Econometrica, Vol. 59, No. 5 (September, 1991), 1455–1485

DURABLE GOODS MONOPOLY WITH ENTRY OF NEW CONSUMERS

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This paper analyzes a model of a dynamic monopolist who produces at constant unit cost. Each period a new cohort of consumers enters the market. Each entering cohort is identical. Consumers within a cohort have different tastes for the good. My main results are: If players are sufficiently patient, any positive average profit less than the maximum feasible level can be attained in a subgame-perfect equilibrium; in the subset of subgame-perfect equilibria in which players use stationary strategies, the seller cannot make sales at prices significantly greater than the lowest willingness to pay when period length goes to zero; and the seller attains the maximum profit when commitment is feasible by charging the same (static monopoly) price in every period.

KEYWORDS: Dynamic monopoly, repeated games, folk theorems, durable goods, Coase conjecture.

1. INTRODUCTION

THIS PAPER ANALYZES a model of a dynamic monopolist who operates in a market in which there is a regular flow of new consumers. The seller produces a durable good at constant unit cost. Each period a new cohort of consumers enters the market. Consumers wish to buy exactly one unit of the item, but are willing to wait. Resales are not allowed. Each entering cohort is identical. Consumers within a cohort have different tastes for the good.

Allowing entry of new consumers changes the character of equilibria in the dynamic monopoly model. For interesting parameter values, the equilibrium specifies that the seller charge a relatively high price in most periods, selling only to buyers with high valuations. Periodically she cuts her price to sell to a large accumulation of buyers with lower valuations. After such a market-clearing sale, the pricing cycle begins again. My main result is that, if players are sufficiently patient, any positive average profit less than the maximum feasible level can be attained in a subgame-perfect equilibrium. I also show that the subset of subgame-perfect equilibria in which players use stationary strategies have the property that when the length of the time period goes to zero, the monopolist seller cannot make sales at prices significantly greater than the lowest willingness to pay. This result agrees with the arguments of Coase (1972)

[1] I am happy to thank people who have helped me with this paper. Gene Grossman and Michael Katz were the first to point out that the equilibrium of Conlisk, Gerstner, and Sobel (1984) need not be subgame perfect. Dilip Abreu, Bob Anderson, Andreas Blume, John Conlisk, Vincent Crawford, Ray Deneckere, Darrell Duffie, Ian Gale, Oliver Hart, Mark Machina, Garey Ramey, Valerie Ramey, Hugo Sonnenschein, Max Stinchcombe, Kit Woolard, and seminar audiences at Northwestern, Pittsburgh, Toronto, UC Berkeley, UC San Diego, Wisconsin, and York made useful suggestions. I am especially grateful to two referees and a co-editor who carefully read the manuscript and provided detailed comments that greatly improved the form and content of the paper. MSRI, NSF, and the Sloan Foundation provided financial support. I finished the first draft of this paper while visiting the University of Wisconsin. I thank my colleagues there for helping to make my visit enjoyable and productive.

and the analyses of Stokey (1981), Bulow (1982), and Gul, Sonnenschein, and Wilson (1986) who analyze durable-goods monopoly models without entry of new customers.

Ausubel and Deneckere (1989a) prove a folk theorem for the dynamic monopoly problem without entry of consumers. They show that any positive level of average profit less than the static monopoly profit can be achieved in a subgame-perfect equilibrium if players are sufficiently patient, the market demand function satisfies a mild regularity condition, and the lowest buyer valuation is not greater than the seller's (constant marginal) cost or production.[2] My result is similar. The assumption that there is a stationary inflow of new consumers replaces the assumption that the marginal cost curve intersects the demand curve. Like Ausubel and Deneckere, I prove the folk theorem by supporting an equilibrium path with a punishment that leads to lower profits. In my treatment in order to approximate the static monopoly average profit the punishment must be more severe than reverting to the stationary equilibrium. The stationary equilibrium profit is not a lower bound for the seller's equilibrium profit. If players are sufficiently patient, then there exist equilibrium outcomes where the seller charges strictly less than the lowest valuation in every period.

Bond and Samuelson (1984 and 1987) analyze another variation of the dynamic monopoly problem. They assume that the durable good depreciates; they permit resales. The monopolist producer must make replacement sales in order to maintain the stock of the good. Therefore equilibria necessarily involve transactions over infinitely many time periods. Bond and Samuelson (1984) show that the stationary equilibria of their model satisfy a version of the Coase conjecture: As the period length shrinks to zero, the time it takes the seller to supply the competitive quantity approaches zero. Their 1987 paper constructs nonstationary equilibria in which the seller is able to restrict supply to the monopoly profit maximizing level.

Ausubel and Deneckere, Bond and Samuelson, and I identify different models in which the monopolist makes sales over infinitely many time periods. Under these circumstances, nonstationary equilibria exist because behavior at the tail end of the game is not determined. While stationary equilibria of these models exhibit the Coase property, the nonstationary equilibria generally do not. The essential difference between these papers and mine is that by assuming a steady inflow of new consumers and the absence of resales, equilibria in my model involve cyclic variations in price.

Prohibiting resales plays a critical role in my model. If an efficient resale market existed, then the entire stock of the good is on the market at each point. Since consumers with the highest valuations keep the item, the seller need only

[2] In other work, Ausubel and Deneckere (1985) analyze a model in which a monopolist produces a sequence of goods and faces a growing market demand. They assume that consumers are uncertain about the seller's constant marginal cost of production, and construct an equilibrium in which the low cost type of seller earns high profits by imitating the strategy of the highest cost type of seller.

keep track of total sales to determine the residual demand. Without resales, the seller must know the number of consumers with each willingness to pay that remain in the market. Since buyers have different tastes, the state of the market in this model is at least two dimensional. Increasing the dimensionality of the state space adds a complication to the analysis not found in the Ausubel-Deneckere and Bond-Samuelson papers.

I describe the model in Section 2. Section 3 discusses stationary equilibria. Stationary equilibria exist in the model. I discuss the effect of shrinking the period length, and show that the number of periods between market-clearing sales is bounded above by a number that does not depend on the period length or the discount factor. Consequently, as the period length shrinks to zero, the length of time between sales also shrinks to zero. This observation proves that the Coase conjecture holds for stationary equilibria in my model. I prove the folk theorem for seller's payoffs in Section 4. In Section 5 I describe the maximum feasible level of average profits. The seller's maximum average profit if she can commit to a selling mechanism is her monopoly profit in the static model. She attains the maximum profit by charging the same price in every period. Section 6 discusses related papers. Conlisk, Gerstner, and Sobel (1984), henceforth CGS, were the first to study this model. They use a different solution concept, and obtain different results. I explain the differences in Section 7.

2. THE MODEL

The monopolist faces a uniform group of nonatomic consumers indexed by $t \geqslant 0$ and by $j = 1$ or 2. Time periods are discrete and indexed by the nonnegative integers. In period n new consumers with indices (t, j) for $n \leqslant t < n+1$ and $j = 1$ and 2 enter the market. The measure of an interval (t_1, t_2) of consumers with $j = 1$ is $\alpha(t_2 - t_1)$ and of consumers with $j = 2$ is $(1-\alpha)(t_2 - t_1)$, where α is strictly between zero and one. Each consumer wishes to buy just one unit of the product. Once a consumer buys the product, he leaves the market forever. Resales are forbidden[3] and the good does not depreciate. Consumers cannot buy before they enter the market. The utility of a consumer indexed by t and j who buys in period n at the price p is $\beta^n(V_j - p)$ provided that $n \geqslant [t]$, where $[t]$ is the greatest integer less than or equal to t. The discount factor β is assumed to be strictly between zero and one. A consumer who never makes a purchase receives utility zero. V_j as the maximum willingness to pay of a consumer indexed by j. I assume that $V_1 > V_2 > 0$. When a consumer of type (t, j) first enters the market, consumers of type (s, i) for $s < t$ and $i = 1$ or 2 who have yet to make a purchase are still in the market.[4] Consumers differ in at

[3] I can justify the no-resales assumption in several ways. First, there is nothing in the model that requires the good to supply a flow of services. Buyers may consume the good completely in the period that they buy it. Second, it may be too costly for an individual to market a single item. Third, buyers may doubt the quality of the good unless it is supported by the monopolist's reputation.

[4] When $[t] = [s]$, consumers of type (t, j) and (s, j) may first buy in the same period, $n = [t]$. However, if $s < t$, then the type (t, j) buyer enters the market after the type (s, j) buyer. The distinction simplifies the definition of stationary strategies.

most two ways: when they enter the market, and how much they are willing to pay for the item. Two consumers with the same j index have cardinally equivalent preferences once they are both in the market. The monopolist can produce at constant unit cost, assumed without further loss of generality to be zero. The monopolist maximizes the expected present value of revenue; she shares the consumers' discount factor β.

In every period, following the entry of new consumers, the monopolist sets a price. Consumers in the market then decide whether to accept or reject this price. A history at date n is a complete description of what has happened in the past. It includes all past prices and purchase decisions of consumers. The monopolist's strategy specifies a price to charge in each period as a function of the history of the game. A strategy for a consumer specifies whether or not the consumer will accept the monopolist's current price given a history and the current price. I am interested in characterizing the subgame-perfect Nash equilibria (henceforth simply equilibria) of this game.

As in Gul, Sonnenschein, and Wilson (1986), I make a technical assumption that restricts the set of equilibria and makes it easier to describe the equilibria that remain. I assume that the equilibrium actions of each agent are constant on histories in which prices offered are the same and the sets of consumers who accept in each time period differ by at most sets of measure zero. This assumption guarantees that a unilateral deviation by a consumer cannot change the actions of other agents; only the monopolist's unilateral deviations can influence the course of the game. Gul, Sonnenschein, and Wilson (1986) present an example to show that this assumption does restrict the set of equilibria. Similar examples exist for the model of this paper.

3. STATIONARY EQUILIBRIA

This section describes the equilibria of the model when each consumer uses a strategy that depends only upon his valuation, the measures of high- and low-valuation consumers in the market when he entered, the measures of high- and low-valuation consumers in the market currently (the current state), and the current price of the monopolist. I call these strategies stationary. The stationarity assumption guarantees that consumers do not condition their behavior on prices charged before they entered the market. Stationary equilibria are important for two reasons. First, Theorem 2 shows that equilibria in stationary strategies satisfy the Coase conjecture, which has been the focus of much attention in the literature on dynamic monopoly problems. More importantly, I use properties of stationary equilibria to construct nonstationary equilibria in Section 4.

Denote by (C, c) the state when the measure of high-valuation buyers in the market is αC and the measure of low-valuation buyers in the market is $(1-\alpha)c$. When the state of the market is (C, c), I say that the mass of high- (low-) valuation consumers is C (c). The characterization of the stationary equilibria requires three preliminary results.

LEMMA 1: *In any stationary equilibrium and after any history, if the state of the market is* (C, c), *then the present value of the seller's expected profit is at least* $[C\alpha + c(1-\alpha)]V_2 + \beta V_2/(1-\beta)$, *and the seller's price specified by the equilibrium is at least* V_2.

Lemma 1 states that the price of the monopolist never falls below V_2, the willingness to pay of the low-valuation consumers. This result is standard in durable-good monopoly models without entry of consumers (for example, Gul, Sonnenschein, and Wilson (1986)) and in the formally related bargaining models with one-sided incomplete information (for example, Fudenberg, Levine, and Tirole (1985)). I do not provide a proof of the result; it is essentially identical to the arguments of Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986). The stationarity assumption is necessary for Lemma 1. There exist nonstationary equilibria, described in Section 4, in which the seller charges less than V_2 on the equilibrium path.

