ for $k \leqq \bar{\imath}$,

$$r_i = \min_k \{(1 - q_k)x_i + q_k t_k\} > \min_{k \leqq \bar{\imath}} \{(1 - q_k)x_j + q_k t_k\} = r_j. \quad ‖$$

Lemma 20. *If $i \to k$, and $j \to m$, $0 < q_m < 1$ and $i > j$, then $k \geqq m$.*

Proof. Together $i \to k$ and $IC(i, m)$ imply that

$$q_k(x_i - t_k) = u_i \geqq q_m(x_i - t_m). \quad (1)$$

Similarly, $j \to m$ and $IC(j, k)$ imply that

$$q_m(x_j - t_m) = u_j \geqq q_k(x_j - t_k). \quad (2)$$

Combining (1) and (2) and rearranging terms yields

$$x_i(q_k - q_m) \geqq q_k t_k - q_m t_m \geqq x_j(q_k - q_m). \quad (3)$$

Consequently, if $i > j$, then $q_k \geqq q_m$. The lemma now follows from Lemmas 12 and 13. ||

Lemma 21. *Let $\underline{i} = \min\{i: q_i > 0\}$. If $q_1 < 1$, then $u_j = 0$ if and only if $j \leqq \underline{i}$.*

Proof. By Lemma 15, there is no k with $1 \to k$, so by Lemma 6, $u_1 = 0$. Suppose $1 < j \leq \underline{i}$. By Lemmas 18 and 15 there is some $k < j$ with $j \to k$, but $k < j$ implies $q_k = 0$, so $j \to k$ implies $u_j = 0$. If $j > \underline{i}$, then $IC(j, \underline{i})$ implies $u_j \geqq q_{\underline{i}}(x_j - t_{\underline{i}}) > 0$. ||

Sufficiency

We now prove the sufficiency part of Theorem 1. In fact, only properties (5.1a), (5.2a), (5.3a), (5.4a, b, d) and (5.5) are used. In this section efficiency of (r, t, q) is not assumed.

Lemma 22. *If (r, t, q) and (r', t', q') are feasible and (r, t, q) satisfies* (5.2a) *and* (5.4b) *and $r = r'$ and $q' \geqq q$, then $q' = q$.*

Proof. Suppose $q'_j > q_j$ for some j. Then either

$$q'_j(x_i - t'_j) > q_j(x_i - t_j) \quad \text{for any } i > j \quad (1)$$

or

$$t'_j > t_j. \quad (2)$$

By 5.4b, since $q'_j > q_j$, there is some $i > j$ for which $x_i - r_i = q_j(x_i - t_j)$. If (1) holds, then $x_i - r'_i = x_i - r_i = q_j(x_i - t_j) < q'_j(x_i - t'_j)$, contradicting $IC(i, j)'$. On the other hand if (2) holds, then $r'_j \geqq q'_j t'_j > q_j t_j = r_j$, where the equality follows from (5.2a) and $x_j \geqq t'_j > t_j$. This contradiction establishes the lemma. ||

Lemma 23. *If (r, t, q) and (r', t', q') are feasible and (r, t, q) satisfies* (5.2a), *and $q' \geqq q$, and if $r_j \geqq r'_j$ and $i \to j$, then $r_i \geqq r'_i$.*

Proof. It suffices to show that

$$q'_j(x_i - t'_j) \geqq q_j(x_i - t_j), \quad (1)$$

for then $i \to j$ and $IC(i, j)'$ yield $x_i - r_i = q_j(x_i - t_j) \leqq q'_j(x_i - t'_j) \leqq x_i - r'_i$, so $r_i \geqq r'_i$.

If $t'_j > t_j$, then by (5.2a), $r_j = q_j t_j$. Also $q'_j t'_j \geqq q_j t_j$ with equality only if $q'_j = q_j = 0$. But $r'_j \geqq q'_j t'_j \geqq q_j t_j = r_j \geqq r'_j$, so $q'_j = q_j = 0$, so (1) holds.