When the monopolist drops her price to V_2 and clears the market, she is said to hold a sale. Lemma 2, stated below, shows that eventually enough low-valuation consumers enter the market to induce the seller to drop the price to V_2, at which time all consumers currently in the market purchase the good. In particular, it shows that in every stationary equilibrium there is a sale. It follows that every buyer eventually purchases in a stationary equilibrium.

Stationary equilibria can be characterized by first analyzing equilibria of a game that ends as soon as the seller holds a sale. In these auxiliary games the strategies and preferences of the buyers and the seller are the same as in the original game except that when the seller first holds a sale there is no further entry and the seller receives a value W (discounted from the period after the sale) instead. Since no one will buy at a price greater than V_1, equilibrium profit is bounded above by $V_1/(1-\beta)$. It follows from Lemma 1 that the equilibrium profit is an element of $[V_2/(1-\beta), V_1/(1-\beta)]$. I assume that continuation values to auxiliary games are also in this interval. Lemma 2 states also that these games last a finite number of periods in the sense that in any equilibria of these games there is a bound on the length of time before the seller has a game-ending sale.

LEMMA 2: *Given* $\beta \in (0,1)$, *there exists a* M^* *such that, in any stationary equilibrium, after any history, there are never more than* M^* *periods until the next sale.* M^* *also bounds the number of periods until the seller holds a sale in any equilibrium, after any history, of any auxiliary game with continuation value* $W \in [V_2/(1-\beta), V_1/(1-\beta)]$.

There is a proof of Lemma 2 in the Appendix.

It follows from Lemma 2 that any stationary equilibrium gives rise to an equilibrium to an auxiliary game where the continuation value is equal to the seller's equilibrium discounted profit. Conversely, if W is the discounted profit in an equilibrium to an auxiliary game with continuation value W, then I obtain a stationary equilibrium by repeating the pre-sale portion of the auxiliary game. To characterize stationary equilibria it suffices to do two things: prove the existence of equilibria to auxiliary games with continuation values $W \geqslant V_2/(1-\beta)$, and prove that there exists a W^* such that the profit of the equilibrium with the continuation value W^* is equal to W^*. Benabou (1989) uses similar arguments to construct equilibria in his model.

Lemma 3 characterizes equilibria of the auxiliary games.

LEMMA 3: *For each $W \in [V_2/(1-\beta), V_1/(1-\beta)]$, the auxiliary game determined by W has an equilibrium. In all equilibria of the auxiliary game the seller earns the same profit, which is a continuous function of W. The equilibrium path of prices is uniquely determined by the first price; following the first period, prices on the equilibrium path are deterministic and, when the sale is to occur i periods in the future, the seller charges $(1-\beta^i)V_1 + \beta^i V_2$. A positive mass of buyers makes purchases in each period.*

I prove Lemma 3 by constructing equilibria to auxiliary games in which the seller must hold a sale in the next i periods (and after the sale she earns W). Lemma 2 implies that requiring the seller to hold a sale in the next i periods is not a binding constraint if i is greater than M^*. Therefore, by starting with $i = 1$, and inductively increasing the maximum time until the next sale from i to $i+1$ periods, an equilibrium to the auxiliary game is determined in a finite number of steps. The argument that I use is close to that of Fudenberg, Levine, and Tirole (1985), although some modifications must be made because there is entry of consumers. The Appendix contains a sketch of the proof. Next I describe equilibrium strategies.

In the Appendix I construct several functions associated with the equilibrium to the auxiliary game. When (C, c) is the state of the market, $\pi^*(C, c; W)$ is the profit of the monopolist; $S^*(C, c; W)$ is the set of profit maximizing prices; $P^*(C, c, p; W)$ is the expected value of the seller's next price if she charged p in the previous period (on the equilibrium path the distribution of seller's next price is degenerate); and $D^*(c, i; W)$ is the smallest value of C such that the seller wishes to hold a sale in i or more periods. For each $i = 1, 2, 3, \ldots,$ define p_i by

$$p_i = (1-\beta^{i-1})V_1 + \beta^{i-1}V_2. \tag{1}$$

Since $V_1 - p_i = \beta^{i-1}(V_1 - V_2)$, a high-valuation consumer obtains the same utility from buying immediately at p_i or waiting $i-j$ periods and buying at p_j. Given the profit functions, I construct the strategies of the consumers. I need to explain how consumers respond to prices as a function of the state of the market. Low-valuation consumers purchase if and only if the price is less than or equal to V_2. The fraction of high-valuation buyers accepting the price $p \in (p_i, p_{i+1}]$ is as large as possible consistent with the constraint that it would be optimal for the seller to charge at least p_i in the next period. That is, if there are currently accumulated masses of C high-valuation buyers and c low-valuation buyers, then $C+1-D^*(c+1,i;W)$ is the mass of high valuers who buy at $p \in (p_i, p_{i+1}]$. No one buys if $C+1-D^*(c+1,i;W)$ is less than zero. The construction does not explicitly determine strategies of the buyers; it simply describes aggregate behavior. I could assume that purchases are made on a first-in, first-out basis. Specifically, when there have been c periods since the last sale, and N is the date of the last sale, then a consumer of type $(t,j)=(s+N+1,1)$ buys at the price $p \in (p_n, p_{n+1}]$ if and only if

$$s \leqslant c+1-D^*(c+1,n;W). \tag{2}$$

If buyers play the strategy defined in (2), then their aggregate behavior coincides with that described above. The strategy (2) depends only on the mass of low-valuation buyers in the market when the buyer arrived (s), the number of low-valuation buyers that have accumulated since the last sale (c), and the current price (which determines n).

$S^*(C,c;W)$ gives the optimal responses to the buyers' strategies. If it is single-valued, then the seller charges that price. If it contains more than one element, then the seller chooses a randomization that is determined by the last price charged. A randomization exists that is consistent with the expectation described by $P^*(\cdot)$. The behavior of the seller on the equilibrium path is uniquely determined following her choice of initial price. The initial price may be any element of $S^*(1,1;W)$. In subsequent periods (on the equilibrium path) the seller must charge the largest price in $S^*(\cdot)$. The monopolist will need to randomize following certain defections (past prices that were not equal to p_i for some i).

Existence of stationary equilibria is a consequence of Lemma 3.

THEOREM 1: *There exists a stationary equilibrium. In all stationary equilibria, the interval between sales is no greater than M^*; following the first period after a sale, prices on the equilibrium path until the next sale are deterministic and, when the sale is to occur i periods in the future, the seller charges $(1-\beta^i)V_1+\beta^i V_2$. A positive mass of buyers makes purchases in each period. Furthermore, if $\alpha V_1 < V_2$, then the stationary equilibrium outcome is unique. In it, the seller charges V_2 in each period. If $\alpha V_1 > V_2$, then the seller charges prices greater than V_2 on the equilibrium path of any stationary equilibrium.*

PROOF: Lemma 3 constructs an equilibrium given the continuation profit available to the seller. Existence of equilibrium reduces to finding a value, W^*, such that $\pi^*(1,1;W^*)=W^*$. The existence of W^* follows from the continuity of $\pi^*(1,1;\cdot)$, and from

$$V_2/(1-\beta) \leqslant \pi^*(1,1;V_2/(1-\beta)) \quad \text{and} \tag{3}$$

$$V_1/(1-\beta) \geqslant \pi^*(1,1;V_1/(1-\beta)). \tag{4}$$

Inequality (3) follows because the seller can always hold a sale when $(C,c)=$

(1, 1); hence $\pi^*(1,1;W) \geqslant V_2 + \beta W$ for all W. Since no one buys if the seller charges more than V_1, $V_1/(1-\beta) \geqslant \pi^*(1,1;W)$ for all W. Therefore, (4) holds.

In any stationary equilibrium the seller can sell to all high valuers currently in the market if she charges p_2. By doing so she earns at least

$$(5) \qquad p_2\alpha + \beta p_1[\alpha + 2(1-\alpha)] + \beta^2 W,$$

when the state of the market is (1, 1). If the seller charges V_2 in every period on the equilibrium path, then $W = V_2/(1-\beta)$. Direct computation reveals that (5) is strictly greater than $V_2/(1-\beta)$ if $\alpha V_1 > V_2$. This proves that if $\alpha V_1 > V_2$, then the seller must sell only to high-valuation buyers in some periods. Now assume $\alpha V_1 < V_2$. Another computation shows that if $C \leqslant c$ and $W \geqslant V_2/(1-\beta)$, then

$$(6) \qquad p_1[C\alpha + c(1-\alpha)] + \beta W > p_2 C\alpha + \beta p_1[\alpha + (c+1)(1-\alpha)] + \beta^2 W.$$

(6) guarantees that the seller strictly prefers to hold a sale immediately rather than to charge p_2. Furthermore, no high buyer will purchase at a price greater than p_2, since he expects the price in the following period to drop to V_2. Consequently the seller must charge V_2 in every period.

Several features of stationary equilibria are of interest. Following the first period after a sale, the equilibrium path generates a determinate sequence of prices. These prices are of the form (1). Low-valuation customers buy at the first sale date after they arrive in the market. They receive no surplus. High-valuation buyers need not buy as soon as they enter the market. Instead, in equilibrium, a fraction of the high-valuation buyers may wait to make a purchase. Although there is no need for buyers to randomize in equilibrium, it is essential to allow different high valuers in the market at the same time to behave differently. These buyers receive the same utility if they buy on any day until the next sale. The seller needs to keep a significant number of high-valuation customers in the market to make the policy of delaying a sale credible. The fewer high-valuation buyers in the market, the more attractive it is to hold a sale. That consumers with the same preferences behave differently is not a novel feature of this model. It appears in the model of Gul, Sonnenschein, and Wilson (1986) when the market demand curve is a step function; and it appears in the qualitatively similar bargaining model of Fudenberg and Tirole (1983) in the form of randomization by the high-valuation type of buyer.

Theorem 1 also provides conditions when stationary sales cycles are nontrivial. If $\alpha V_1 > V_2$, then the seller would prefer to charge a high price rather than a low price if the game lasted for only one period. To see this, observe that in the one-period game the seller has two sensible strategies: she can sell to buyers with high and low valuations, or she can sell only to the high valuers. In the first case, the highest price she can charge is V_2, and she earns V_2. In the second case, the highest price she can charge is V_1, and she earns αV_1. CGS identify this condition. It is also well known that for the durable-goods monopolist problem with no entry and two valuations the monopolist charges more than V_2 in equilibrium only if $\alpha V_1 \geqslant V_2$.

The recent theoretical models of the durable-good monopolist discuss the observation made by Coase (1972) that a monopolist loses her monopoly power if the time between offers shrinks to zero. Gul, Sonnenschein, and Wilson (1986) establish the result for markets with arbitrary demand curves (satisfying mild regularity conditions) and no entry of new consumers, provided that the minimum consumer valuation is strictly greater than the constant marginal cost of production. Earlier papers by Stokey (1981) and Bulow (1982) establish the result in special cases. A rough intuition for the theorem is that buyers know that the price will fall to the lowest valuation in the market eventually and if the interval between offers is short, then it will not take long for the prices to drop.

When there is no entry of new consumers, any incentive to charge a high price disappears as soon as high valuation buyers leave the market. When new consumers with high valuations enter the market in each period, the seller always has some incentive to charge high prices. This observation suggests that the seller retains some power to extract the surplus of high valuers as the time between offers shrinks to zero. In Section 4, I show that this intuition is correct: There exist equilibria in which the seller extracts monopoly profits from the buyers. Nonstationary strategies are necessary for the result. Theorem 2 below states that the number of periods until the next sale in any stationary equilibrium has an upper bound that does not depend upon the discount factor or the interval between periods. It follows that as the time between periods shrinks to zero, no buyer will need to wait long before the next sale. Therefore, no buyer will be willing to pay much more than V_2, the lowest valuation. If the time between periods is tiny, then the amount of new entry in any single period is tiny. The following provides some intuition for the result. When there is any real time delay between sales, the number of new entrants is negligible relative to the number of low-valuation customers who have been waiting. In a stationary equilibrium, the temptation to hold a sale becomes irresistible unless there is a large backlog of high-valuation buyers waiting as well. But if there is a large backlog of high-valuation buyers, then the arguments of Gul, Sonnenschein, and Wilson (1986) suggest that the seller would do better by cutting prices more rapidly. The Appendix contains a direct proof of Theorem 2. The bound derived in the proof is equal to $(M+1)\alpha/[M^2(1-\alpha)]+1$, where $M = V_2/(V_1 - V_2)$.

THEOREM 2: *There exists a finite value K that does not depend on the interval between periods or the discount factor such that, after any history leading to a market state (C, c) with $C \leqslant c$,*[5] *the number of periods until the next sale in any stationary equilibrium is bounded above by K.*

Unlike the bound M^* of Lemma 2, which depends on β, K depends only on V_1, V_2, and α.

[5] If a low-valuation consumer prefers to buy in the current period, then high-valuation consumers will strictly prefer to buy. Consequently, market states in which $C > c$ occur only when there has been a simultaneous deviation by a positive mass of buyers. Behavior that follows simultaneous deviations does not influence the equilibrium path.

Hart (1989) shows that there is a finite upper bound to the number of periods before which bargainers will reach agreement with probability one in a two-type model of sequential bargaining. His result applies directly to the durable-good monopoly problem when the market demand function is a step function. Theorem 2 extends the result to a model where new consumers enter the market.

In order to analyze the effect of period length on the stationary equilibrium, I let Λ be the length of a period. Λ is also the flow of new entrants into the market per period. The relevant discount factor is β^{Λ}. If there are no more than K periods between sales in a stationary equilibrium, then (1) implies that the seller charges prices no greater than

$$V_2 + (1-\beta^{\Lambda K})(V_1 - V_2). \tag{7}$$

Since the second term in (7) goes to zero as Λ approaches zero, Theorem 2 implies a version of the Coase conjecture valid for my model.

COROLLARY: *Given any $\varepsilon > 0$, there exists $\delta > 0$ such that if the period length Λ is less than δ, then the seller charges prices no higher than $V_2 + \varepsilon$ and earns profit no greater than $(V_2+\varepsilon)/(1-\beta)$ in a stationary equilibrium.*

4. THE FOLK THEOREM

My main result is that the model of this paper admits multiple, qualitatively different, equilibrium outcomes. Theorem 3 demonstrates that if the players are sufficiently patient, then any positive average profit level less than $\max\{\alpha V_1, V_2\}$ is attainable in an equilibrium (average profit is $1-\beta$ times the present discounted value of profit). In Section 5, I show that this upper bound is the best possible: Using the optimal selling mechanism the seller earns average profit equal to $\max\{\alpha V_1, V_2\}$.

THEOREM 3: *Given $\varepsilon > 0$, there exists $\beta^* \in (0,1)$ such that if $\beta \in (\beta^*, 1)$ and $V \in [\varepsilon, \max\{\alpha V_1 - \varepsilon, V_2\}]$, then there exists an equilibrium where the seller's discounted profit is $V/(1-\beta)$.*

I describe strategies that support equilibria yielding average profit between ε and $\max\{\alpha V_1 - \varepsilon, V_2\}$ assuming, without loss of generality, that ε is "small" ($2\varepsilon < \alpha V_2$ is sufficient). I close the section with a comparison of the result to related work and a discussion of how one might select an equilibrium outcome in the model.

Equilibria have a simple path. More complicated strategies are specified off the equilibrium path. I describe first the strategies used on the equilibrium path. Then I describe the punishment equilibria. Throughout this section, I assume $\alpha V_1 > V_2$. The strategies below apply only for this case. A slight modification, given in the Appendix, is needed when $\alpha V_1 \leqslant V_2$.

The equilibrium path behavior depends on whether the seller's equilibrium profit exceeds $V_2/(1-\beta)$. On the path of an equilibrium in which the seller earns $V/(1-\beta)$ for $V \in (0, V_2]$, the seller charges V in every period. Buyers purchase as soon as they enter the market.

To attain equilibrium average profit equal to $V \in (V_2, \alpha V_1)$, the seller holds a sale every n periods and charges $p_{n-c+1} = (1-\beta^{n-c})V_1 + \beta^{n-c}V_2$ when there are c periods of low-valuation buyers in the market. These prices are of the form (1) used in the stationary equilibrium. On the equilibrium path high-valuation buyers purchase as soon as they enter the market; low-valuation buyers purchase the first time after they enter the market that the price falls to V_2. The time between sales, n, depends on the desired equilibrium average profit, V. To compute the precise relationship, I use a result found in CGS. CGS compute the profit of the seller if c periods of low-valuation buyers are in the market, sales occur every N periods, prices are given by (1), and buyers purchase the first time the price is no higher than their valuation. They find that this value, call it $\phi(N, c)$, is equal to $\alpha V_1/(1-\beta) - [(\alpha V_1 - V_2)N\beta^{N-c}]/(1-\beta^N) + \alpha(V_1 - V_2)\beta^{N-c}(c-1)$. $\phi(N, 1)$ is what the seller earns on the path of the equilibrium when sales occur every N periods. When $\alpha V_1 > V_2$, $\phi(\cdot, 1)$ is strictly increasing, $\phi(1,1) = V_2/(1-\beta)$, and $\lim_{N\to\infty} \phi(N,1) = \alpha V_1/(1-\beta)$. I define n to be the solution to $\phi(n,1) = V/(1-\beta)$; n is defined implicitly by

$$(8) \qquad (\alpha V_1 - V)/[(\alpha V_1 - V_2)(1-\beta)] = n\beta^{n-1}/(1-\beta^n).$$

The parameter n depends on β and on the profit level of the equilibrium; provided that $V \in (V_2, \alpha V_1)$, it is uniquely determined by (8) and is an increasing function of V. Since n is the number of periods between sales, it should be an integer. I describe the equilibrium assuming that n can take on any positive real value. There are several ways to avoid the integer problem. For example, one could specify that equilibrium cycles alternate in length between the integer values closest to the solution of (8). Properly chosen, these cycles would lead to profit that is arbitrarily close to $V/(1-\beta)$.[6]

If the seller is to attain average profit greater than V_2, then there should be a long interval between sales. If the length of time between sales is n, and β^n converges to one, then the prices given by (1) are not significantly greater than V_2. Cycles are not long enough to generate average profit greater than V_2. Since $\lim_{\beta\to 1}\beta^n = \lim_{\beta\to 1} e^{-(1-\beta)n}$, the asymptotic properties of $(1-\beta)n$ determine those of β^n. It is straightforward to show that if (8) defines n, then $\lim_{\beta\to 1}(1-\beta)n$ exists and

$$(9) \qquad \lim_{\beta\to 1}(1-\beta)n = L \quad \text{where} \quad (e^L - 1)/L = (\alpha V_1 - V_2)/(\alpha V_1 - V),$$

so n increases at a rate proportional to $1/(1-\beta)$. (9) implies that β^n converges

[6]Alternatively, one could allow public correlation, which would make it possible to attain any equilibrium value in the convex hull of the values attainable through cycles of integer length. Or one could pick the least integer greater than the value of n that solves (8) and lower the prices on the equilibrium path to make the seller's profit equal to $V/(1-\beta)$.

to something strictly less than one; the greater is V, the smaller is the limit. The result is intuitive because in order to approximate average profit of αV_1, high-valuation buyers must be induced to pay V_1; they will do so only if the discounted surplus available to a high valuer who waits for a sale, which is at least $\beta^n(V_1 - V_2)$, converges to zero.

If the seller deviates from the equilibrium path, then a punishment begins. The nature of the punishment depends on how many buyers are in the market at the time of a deviation; punishment strategies are independent of V, the average profit generated on the equilibrium path. Let $G(C,c) = C\alpha + c(1-\alpha)$ be the measure of consumers when (C,c) is the state of the market. A deviation triggers a qualitatively different punishment depending on whether $G(C,c)$ is greater than or less than m, where m is a number determined by β (but independent of V). I define m precisely below. For now it is enough to know that m is so large (relative to β) that $\lim_{\beta \to 1} \beta^m = 0$ or, equivalently,

$$\lim_{\beta \to 1} (1-\beta)m = \infty. \tag{10}$$

Equations (9) and (10) imply that $m > n$. If $G(C,c) > m$ in the current period or in the event of simultaneous deviations, then players follow strategies from a fixed stationary equilibrium. Since $m > n$, this punishment occurs only after histories in which there have been many deviations from the equilibrium. If $G(C,c) \leqslant m$, then a deviation triggers a (state-dependent) two-stage punishment equilibrium, which is necessary because the stationary equilibrium is not a harsh enough punishment to encourage the seller to follow an equilibrium path that leads to average profit approaching αV_1 when $G(C,c) \leqslant m$.

To get an idea of why the stationary equilibrium is not a severe enough punishment, argue as follows. The attraction of cutting prices in the middle of a cycle is that the seller can move forward the revenue she obtains from the accumulated low-valuation consumers; if, as in the stationary equilibrium, everyone in the market buys when the seller charges V_2, then the gain from selling in the middle of a cycle (when there are, say, $(1-a)n$ accumulated periods of low-valuation buyers from $a \in (0,1)$) is approximately $(1-\alpha)(1-\beta^{an})(1-a)nV_2$ because the seller earns V_2 from the accumulated $(1-\alpha)(1-a)n$ measure of low valuers in the current period rather than earning V_2 in periods in the future. On the other hand, after the market clears the seller earns an average profit of approximately V_2 in the stationary equilibrium, while the average profit on the equilibrium path can be no larger than αV_1. Consequently, a deviation is attractive if $(1-\alpha)(1-\beta^{an})(1-a)nV_2 > (\alpha V_1 - V_2)/(1-\beta)$, which, by (9), will hold if V is close to αV_1.

Since reverting to the stationary equilibrium in the event of a deviation cannot support a high-profit outcome, a harsher punishment is needed when $G(C,c) \leqslant m$ at the time of a deviation. The punishments are qualitatively similar to the two-stage punishments used by Abreu (1986) (see also Abreu (1989) and Fudenberg and Maskin (1986) for related approaches) to study equilibria in repeated games with discounting. The first stage of the punishment is severe. The second stage is a reward for participating in the first stage. The first phase of the punishment in the equilibria that I construct takes place in the period that the deviation occurs. The equilibrium specifies that the seller charge a low price, denoted by $g(C,c)$; everyone in the market buys at this price. This phase of the punishment lasts for only one period. The second phase of the punishment is a reward that involves following a path, selected from the class of candidate equilibria described above, for the remainder of the game. The second-phase equilibrium yields profit $v(C,c)$.[7] The equilibrium specifies a different punishment depending on the state of the market when a deviation took place. There is only one punishment equilibrium in Abreu's work. The reason for the difference is that the game studied here is not a simple repeated game. Only the seller is an active player in every period. The game repeats after a market clearing sale, but sales occur at endogenously determined intervals. Most important, there is a qualitatively different subgame for each market state (C,c), and there are an infinite number of potential states.