If $t'_j \leqq t_j$, then (1) holds unless $x_i - t'_j < 0$. But then $i \to j$ implies $0 > x_i - t'_j \geqq x_i - t_j \geqq x_i - r_i$, which violates feasibility. ||

Lemma 24. *Let (r, t, q) and (r', t', q') be feasible and let (r, t, q) satisfy* (5.1a), (5.2b), (5.3a), (5.4a, b, d) *and let $q' \geqq q$. If $r_1 \geqq r'_1$, then for all i, $r_i \geqq r'_i$.*

Proof. The proof is inductive. Suppose $r_j \geqq r'_j$ for all $j \leqq k$. If $q'_k = q_k = 1$, then (5.1a) and (5.3a) imply that for any $i > k$, $q_i = 1$ and $r_i = r_k \geqq r'_k \geqq r'_i$, where the last inequality follows from $IC(i, k)'$. If $q_k < 1$, then by (5.4b) there is some $i > k$ with $i \to k$, so by Lemma 23, $r_i \geqq r'_i$. Now let $k < l < i$. By (5.4a), for some j, $l \to j$, then by (5.4d), $j \leqq k$, so again Lemma 23 implies $r_l \geqq r'_l$.

Proof of Sufficiency in Theorem 1. *Let (r, t, q) be feasible and satisfy* (5.1a), (5.2a), (5.3a), (5.4a, b, d), (5.5). *Then it is audit efficient.*

Proof. If $q_1 = 1$, then by (5.3a), $q_i = 1$ for all i, so (r, t, q) is audit efficient.

If $q_1 < 1$, let (r', t', q') be feasible and satisfy $h \cdot r' \geqq h \cdot r$ and $q' \geqq q$. By (5.5) $u_1 = 0$, so $r_1 = x_1 \geqq r'_1$. By Lemma 24, $r \geqq r'$. But $h > 0$, $r \geqq h \cdot r$ imply $r = r'$. Therefore by Lemma 22, $q' = q$. Thus $(r', q') = (r, q)$, so (r, t, q) is audit efficient. ||

Proof of Corollary 2. *If (r, t, q) and (r', t', q) are audit efficient and $q_1 < 1$, then $r = r'$ and $t = t'$.*

Proof. By Lemma 21, $q_1 < 1$ implies that $z_1 = z'_1 = x_1$. Thus, $t_1 = t'_1 = x_1$. The result now follows from Lemma 24. ||

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NOTES

1. More generally, a mechanism would provide a lottery on taxes, etc. for each message, but with a risk neutral agent there is no point in doing this.
2. This point was made by Green and Laffont (1986) who examined problems in which exogenous restrictions may limit the set of messages available to a type. In this model, the limitation on the set of messages that an agent may use is determined by the mechanism.
3. In the context of a model in which the IRS chooses only the audit policy and is not able to make commitments, Graetz, Reinganum and Wilde (1986) show that if some taxpayers are honest, then there is a mixed strategy equilibrium which involves high income taxpayers misreporting their income with some positive probability.
4. Part 4c implies that we can delete, without loss of generality, constraints of the form $IC(j, i)$ for $i > j$ from the optimization problem provided that $0 < p_j < 1$. If $p_j = 0$, then $t_j = x_j$. Thus, for $i > j$, by (5.2b) $t_i > t_j = x_j$ and $IC(j, i)$ is automatically satisfied. If $p_j = 1$, then an efficient scheme involves $p_i = p_j$, $t_i = t_j$ for all $i > j$ whether or not we include $IC(j, i)$ in the set of constraints.
5. If the spacing between income levels is uniform, say $x_{i+1} - x_i = 1$, and the distribution of incomes is decreasing and exponential, so that $h_{i+1} = \alpha h_i$ for $0 < \alpha < 1$, then there always exists a nontrivial interval of costs for which stochastic auditing dominates deterministic policies.
6. For example, the scheme below is audit efficient, but IC (3, 2) does not bind.

x	t	p	f
1	1	$\frac{1}{2}$	1
2	2	$\frac{1}{4}$	0
3	$2\frac{2}{7}$	$\frac{1}{8}$	0
4	$2\frac{1}{2}$	0	—

7. Mookherjee and P'ng use a different objective function than we do; they look for schemes that maximize the sum of a concave function of individual utilities. This modification does not change what you can say about efficient auditing schemes.
8. When agents are risk averse it may be optimal to have taxes be a random function of reports. We ignore this possibility. When there are only two income levels, the monotonicity results hold.
9. This was once suggested by Ed Green.

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