I give explicit formulas for $g(\cdot)$ and $v(\cdot)$ below. First, I give an explanation of why the punishments depend nontrivially on the state of the market. Imagine strategies which specify that in the event of a deviation, the seller punishes herself by charging a low price $p < V_2$, independent of the market state at the time of the deviation. Buyers would then never expect the price to fall below p, and the seller could induce everyone in the market to buy at any price less than $\beta p + (1-\beta)V_2$. As a result, the seller prefers to charge prices slightly higher than p rather than to follow the equilibrium. The same reasoning suggests that $g(\cdot)$ must increase to encourage the seller to participate in her own punishment. The function that I use actually increases without bound. No buyer will purchase at a price greater than his valuation, so the seller will not be able to clear the market when the market state (C,c) is large. That is why I use the stationary equilibrium as a punishment when $G(C,c) \geqslant m$. $g(\cdot)$ can, however, be made arbitrarily small for all states (C,c) such that $G(C,c) \leqslant n$. Formally, there exists $\beta^* \in (0,1)$ such that for all $\beta \in (\beta^*,1)$ and the n corresponding to β given by (8),

$$\text{(11)} \qquad \text{if} \quad G(C,c) \leqslant n, \quad \text{then} \quad g(C,c) < \varepsilon.$$

Since on the path of the equilibrium the state of the market is of the form $(1,c)$ for $c \leqslant n$, (11) limits the seller's profits when she deviates from the equilibrium path and clears the market before the prescribed date.

If the seller participates in her punishment and charges the price $g(C,c)$, then everyone in the market buys. The second phase of the punishment begins with the state of the market $(1,1)$; for the rest of the game the players use strategies that lead to the payoff of $v(C,c)$ for the seller. The construction requires that $v(\cdot)$ satisfy several properties. If $\beta v(1,1) > \varepsilon/(1-\beta)$ it would not

[7] (C,c) refers to the state of the market at the time of the deviation; $v(C,c)$ is the continuation value to the seller when there is only a single period's accumulation of high and low consumers in the market immediately following a period in which the seller participates in her punishment by charging $g(C,c)$.

be possible to support an equilibrium in which the seller earned average profit ε; instead, the seller would prefer to deviate in the very first period and receive a punishment that is better than the equilibrium itself. In fact, I require that there exists $\beta^* \in (0,1)$ such that for all $\beta \in (\beta^*, 1)$,

$$\text{(12)} \qquad \text{if} \quad G(C,c) \leqslant n, \quad \text{then} \quad v(C,c) = \varepsilon/(1-\beta).$$

Recall that on the equilibrium path, $G(C,c) \leqslant n$. Provided that (11) holds, so that $g(C,c)$ is less than ε on the equilibrium path, (12) guarantees that the seller has no incentive to deviate from the equilibrium path. In view of (12), one might hope that it is sufficient to set $v(C,c) = \varepsilon/(1-\beta)$ for all (C,c). Unfortunately, I cannot take $v(C,c)$ to be a constant. Unless $v(C,c)$ is as large as the stationary equilibrium payoff when $G(C,c) = m$, the seller would not participate in her punishment; instead she could defer sales entirely for a period in order to earn the high profits of the stationary equilibrium that follows. For this reason, I define $U = (\alpha V_1 + V_2)/2 > V_2$ and require that

$$\text{(13)} \qquad \text{if} \quad G(C,c) = m+1, \quad \text{then} \quad v(C,c) = U/(1-\beta).$$

I next describe the strategies of the buyers in the punishment portion of the equilibrium. If $G(C,c) > m+1$, then players follow strategies from the fixed stationary equilibrium in the continuation game. When $G(C,c) \in (m, m+1]$, the equilibrium specifies that the seller charge $p = g(C,c)$. If she does, then everyone in the market buys; if she does not, then players follow strategies from the fixed stationary equilibrium in the continuation. If the seller charges $p \neq g(C,c)$ when $G(C,c) \leqslant m$, then the new market state is

$$\text{(14)} \qquad (D,d) = \begin{cases} (C+1, c+1) & \text{if } p > (1-\beta)V_1 + \beta g(C+1, c+1), \\ (F, c+1) & \text{if } p = (1-\beta)V_1 + \beta g(F, c+1) \\ & \qquad \text{for } 1 \leqslant F \leqslant C+1, \\ (1, c+1) & \text{if } p \in \big((1-\beta)V_2 + \beta g(1, c+1), \\ & \qquad (1-\beta)V_1 + \beta g(1, c+1)\big), \\ (1, f) & \text{if } p = (1-\beta)V_2 + \beta g(1, f) \\ & \qquad \text{for } 1 \leqslant f \leqslant c+1, \\ (1,1) & \text{if } p < (1-\beta)V_2 + \beta g(1,1). \end{cases}$$

I can use the description of how the state changes to specify the behavior of individual buyers. In the top line of (14), no one buys; in the second line a fraction of the high valuers buy; in the third line all of the highs and none of the lows buy; and so on. For prices in the ranges given in the second and fourth lines of (14) there are many ways to specify behavior of individuals that give rise to the same change in the state of the market.

To interpret (14), notice that if the seller does not participate in the first phase of the punishment, then the punishment begins again in the next period. The price in the next period is of the form $g(D,d)$ for some (D,d). If all buyers in the market buy either in the current period or the next one, then the state must change according to (14). Specifically, no high valuers buy at p if the surplus from waiting is greater (that is, when $V_1 - p < \beta[V_1 - g(C+1, c+1)]$ or $p > (1-\beta)V_1 + \beta g(C+1, c+1)$); some high valuers buy if they obtain the same surplus whether they buy right away at p or in the next period at $g(D, d)$; all the high valuers buy if they are better off accepting p today than $g(1, c+1)$ tomorrow. Similar arguments apply to the low-valuation buyers.

To complete the description of the equilibrium, I give explicit formulas for $v(\cdot)$ and $g(\cdot)$. Let $g(C,c) = [(1-\beta)V_1 G(C,c)\eta]/(\beta\eta)$, and let η take on any value in the interval $(1/2, 1)$. Since $\eta < 1$, (9) implies (11). Define the parameters m and m' by

$$(15) \qquad \left[(1-\beta)V_1(m')^{\eta}\right]\big/(\beta\eta) = \varepsilon \quad \text{and} \quad \left[(1-\beta)V_1(m+1)^{\eta}\right]\big/(\beta\eta) = V_2;$$

$m > m'$ and $g(C,c)$ is less than, equal to, or greater than $\varepsilon(V_2)$ depending on whether $G(C,c)$ is less than, equal to, or greater than m' $(m+1)$. m' and m depend on the discount factor, but, unlike n, do not depend on the profit of the equilibrium path. Since $\eta < 1$, (10) holds. I use the particular function form of $g(\cdot)$ to define $v(\cdot)$ and to show that it is in the interest of the seller to charge $p = g(C,c)$ in the first phase of the punishment portion of the equilibrium. Lemmas 5 and 6 contain the details of the argument. I give a sketch following the definition of $v(\cdot)$.

Let $v(C,c) = (1-\beta)^{-1}\max\{\varepsilon, g(C,c) + (U - V_2)(G(C,c) - m')/(m+1-m')\}$, which, since $g(C,c) \leqslant \varepsilon$ if and only if $G(C,c) \leqslant m'$, implies that

$$(16) \quad v(C,c) = \begin{cases} \varepsilon/(1-\beta) & \text{if } G(C,c) \leqslant m', \\ \dfrac{\{g(C,c) + (U-V_2)(G(C,c)-m')/(m+1-m')\}}{(1-\beta)} & \\ & \text{if } G(C,c) \geqslant m'. \end{cases}$$

$v(\cdot)$ is nondecreasing and, since $m' > n$ for β sufficiently close to one, satisfies (12). (13) follows directly from (15) and (16). The proof of Theorem 3 uses two additional properties of $v(\cdot)$ that must hold for β sufficiently close to one:

$$(17) \qquad g(C,c) \leqslant (1-\beta)v(C,c) \quad \text{and}$$

$$(18) \qquad v(C,c) - \beta v(C+1, c+1) \geqslant \varepsilon/2.$$

(17) follows immediately from the definitions of $v(\cdot)$, $g(\cdot)$, and m'. The inequality is used to show that when the seller charges $g(\cdot)$, the market clears because buyers do not expect to have an opportunity to buy at a lower price.

(18) is an immediate consequence of the definition of $v(\cdot)$ when $G(C,c) \leqslant m' - 1$. Lemma 4 (in the Appendix) proves it in general. I use (18) to show that the seller responds optimally when she participates in her punishment. If the seller participates in her punishment, then she receives the reward $v(C,c)$ starting in the next period. If she delays the punishment for only period, then her largest possible reward is $v(C+1, c+1)$, but it must be discounted since it begins one period later. (18) states that the loss associated with postponing the reward for a period is bounded away from zero.

Now I give an informal explanation of why it is in the interest of the seller to charge $p = g(C, c)$ in the first phase of the punishment portion of the equilibrium. There are three possible situations to consider. First, if players are following a stationary equilibrium, then there is nothing to prove.

Second, if $G(C, c) \in (m, m+1]$, then the equilibrium specifies that the seller clear the market by charging $g(C, c)$, which is approximately V_2. By the definition of $v(\cdot)$ and (13), when β is close to one she earns an average profit of nearly U in the reward phase of the equilibrium, which begins in the next period. The attraction of a deviation to the seller is that she can sell to high valuers at prices greater than V_2. Since, by Theorem 2, there are at most K periods until the next sale in a stationary equilibrium, the seller can charge no more than $(1-\beta^K)V_1 + \beta^K V_2 \leqslant V_2 + K(1-\beta)(V_1 - V_2)$. Even if the seller could earn the extra $K(1-\beta)(V_1 - V_2)$ on each of the high valuers in the market, her gain would be on the order of $m(1-\beta)$; her continuation profit would be the stationary equilibrium profit of roughly $V_2/(1-\beta)$. The punishment for a deviation is therefore approximately $(U - V_2)/(1-\beta)$. Since $\eta > 1/2$, (15) implies that $m(1-\beta)$ is small relative to $1/(1-\beta)$; the potential profits that the seller could make on the accumulated highs is not large enough to compensate for a reduction in future average profits. Lemma 5, presented in the Appendix, gives a precise version of this argument.

Finally, assume $G(C, c) \leqslant m$. If the seller does not charge $g(C, c)$ when the equilibrium specifies that she should, buyers expect her to charge a price of the form $g(D, d)$ and clear the market in the next period. As a result, there are two effects of a one-shot deviation. First, the seller clears the market in two periods rather than one. Second, the seller postpones the ultimate reward phase. (18) guarantees that the second effect encourages the seller to follow the equilibrium. The first effect may allow the seller to earn more from the sellers currently in the market than the equilibrium, but it is always dominated by the second effect. Lemma 6, which is stated and proved in the Appendix, gives a formal proof. Here is an intuition. If the seller is expected to charge $g(D, d)$ in the next period, then no one will buy at prices greater than $(1-\beta)V_1 + \beta g(D, d)$ in the current period. These prices cannot be much greater than $g(C, c)$, particularly when the market is crowded (due to the concavity of $g(\cdot)$), so the deviation is not profitable enough to compensate for the decrease in payoff caused by delaying the reward.

Proof of Theorem 3: I have already argued that the profit generated by the equilibrium path is what I claim it to be. Lemmas 5 and 6 establish that the seller's strategy responds optimally to the buyers' strategies specified for the punishment portion of the equilibrium. I show below that it is never advantageous for the seller to deviate from the equilibrium path and that the buyers respond optimally to the seller's strategy.

I first show that the seller responds optimally to the buyers' equilibrium strategies if she follows the equilibrium path. There are two cases, depending on whether or not the average profit on the equilibrium path is greater than V_2. In order for the seller to prefer to follow the equilibrium path rather than defect when average profit exceeds V_2, it must be the case that, for $c \leqslant n$, $\phi(n,c)$ is greater than or equal to the profit that the seller would receive from a defection. If the seller chooses to deviate from the equilibrium path, then the best she can do is charge $g(1,c)$ because as soon as the seller deviates from the equilibrium path, the punishment portion begins, and the optimal behavior for the seller when $G(C,c) < m$ (which it must be following the first deviation since on the equilibrium path $G(C,c) \leqslant G(1,n) < n$ and $n < m$) is to charge $g(C,c)$ as specified by the punishment equilibrium. Her profit is bounded above by

$$g(1,c)G(1,c) + \beta v(1,c) \leqslant \left[(1-\beta)V_1 n^{1+\eta}\right]/(\eta\beta) + \beta v(1,c). \tag{19}$$

On the other hand, since on the path of the equilibrium that yields average profit greater than V_2 the seller charges at least V_2 in every period, and high valuation buyers purchase as soon as they enter,

$$(1-\beta)\phi(n,c) \geqslant \alpha V_2. \tag{20}$$

Since, by (9), $\lim_{\beta \to 1}(1-\beta)^2 n^{1+\eta} = 0$, and by (12), $(1-\beta)v(1,c) = \varepsilon$, (19) and (20) imply that if β is sufficiently close to one and ε is small, then

$$\phi(n,c) \geqslant g(1,c)G(1,c) + \beta v(1,c) \quad \text{for} \quad c \leqslant n. \tag{21}$$

(21) implies that the seller prefers to follow the equilibrium path than to defect.

When $V \leqslant V_2$, the state of the market on the equilibrium path is always $(C,c) = (1,1)$. The most the seller can earn if she deviates from the equilibrium path is

$$g(1,1) + \beta v(1,1), \tag{22}$$

which she earns if she charges $g(1,1)$. Since $g(1,1) < \varepsilon$ when β is close enough to one, (12) implies that (22) is bounded above by $\varepsilon + \beta\varepsilon/(1-\beta) = \varepsilon/(1-\beta)$, which is less than or equal to the $V/(1-\beta)$ that the seller earns when she follows the equilibrium.

I need to check that the buyers behave optimally when they follow the specified strategies on the equilibrium path. High valuers obtain the same surplus from buying any day until the next time the seller cuts her price to V_2. They respond optimally when they buy in the period that they enter the market. Low valuers also respond optimally when they buy at the first sale after entering the market.

It is optimal for a buyer to purchase whenever the seller charges $p = g(C,c)$ and the state of the market (C,c) satisfies $G(C,c) \leqslant m+1$. To see this, note that after the seller charges $g(C,c)$, the equilibrium specifies that she charge $(1-\beta)v(C,c)$ in all of the remaining periods (if $(1-\beta)v(C,c) \leqslant V_2$) or prices at least as large as V_2 in all of the remaining periods (if $(1-\beta)v(C,c) > V_2$). It follows from (17) in the first case and $g(C,c) \leqslant V_2$ in the second that she always charges prices greater than or equal to $g(C,c)$ in the future. Since all buyers obtain nonnegative surplus from a purchase at $g(C,c)$ and they do not expect the seller to charge a lower price thereafter, it is optimal for every buyer in the market to purchase when the seller charges $g(C, c)$.

If the seller charges $p \neq g(C, c)$ and $G(C, c) > m$, then the equilibrium specifies that all players follow stationary equilibrium strategies, so buyers are responding optimally. If the seller charges $p \neq g(C, c)$ and $G(C, c) \leqslant m$, then given the strategies of the other players, the equilibrium specifies that the seller charge $g(D, d) \leqslant V_2$ in the next period, where (D, d) is the next state of the market, and thereafter never charge a price below $g(D, d)$. Consequently all buyers in the market plan to buy either in the current period or the next one. The reaction specified in (14) guarantees that all buyers purchase in a period that maximizes their surplus.

I conclude this section with some comments on related literature and remarks on the properties of equilibria. Ausubel and Deneckere (1989a) prove that if the demand curve of the buyer intersects the cost curve of the seller, then any level of profit strictly between zero and the static monopoly profit may be attained in equilibrium if the interval between time periods is small enough (or, equivalently, the discount factor approaches one). If the lowest buyer valuation is strictly greater than the cost of production, then Gul, Sonnenschein, and Wilson (1986) show that there is only one equilibrium and in it the seller is unable to charge prices much higher than the lowest valuation when the time between periods shrinks. It may be more useful to describe these results differently. Gul, Sonnenschein, and Wilson (1986) show that stationary equilibria have the Coase property.[8] If the cost of production is strictly less than the lowest valuation, then the unique equilibrium is stationary. If the demand curve intersects the cost curve, then there are multiple equilibria. The stationary equilibria have the Coase property; the nonstationary equilibria need not. The relationship between stationarity and the Coase property is what carries over to my model.[9] All stationary equilibria have the Coase property but, because there is an inflow of new consumers, the market does not end in finite time so there are nonstationary equilibria even when every buyer's valuation exceeds the seller's cost.

Bond and Samuelson's (1984 and 1987) models of replacement sales have similar properties. As the period length shrinks to zero, the stationary equilibrium outcome converges to the competitive one. Because the monopolist is always producing to meet the demand for replacement goods, there also exist nonstationary equilibria in which the seller makes monopoly profits.

There are also equilibria that are worse for the seller than the stationary equilibrium: If the discount factor is close to one, then there exist equilibria in

[8] Gul and Sonnenschein (1988), in the related context of alternating offer bargaining with one-sided uncertainty and common knowledge of gains from trade, show that the length of time needed to reach an agreement in stationary equilibria shrinks to zero as the period length shrinks to zero.

[9] The connection between stationary equilibria and the Coase property should not be taken too far. Ausubel and Deneckere (1988a and 1988b) analyze the set of stationary equilibria in infinite-horizon bargaining games with two-sided incomplete information. They show that the set of stationary equilibrium outcomes is large. It includes efficient outcomes. They also show that when there is two-sided incomplete information stationary equilibria need not satisfy the Coase property.

which the seller earns an arbitrarily small amount in every period. It is therefore misleading to say that the seller loses all monopoly power in stationary equilibria. The seller makes even less in other equilibria.

The equilibrium paths I construct are stationary. If the seller's average profit is less than V_2, then each period she charges the same price. If the seller's average profit is greater than V_2, then she follows the cyclic pricing pattern (1). In both cases, the behavior of the players on the equilibrium path depends only on the market state. Also, in contrast to the stationary equilibrium, there is never a cumulation of high valuers on the equilibrium path nor does the seller randomize.

The seller charges strictly positive prices in the equilibria I consider. There exist equilibria in which the seller periodically pays customers to buy her product in order to earn a reward for doing so in the future. Allowing the seller to charge negative prices does not expand her set of equilibrium payoffs.

The existence of multiple equilibria that have different payoffs for the monopolist may be counterintuitive. One might expect that if the seller really has market power, then she would be able to influence the market enough to select the most profitable equilibrium. In practice the seller may have this power. Several arguments could be used to select an equilibrium in my model. I mention some possibilities below. The discussion is only suggestive; I have no results that select a unique equilibrium.

If the buyers learn about the behavior of the seller by studying prices she has charged in the past, then it may be in the seller's interest to follow the path that leads to high profits in order to convince future entrants that sales occur infrequently. In a different context, Laffont and Maskin (1987) show that a monopolist can induce naive buyers, who use the monopolist's past behavior to forecast her future behavior, to have beliefs consistent with her most profitable equilibrium.

A different approach to equilibrium selection is the reputation model of Fudenberg and Levine (1989) in which a long-lived player is able to approximate the payoff of a player able to commit to an action in each stage game if her short-lived opponents are uncertain about her preferences.[10] For the equilibria that I have discussed, the monopolist is the only player that remains active for more than a finite number of periods. It would be natural to assume that buyers are uncertain about the state of the market, the seller's cost of production, or the ability of the seller to commit to a policy of never holding a sale.

Still another approach to equilibrium selection is the idea of "money burning" introduced by Ben-Porath and Dekel (1988) and van Damme (1989). These papers provide conditions under which one player's most preferred outcome is the unique equilibrium of an auxiliary game where that player can publicly burn money prior to the play of the original game. Noninformative advertising may play the role of money burning in the dynamic monopoly problem. It seems

[10] Fudenberg, Kreps, and Maskin (1990) prove folk theorems for games with both long- and short-lived players.

natural to assume that the monopolist is the only player in the game who has access to a money-burning technology.

5. THE PROFIT-MAXIMIZING MECHANISM

In this section I determine the seller's maximal earnings if she were able to commit to an arbitrary selling strategy. A simple way to think about commitment power is to imagine that the seller is a Stackelberg leader in the game. She chooses a strategy that maximizes her discounted profits assuming that the buyers respond optimally. The announced policy need not be an optimal response to the buyers' behavior. For example, the seller could announce that she will charge a price slightly less than V_1 in every period; high valuers can do no better than purchase in the period they enter, so the seller earns $\alpha V_1/(1-\beta)$. When $\alpha V_1 > V_2$, this policy is more profitable than the stationary equilibrium.[11] In this section I show that there is no selling strategy available to the seller that allows her to earn more than she would if she was committed to charging a constant price in every period. When $\alpha V_1 > V_2$, the profit maximizing constant price is V_1; when $V_2 > \alpha V_1$, the profit maximizing price is V_2. This result shows that the upper bound to the seller's profit in noncooperative equilibrium given in Theorem 3 is the best possible.

In order to find the maximum profit available to the seller, I need to describe feasible selling mechanisms. In a direct selling mechanism the seller chooses functions $r(t,j)$ and $q(t,j)$ for $t \geqslant 0$ and $j = 1$ or 2. Each buyer reports a type (s,k) to the seller. He is able to report any valuation, $k = 1$ or 2, and any date of entry later than his actual arrival time, $s \geqslant [t]$. A buyer who reports (s,k) pays $r(s,k)$ (in period zero dollars) and obtains the item with discounted probability $q(s,k)$ (that is, if the buyer receives the item with probability Q_n in period n, then $q(s,k) = \sum_{n=0}^{\infty} \beta^n Q_n$). The mechanism must satisfy, for $j, k = 1$ and 2, and $t \geqslant 0$:

$$V_j q(t,j) - r(t,j) = \max_{s \geqslant [t]} \{V_j q(s,k) - r(s,k)\}, \tag{23}$$

$$V_j q(t,j) - r(t,j) \geqslant 0, \quad \text{and} \tag{24}$$

$$0 \leqslant q(t,j) \leqslant \beta^{[t]}. \tag{25}$$

(23) is an incentive compatibility condition which guarantees that each buyer would prefer to report his own type (t,j) than any other type of the form (s,k) for $s \geqslant [t]$: No one can gain by imitating someone who enters the market after he does. Condition (24) is the individual rationality restriction; a buyer could earn zero surplus if he does not buy. The third condition, (25), places an upper bound on the discounted probability of a buyer who enters in period $[t]$; at best a buyer obtains the item with probability one in the period that he enters.

[11] CGS show that if $\alpha V_1 > V_2$ and high valuers buy in the period that they enter the market, then the profits of the seller are increasing in the length of sales cycles. Consequently profits from the stationary equilibrium are always strictly less than $\alpha V_1/(1-\beta)$.

The seller's optimal selling mechanism maximizes

$$(26) \qquad \int [\alpha r(t,1) + (1-\alpha) r(t,2)]\, dt,$$

subject to (23), (24), and (25). Although there are selling mechanisms in which buyers report an element from an arbitrary set of messages rather than choosing an element directly from the set of indices, the revelation principle implies that the seller does not gain from using them.[12]

The formulation does depend on the assumption that all buyers and the seller share a common discount factor. This assumption guarantees that the discounted value of payments, $r(\cdot)$, is the same for all players. Specifically, assume that the buyers all have the discount factor β, but the seller uses the discount factor $\gamma \neq \beta$. If a buyer who reported (s,k) could purchase the item by paying $p(t;s,k)$ in period t for $t = 0,1,2,\ldots$, then the buyer's discounted price is $\sum_{t=0}^{\infty} \beta^t p(t;s,k)$, which is not necessarily equal to the seller's valuation of the stream of payments, $\sum_{t=0}^{\infty} \gamma^t p(t;s,k)$. The seller can take advantage of the difference in time preferences and earn profit by borrowing and lending even if she did not sell anything.

Theorem 4 describes what the monopolist could earn using a selling mechanism.

THEOREM 4: *Using the optimal selling mechanism the seller earns* $(1-\beta)^{-1} \max\{\alpha V_1, V_2\}$.

PROOF: Consider the problem of finding $r(\cdot)$ and $q(\cdot)$ to maximize (26) subject to (25),

$$(27) \qquad V_1 q(t,1) - r(t,1) \geqslant V_1 q(t,2) - r(t,2), \quad \text{and}$$

$$(28) \qquad V_2 q(t,2) - r(t,2) \geqslant 0.$$

(27) and (28) are implied by (23) and (24). I first show that the maximum value of (26) subject to the constraints (25), (27), and (28) is equal to $(1-\beta)^{-1} \max\{\alpha V_1, V_2\}$. Then I show that there is a selling mechanism satisfying (23)–(25) that attains this value.

(26) is increasing in $r(t,1)$ and $r(t,2)$. Since (27) is weakened when $r(t,2)$ is increased, (28) must bind at the optimum. Similarly, (27) must bind or the value of the objective function can be increased by increasing $r(t,1)$. Using these

[12] The standard revelation principle must be modified because in any feasible mechanism a buyer cannot receive an item with discounted probability greater than β^t, where t is the period that he enters the market. Therefore the set of feasible reports depends on the type of the buyer. It is simple to verify that the revelation principle still applies.

constraints (taken as equations) to solve for $r(\cdot)$ in terms of $q(\cdot)$, (26) becomes

$$\int [\alpha V_1 q(t,1) + (V_2 - \alpha V_1) q(t,2)]\, dt. \tag{29}$$

Maximizing (29) subject to (25) is easy since the objective function is linear: The upper bound on $q(t,1)$ must bind almost everywhere, and $q(t,2)$ should equal its upper or lower bound depending on whether $V_2 - \alpha V_1$ is positive or negative. The value at the solution is $(1-\beta)^{-1}\max\{\alpha V_1, V_2\}$.

I constructed the upper bound on profit by ignoring some of the incentive constraints faced by the seller. However, there are mechanisms that solve (26) subject to (25), (27), and (28) that also satisfy (23) and (24). When $V_2 > \alpha V_1$ the solution requires that buyers purchase the item at the price V_2 in the period that they enter ($r(t,k) = \beta^{[t]} V_2$ and $q(t,k) = \beta^{[t]}$). The seller can implement this outcome by charging V_2 in every period. When $V_2 < \alpha V_1$ the solution requires that high buyers purchase at the price V_1 in the period that they enter the market ($r(t,1) = \beta^{[t]} V_1$, $r(t,2) = 0$, $q(t,1) = \beta^{[t]}$, and $q(t,2) = 0$). The seller can implement this outcome by charging V_1 in every period. It follows that the seller can obtain the upper bound to profit, so the proof is complete.

If there was no entry of new consumers and the seller could set a single take-it-or-leave-it price, then she would charge either V_2 and sell to everyone, or V_1 and sell only to the high valuers. The theorem states the seller can do no better than repeat her best one-shot strategy even when there is entry of new consumers and more elaborate selling strategies are feasible. The seller implements the optimal selling strategy by promising to charge the same price (either V_1 or V_2) in every period.

Theorem 4 is reminiscent of Stokey's (1979) result that a monopolist in a multiperiod market without entry of new consumers can do no better than commit to the single static monopoly price if she discounts at the same rate as the buyers. In a bargaining model formally analogous to the monopoly problem, Riley and Zeckhauser (1983) show that a take-it-or-leave-it offer is the optimal bargaining mechanism.

CGS show that if the seller charges prices of the form (1) and high valuers buy in the period that they enter the market, then profits are monotonically increasing (decreasing) in the interval between sales if $\alpha V_1 > V_2$ ($\alpha V_1 < V_2$). Coles (1989) obtains a similar result.

If the seller commits to the strategy of charging V_1 in every period, then eventually she must resist the temptation to sell to an enormous backlog of low valuers. The rationality of high valuers makes it profitable ex ante for the seller to try to keep the price high; if the buyers expect the price to fall in the future, then they will not pay V_1 when they enter the market. Theorem 4 demonstrates that (if $\alpha V_1 > V_2$) the profit the seller makes from cutting her price to sell to low valuers does not compensate for her reduced ability to discriminate against high valuers. The result demonstrates the power of subgame perfection. After high valuers have left the market, the seller no longer has an incentive to keep her price high; in a subgame that begins with a large enough accumulation of low valuers in the market the seller must hold a sale.

6. RELATED LITERATURE

An interesting feature of the model is that equilibrium behavior involves cyclic fluctuation of prices and purchases in a stationary environment. CGS and Sobel (1984) discuss how the model captures several common features of sales. There are other models with similar qualitative properties.

Coles (1989) studies a durable-good monopolist whose products break down at a constant rate. No new consumers arrive, but old consumers reenter the market when their product fails.[13] Coles works with a continuous time model. He shows that (stationary) equilibria have the Coase property and proves that if commitment were possible a revenue-maximizing seller would charge the same price in every period. Theorem 4 of this paper is an analogous result.

In Fershtman and Fishman's (1989) model a constant flow of identical consumers enter a market. There are many suppliers. Consumers must obtain a price quotation before they make a purchase. The information is costly to obtain. Fershtman and Fishman show that there exist equilibria in which no buyers search in certain periods. Instead they accumulate and buy in "boom" periods, which may occur at fixed intervals.

The monopolist in Benabou's (1989) paper faces a cost of adjusting its nominal price in an inflationary economy. The good produced lasts for two periods. In the Markov-perfect equilibrium the seller typically uses a mixed strategy to determine when to increase her nominal price. Some consumers speculate on the timing of price adjustments; they buy in large quantities for future resale when they expect the price to increase. While Benabou concentrates on different issues, the price dynamics of his model have several features in common with mine. In particular, real prices decline in most periods, but periodically they rise sharply. On average demand is higher just before the price rises. Benabou's price dynamics are typically stochastic; mine are deterministic.

7. COMPARISON WITH CGS

CGS analyze the model of this paper but they obtain different results. In this section I briefly review how CGS characterize their equilibrium and explain how it differs from the subgame-perfect equilibria that I discuss.

CGS do not specify complete strategies for the players of the game. They reduce the problem to one where the seller selects the length of a sale cycle. The seller picks a sale date, and charges prices of the form (1). High-valuation buyers purchase as soon as they enter the market, and low-valuation buyers

[13] Kyle Bagwell and Garey Ramey and Gene Grossman and Michael Katz also have suggested this model.

purchase at the first sale date after they enter the market. A dynamic choice problem determines the length of the equilibrium sale cycle. Each period the seller may either have a sale or wait. If she has a sale, then all future sales cycles occur at the same interval. If she waits, then in the next period she must again decide whether or not to have a sale. CGS show that if the seller goes long enough without having a sale, then she cannot resist holding one. From that date they use backward induction to determine the equilibrium cycle length.

CGS assume that high-valuation buyers purchase immediately if they do not lose surplus by doing so. Therefore there will never be a backlog of high valuers in equilibrium. They also assume that the timing of the first sale influences expectations about the timing of future sales. Hence the seller maintains an incentive to keep her price high even when the interval between periods is short. By delaying a sale the seller convinces future entrants that future cycles will be long. This effect does not enter into computations when new consumers do not come into the market or, as in the stationary equilibria of this paper, pricing behavior prior to a sale does not influence equilibrium behavior after the market clears. It is precisely this kind of consideration that allows me to construct nonstationary equilibria where the seller charges prices significantly greater than V_2 even when the interval between offers is arbitrarily small.

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Manuscript received May, 1989; final revision received October, 1990.

APPENDIX

PROOF OF LEMMA 2: Fix an equilibrium and a subgame in which there have been N periods since the previous sale. Assume that there is no sale until a mass of at least n low valuers have accumulated. Denote by C_k the mass of high-valuation buyers in the market k periods after the sale. It follows that the mass of buyers in period k is $C_k - C_{k+1} + 1$. Consequently, in equilibrium it must be that

$$\text{(A1)} \qquad \sum_{i=0}^{n-k} (C_{k+i} - C_{k+1+i} + 1)\beta^i \alpha V_1 + \beta^{n-k} n(1-\alpha)V_2 + \beta^{n-k+1}V_1/(1-\beta)$$
$$\geqslant \alpha C_k V_2 + kV_2(1-\alpha) + \beta V_2/(1-\beta)$$

for all $N \leqslant k \leqslant n$. The left-hand side of the inequality is an upper bound on the profit available if there have been k periods since the last sale and the seller waits until there are at least n periods of low-valuation consumers before clearing the market. The first term on the left is a bound on the profit available from the high-valuation buyers who enter prior to the sale. It assumes that all sales are made at the price V_1, the highest price that any consumer will pay for the good. The second term is the discounted profit obtained from the low consumers when there is an accumulation of n periods of low consumers. The third term is an upper bound for the continuation profit. Since all consumers buy as soon as they enter the market if the seller charges V_2, the right-hand side is a lower bound for the monopolist's profit. It follows from algebraic manipulation of inequality (A1) that

$$\text{(A2)} \qquad \alpha(V_1 - V_2)C_k - (1-\beta)\alpha V_1 \sum_{i=0}^{n-k-1} \beta^i C_{k+1+i}$$
$$\geqslant \left(\beta V_2 - \alpha V_1 - \beta^{n-k+1}V_1\right)\Big/(1-\beta) + (1-\alpha)\left(k - n\beta^{n-k}\right)V_2$$

for all $N \leqslant k \leqslant n$.

The following general fact allows me to simplify (A2). Suppose there exist $C_k \geqslant 0$ and D_k for $k = N, N+1, \ldots, n$, and $\delta \in (0,1)$ such that

$$\text{(A3)} \qquad \delta C_k - (1-\beta) \sum_{i=0}^{n-k-1} \beta^i C_{k+1+i} \geqslant D_k \quad \text{for} \quad k = N, N+1, \ldots, n.$$

Since $C_k \geqslant \delta C_k$ for all $k = N, N+1, \ldots, n$, an induction argument demonstrates that

$$\text{(A4)} \qquad \delta C_k \geqslant D_k + (1-\beta) \sum_{i=k+1}^{n} D_i \quad \text{for all} \quad k = N, N+1, \ldots, n$$

(where, by convention, $\Sigma_{i=n+1}^{n} D_i = 0$).

If I let $D_k = \{(\beta V_2 - \alpha V_1 - \beta^{n-k+1}V_1)/(1-\beta) + (1-\alpha)(k - n\beta^{n-k})V_2\}/(\alpha V_1)$, $\delta = (V_1 - V_2)/V_1$, and C_k be as above, then (A2) implies that (A3) holds. Plugging these values of δ and D_k into (A4) yields

$$\alpha(V_1 - V_2)C_k \geqslant (\beta V_2 - \alpha V_1 - \beta V_1)/(1-\beta) + (n-k)$$
$$\times\left\{\left[\beta V_2 - \alpha V_1 - (1-\alpha)V_2\right] + (n+k+1)(1-\beta)(1-\alpha)V_2/2\right\}$$
$$\geqslant -(\alpha+\beta)V_1/(1-\beta) + (n-k)\left[(n+k+1)(1-\beta)(1-\alpha)V_2/2 - V_1\right].$$

Since it must be the case that $C_k \leqslant k$, it follows that if there have been k periods since the last sale and the next sale does not occur in the next $n-k$ periods, then

$$\alpha(V_1 - V_2)k \geqslant -(\alpha+\beta)V_1/(1-\beta) + (n-k)\left[(n+k+1)(1-\beta)(1-\alpha)V_2/2 - V_1\right].$$

When $n-k$ is sufficiently large, this inequality cannot hold. Therefore, there is a constant M^* such that the number of periods until the next sale is always less than M^*.

Proof of Lemma 3 (Sketch): I construct a candidate equilibrium under the assumption that the seller must hold a sale in the next n periods (including the current one), the seller never charges less than V_2, and that the seller's profit is $W \geqslant V_2/(1-\beta)$ after the sale. Let $p_1 = V_2$, $p_2 = (1-\beta)V_1 + \beta V_2$, $S_1(C,c) = P_1(C,c,p) = V_2$, $\pi_1(C,c) = p_1[C\alpha + c(1-\alpha)] + \beta W$, $B_2(C,c,p) = C$ if $p \leqslant p_2$, and $D_1(c,1) = 1$. When $n = 1$, the seller must hold a sale. p_n denotes the highest price at which any consumer buys if there will be a sale in no more than n periods. $\pi_1(C,c)$ is equal to the profit of the seller if there are accumulated masses of C high-valuation buyers and c low-valuation buyers, and the value of the market after a sale is W. $S_n(C,c)$ is the set of prices that an optimizing seller would charge in the current period if the seller must hold a sale in the next n periods. $P_n(C,c,p)$ is the expected value of the seller's price if she charged p in the last period. $S_1(C,c) = P_1(C,c,p) = V_2$ since the seller must hold a sale in the current period when $n = 1$. $B_{n+1}(C,c,p)$ gives the mass of high valuers that buy in the $(n+1)$th step of the construction when p is charged. When $n = 1$, a high valuer compares the surplus from buying at p, $V_1 - p$, to the surplus from buying in the next period at V_2, $\beta(V_1 - V_2)$. Buying immediately is superior to waiting if $p < p_2$. When $p = p_2$, buying and waiting yield the same surplus. I take $B_2(C,c,p_2) = C$ because otherwise the seller's second stage optimization problem need not have a solution. For $j \leqslant n$, $D_n(c,j)$ is the highest value of C below which the seller wishes to charge a price that is strictly less than p_j (assuming that there will be a sale in the next n periods). If the seller wants to charge $p \geqslant p_j$ for all $C \geqslant 1$, then set $D_n(c,j) = 1$. In particular, $D_n(c,1) = 1$ for all n. Although $\pi_k(\cdot)$, $S_k(\cdot)$, $P_k(\cdot)$, $B_{k+1}(\cdot)$, and $D_k(\cdot)$ depend on W, I suppress this dependence in the proof to simplify notation.

The construction continues by induction. At stage k of the induction, no buyer will pay more than p_k. Assume that I have constructed p_{k+1}, $\pi_k(\cdot)$, $S_k(\cdot)$, $P_k(\cdot)$, $B_{k+1}(\cdot)$, and $D_k(\cdot)$ for

$k = 1, \ldots, n-1$ and that these functions satisfy:

E1. $p_{k+1} = (1-\beta^k)V_1 + \beta^k V_2$.

E2. $S_k(C,c) \subset \{p_1, \ldots, p_k\}$.

E3. $$B_{k+1}(C,c,p) = \begin{cases} C & \text{if } p \leqslant p_1, \\ \max\{C+1-D_k(c+1,j),0\} & \text{if } p \in (p_j, p_{j+1}]. \end{cases}$$

E4. If $C > D_k(c,j)$, then $p_{j-1} \notin S_k(C,c)$.

E5. For $j \leqslant j-1$, if the first price charged is less than or equal to p_j, then there is a sale in no more than j periods.

E6. $$\pi_k(C,c) = \begin{cases} p_k(C+1-D_{k-1}(c+1,k-1))\alpha + \beta\pi_{k-1}(D_{k-1}(c+1,k-1),c+1) & \\ & \text{if } C \geqslant D_k(c,k), \\ \pi_{k-1}(C,c) & \text{if } C \leqslant D_k(c,k). \end{cases}$$

E7. $D_k(c,j) = \min\{D_k(c,k), D_{k-1}(c,j)\}$ for $j<k$.

E8. $P_k(C,c,p) = V_1 - (V_1-p)/\beta$ if $p \in (p_j, p_{j+1})$ and $D_k(c+1,j) \in (1, C+1)$.

E9. $D_k(c,j) > D_k(c+1,j-1) - 1$ for $j = 2, \ldots, k$.

E10. $\pi_k(\cdot)$ and $D_k(\cdot)$ are continuous function of W.

I do not verify the induction hypothesis. Fudenberg, Levine, and Tirole's (1985) verification is similar. E1 defines the highest price that any buyer will pay if there will be a sale in the next $k+1$ periods. Buying at p_{k+1} in the current period yields the same utility to a high buyer as waiting k more periods for a sale. E2 states that the optimal price for the seller must be of the form $(1-\beta^i)V_1 + \beta^i V_2$ for $i \leqslant k$. E3 describes the aggregate response of the buyers to the seller's strategy. E4 states that the seller's optimal price does not decrease when the mass of high valuers in the market increases. E5 gives a condition that insures that the artificial constraint on the timing of sales does not bind. E6 implies that if the seller charges p_k, then she sells to as many high valuers as possible consistent with p_{k-1} being optimal in the next period. E7 implies that relaxing the constraint on the time of the next sale does not lower the optimal prices of the seller. E8 is equivalent to requiring that the high valuer be indifferent between buying at p and waiting. E9 guarantees that there are positive purchases in each period. E10 insures that the equilibrium profit function of the auxiliary game is continuous in W.

Since there is a bound M^* on the number of periods until the next sale in all subgames of a stationary equilibrium, the seller does not charge more than p_{M^*+1} in equilibrium. Therefore, E5 guarantees that $\pi_k(\cdot)$, $S_k(\cdot)$, $D_k(\cdot)$, $B_k(\cdot)$, and $P_k(\cdot)$ do not change when $k \geqslant M^*+1$; denote the limiting values by $\pi^*(\cdot)$, $S^*(\cdot)$, $D^*(\cdot)$, $B^*(\cdot)$, and $P^*(\cdot)$. I describe in the text the correspondence between these mappings and equilibria to the auxiliary game.

Proof of Theorem 2: Assume that the equilibrium calls for the seller to hold a sale in k periods. Denote by s_i the mass of sales to high-valuation customers i periods prior to the sale date. Let (C,c) represent the current state of the market. The discounted profit to the monopolist n periods prior to the sale date, for $n = 0, 1, \ldots, k$, is equal to

$$\text{(A5)} \qquad \sum_{i=0}^{n} \alpha\beta^{n-i} p_{i+1} s_i + \beta^n (k+c)(1-\alpha)V_2 + \beta^{n+1} W.$$

If $n > 0$, then the seller is able to get profit equal to at least

$$\text{(A6)} \qquad \alpha p_n(s_n - 1) + \sum_{i=0}^{n-1} \alpha\beta^{n-i-1} p_{i+1} s_i + \beta^{n-1}(k+c-1)(1-\alpha)V_2 + \beta^n W.$$

Expression (A6) is what the seller would earn if she charges p_{i-1} instead of p_i and the mass of high-valuation buyers in the market when a particular price is offered remains the same. The mass of high-valuation buyers needed to make it optimal for the seller to charge p_i is an increasing function of the mass of low-valuation buyers in the market. In other words, if it is optimal to charge a price at least as great as p_i when the market state is (C, c), then it is optimal to charge a price at least as great as p_i when the market state is $(C, c-1)$. For this reason, the seller earns at least (A6) if she begins by charging p_n instead of p_{n+1}. Therefore, in equilibrium, it must be the case that (A5) is greater than or equal to (A6). It follows from (A5) and (A6) that

$$\text{(A7)}\qquad \alpha(p_{n+1}-p_n)s_n-\beta^n(1-\beta)W+\alpha p_n+(1-\alpha)\beta^n V_2$$
$$\geqslant \sum_{i=0}^{n-1}\alpha(1-\beta)\beta^{n-i-1}p_{i+1}s_i+\beta^{n-1}(1-\beta)V_2(k+c-1)(1-\alpha).$$

Because there is no need for the seller to carry a backlog of high valuers into a sale period

$$\text{(A8)}\qquad s_0=1.$$

Consequently, algebraic manipulation of (A7), using $\sum_{i=1}^{n-1}(1-\beta)\beta^{n-i-1}=1-\beta^{n-1}$ and $(1-\beta)W \geqslant V_2$, implies that

$$\text{(A9)}\qquad \alpha(p_{n+1}-p_n)s_n\geqslant \sum_{i=1}^{n-1}\alpha(1-\beta)\beta^{n-i-1}(p_{i+1}s_i-V_1)$$
$$+\beta^{n-1}(1-\beta)V_2(k+c-1)(1-\alpha).$$

It follows from (A9) and the definition of p_i that

$$\alpha s_n\geqslant M(k+c-1)(1-\alpha)+(M+1)\sum_{i=1}^{n-1}\alpha\beta^{-i}(s_i-1)-\sum_{i=1}^{n-1}\alpha s_i,$$

where $M=V_2/(V_1-V_2)$. Consequently, since $\beta<1$,

$$\text{(A10)}\qquad \alpha s_n\geqslant M(k+c-1)(1-\alpha)+M\left(\sum_{i=1}^{n-1}\alpha\beta^{-i}s_i\right)-(M+1)\sum_{i=1}^{n-1}\beta^{-i}\alpha.$$

Now to obtain a contradiction, suppose that

$$\text{(A11)}\qquad k>(M+1)\alpha/\left[M^2(1-\alpha)\right]+1.$$

(A10) when $n=1$ implies that

$$\text{(A12)}\qquad \alpha s_1\geqslant M(k+c-1)(1-\alpha).$$

(A11) and (A12) imply that if $i=1$, then

$$\text{(A13)}\qquad \alpha s_i\geqslant (M+1)\alpha/M+M(c-1)(1-\alpha).$$

Moreover, if (A13) holds for $i=1,2,\ldots,n-1$, then (A10) and (A11) combine to guarantee that (A13) holds for $i=n$ as well. It follows by induction that (A13) holds for all $i=1,2,\ldots,k$. But $\sum_{i=1}^{k}s_i=(k+C)$, since the seller serves all of the high valuers by the time of the sale. (A8) now implies that $\sum_{i=1}^{k}s_i=(k+C-1)$. Hence, summing both sides of (A13) from $i=1$ to k yields $(k+C-1)\alpha \geqslant (M+1)k\alpha/M+kM(c-1)(1-\alpha)$, which, if $C\leqslant c$, implies that $k\leqslant (M+1)\alpha/[M^2(1-\alpha)]$. It follows that the number of periods until the next sale is at most $(M+1)\alpha/[M^2(1-\alpha)]+1$.

The remainder of the Appendix completes the proof of Theorem 3.

LEMMA 4: *Given any $\varepsilon>0$, there exists $\beta^*\in(0,1)$ such that if $\beta\in(\beta^*,1)$, then $v(C,c)-\beta v(C+1,c+1)\geqslant\varepsilon/2$.*

PROOF: Let $h(x)=(1-\beta)V_1x^\eta/(\beta\eta)$. It suffices to show that if $x\geqslant m'-1$, then

$$\text{(A14)}\qquad \liminf_{\beta\to 1}(1-\beta)^{-1}\{h(x)-\beta h(x+1)+(U-V_2)[(x-m')(1-\beta)-\beta]/(m+1-m')\}$$
$$\geqslant\varepsilon/2.$$

If $x \geqslant m' - 1$, then

$$\text{(A15)} \qquad \begin{aligned}[h(x)-\beta h(x+1)]/(1-\beta) &\geqslant \left[(1-\beta)(1+x)^{\eta}-\eta x^{\eta-1}\right]V_1/(\beta\eta) \\ &\geqslant \left[(1-\beta)(m')^{\eta}-\eta(m'-1)^{\eta-1}\right]V_1/(\beta\eta) \\ &= \varepsilon - V_1(m'-1)^{\eta-1}/\beta.\end{aligned}$$

The first inequality follows from the definition of $h(\cdot)$ and $x^{\eta} \geqslant (1+x)^{\eta} - \eta x^{\eta-1}$, the second inequality follows because $(1+x)^{\eta}$ is increasing and $x^{\eta-1}$ is decreasing, and the equation follows from (15) (the definition of m'). Since, from (15), $[(m+1)/m']^{\eta} = V_2/\varepsilon > 1$, (10) implies that $\lim_{\beta \to 1} 1/[(1-\beta)(m+1-m')] = 0$ and $\lim_{\beta \to 1}(m'-1)^{\eta-1} = 0$. Hence, (A15) implies (A14).

LEMMA 5: *There exists $\beta^* \in (0,1)$ such that for all $\beta \in (\beta^*, 1)$ the punishment strategy specified for the seller responds optimally to the buyer when $G(C,c) > m$.*

PROOF: Since players follow stationary equilibrium strategies if $G(C,c) > m+1$ or in the event of simultaneous deviations, it suffices to consider the case where the market state (C,c) satisfies $G(C,c) \leqslant m+1$.

If the seller follows the strategy specified by the equilibrium and charges $g(C,c)$, then, since everyone currently in the market buys, she earns $G(C,c)g(C,c) + \beta v(C,c)$. If $G(C,c) > m$, then $G(C,c)g(C,c) \geqslant (1-\beta)V_1 m^{\eta+1}/(\beta\eta)$. Since $m^{\eta+1} \geqslant (m+1)^{\eta+1} - (\eta+1)(m+1)^{\eta}$ it follows from (15) that

$$\text{(A16)} \qquad G(C,c)g(C,c) + \beta v(C,c) \geqslant V_2(m-\eta) + \beta v(C,c).$$

If the seller instead charges $p \neq g(C,c)$, then players follow the strategies of the stationary equilibrium. Let K be the bound, derived in Theorem 2, on the number of periods until the next sale (at price V_2) in any stationary equilibrium. No one will buy at a price higher than $(1-\beta^K)V_1 + \beta^K V_2$. Consequently the seller earns at most

$$\text{(A17)} \qquad \left[(1-\beta^K)V_1 + \beta^K V_2\right]C\alpha + (1-\alpha)cV_2 + \beta w,$$

if she does not charge $g(C,c)$. The first term in (A17) is the profit from selling immediately to all high valuers currently in the market at the highest price they would accept, the second term is the profit from selling immediately to the low valuers currently in the market at the highest price that they would accept, and w is the profit of the seller in the stationary equilibrium after the market has been cleared. After clearing the market, the seller could earn no more in a stationary equilibrium (when $\alpha V_1 \geqslant V_2$) than she would if she sold to all high valuers as soon as they enter the market at the price p_K and sold to low valuers as soon as they enter the market at the price V_2. Consequently,

$$\text{(A18)} \qquad w \leqslant \left[(1-\alpha)V_2 + \alpha p_K\right]/(1-\beta) \leqslant V_2/(1-\beta) + \alpha K V_1,$$

where the last inequality follows since $p_K = (1-\beta^{K-1})V_1 + \beta^{K-1}V_2 \leqslant (1-\beta)KV_1 + V_2$. Combining the upper bound in (A17) with that of (A18) leads to an upper bound for the seller's profit if she strays from the equilibrium:

$$\text{(A19)} \qquad G(C,c)V_2 + \beta V_2/(1-\beta) + \left[(1-\beta)C\alpha(V_1 - V_2) + \alpha V_1\right]K,$$

where I use $(1-\beta^K) \leqslant K(1-\beta)$. The seller does not wish to deviate if the lower bound in (A16) exceeds (A19). Since $m+1 \geqslant G(C,c)$, $mV_2 + V_2/(1-\beta) \geqslant G(C,c)V_2 + \beta V_2/(1-\beta)$, and therefore it suffices to show that

$$\text{(A20)} \qquad \beta v(C,c) \geqslant [\eta + 1/(1-\beta)]V_2 + \left[(1-\beta)C\alpha(V_1-V_2) + \alpha V_1\right]K.$$

Since $G(C,c) > m$, and $\alpha V_1 > V_2$, (13) and (18) imply that $(1-\beta)[v(C,c) - \beta V_2] > \varepsilon/2 > 0$. It follows from $C\alpha \leqslant G(C,c) \leqslant m+1$ that (A20) holds for β sufficiently close to one if

$$\text{(A21)} \qquad \lim_{\beta \to 1}(1-\beta)\left\{(\eta+1+\beta)V_2 + \left[(1-\beta)(m+1)(V_1-V_2) + \alpha V_1\right]K\right\} = 0.$$

(A21) follows since $\eta > 1/2$ implies that $\lim_{\beta \to 1}(1-\beta)^2 m = 0$.

LEMMA 6: *There exists $\beta^* \in (0,1)$ such that for all $\beta \in (\beta^*, 1)$ the punishment strategy specified for the seller responds optimally to the buyer when $G(C,c) \leqslant m$.*

PROOF: Assume that buyers follow the strategy given in (14). If $p > (1-\beta)V_1 + \beta g(C+1, c+1)$, then no one buys and the seller earns what she would have earned if she had charged $(1-\beta)V_1 + \beta g(C+1, c+1)$. The seller earns strictly more by charging $(1-\beta)V_1 + \beta g(C+1, c+1)$ instead of $p \in ((1-\beta)V_2 + \beta g(1, c+1), (1-\beta)V_1 + \beta g(1, c+1))$; in both cases all of the high valuers and none of the low valuers currently in the market buy and the continuation is identical. If $p < (1-\beta)V_2 + \beta g(1,1)$, then the seller would earn strictly more by charging $(1-\beta)V_2 + \beta g(1,1)$; in both cases everyone in the market buys and the continuation is identical. Therefore, only defections to prices p either of the form

$$(1-\beta)V_1 + \beta g(D, c+1) \quad \text{for} \quad D \in [1, C+1] \tag{A22}$$

or of the form

$$(1-\beta)V_2 + \beta g(1, d) \quad \text{for} \quad d \in [1, c+1] \tag{A23}$$

could be attractive. If the seller charges a price of the form (A22) and subsequently follows the behavior specified by the equilibrium, then her profit will be

$$\begin{aligned} &[(1-\beta)V_1 + \beta g(D, c+1)](C+1-D)\alpha + \beta g(D, c+1)G(D, c+1) + \beta^2 v(D, c+1) \\ &\quad = \beta g(D, c+1)G(C+1, c+1) + \beta^2 v(D, c+1) + (1-\beta)V_1(C+1-D)\alpha, \end{aligned} \tag{A24}$$

since the candidate strategies specify that the mass $(C+1-D)\alpha$ of high valuers purchase immediately, the market empties in the following period when the seller charges $g(D, c+1)$, and then the equilibrium with profit $v(D, c+1)$ begins. The seller earns $g(C,c)G(C,c) + \beta v(C,c)$ if she follows the strategy specified by the equilibrium, so a defection to a price of the form (A22) will not be profitable if

$$g(C,c) \geqslant (G(C,c))^{-1} \max_{D \leqslant C+1} w(C, c, D), \tag{A25}$$

where

$$\begin{aligned} w(C,c,D) = {}& \beta G(C+1, c+1)g(D, c+1) + (1-\beta)V_1(C+1-D)\alpha \\ &- \beta v(C,c) + \beta^2 v(D, c+1). \end{aligned}$$

Similarly, if the seller charges a price of the form (A23) and then follows the behavior specified by the equilibrium, her profit will be

$$\begin{aligned} &[(1-\beta)V_2 + \beta g(1,d)]G(C+1, c+1-d) + \beta g(1,d)G(1,d) + \beta^2 v(1,d) \\ &\quad = \beta g(1,d)G(C+1, c+1) + \beta^2 v(1,d) + (1-\beta)V_2 G(C+1, c+1-d), \end{aligned} \tag{A26}$$

so a defection to a price of the form (A23) will not be profitable if

$$g(C,c) \geqslant (G(C,c))^{-1} \max_{d \leqslant c+1} \mu(C, c, d), \tag{A27}$$

where

$$\begin{aligned} \mu(C,c,d;g) = {}& \beta G(C+1, c+1)g(1,d) + (1-\beta)V_2 G(C, c+1-d) \\ &- \beta v(C,c) + \beta^2 v(1,d). \end{aligned}$$

It suffices to show that (A25) and (A27) are satisfied. Observe that $\mu(C,c,d) - \beta^2 v(1,d)$ is strictly concave in d and the partial derivative of $\mu(C,c,d) - \beta^2 v(1,d)$ with respect to d evaluated at $d = c+1$ is $(1-\beta)(1-\alpha)[V_1 G(C+1, c+1)G(1, c+1)^{\eta-1} - V_2] > 0$. Therefore, the solution to $\max_{d \leqslant c+1}\{\mu(C,c,d) - \beta^2 v(1,d)\}$ is $d = c+1$. Since $v(\cdot, d)$ is nondecreasing in d, the unique solution to $\max_{d \leqslant c+1} \mu(C, c, d)$ is also $d = c+1$. Similarly, the unique solution to $\max_{D \leqslant C+1} w(C, c, D)$ is $D = C+1$. Also, since $g(\cdot)$ is increasing, $V_1 > V_2$, and $v(\cdot)$ is nondecreasing, $\mu(C, c, c+1) \leqslant w(C, c, 1)$. Therefore, $w(C, c, C+1) = \max_{D \leqslant C+1} w(C, c, D) \geqslant w(Cc, 1) \geqslant$

$\mu(C, c, c+1) = \max_{d \leqslant c+1} \mu(C, c, d)$. Hence, (A25) and (A27) hold if and only if $g(C,c) \geqslant w(C, c, C+1)/G(C,c)$ or, equivalently,

(A28) $$\beta G(C+1, c+1) g(C+1, c+1) - G(C,c) g(C,c) \leqslant \beta[v(C,c) - \beta v(C+1, c+1)].$$

Since $G(C+1, c+1) = G(C,c) + 1$, calculus shows that the value of $G(C,c)$ that maximizes the left-hand side of (A28) satisfies $G(C,c) = \gamma/(1-\gamma)$, where $\gamma^{\eta} = \beta$. Substitution of this value for $G(C,c)$ into the left-hand side of (A28) and use of the definition of $g(\cdot)$ yields

(A29) $$\beta G(C+1, c+1) g(C+1, c+1) - G(C,c) g(C,c) \leqslant (1-\gamma^{\eta})[\gamma/(1-\gamma)]^{\eta} V_1/(\beta\eta).$$

The right-hand side of (A29) goes to zero as β converges to one. Hence, the left-hand side of (A28) converges to zero as β converges to one. Since the right-hand side of (A28) is at least $\beta\varepsilon/2 > 0$ by (18), the proof is complete.

I have assumed throughout the proof of Theorem 3 that $\alpha V_1 > V_2$. It remains to show that there exist equilibria in which the seller earns average profit between ε and V_2 when $V_2 \geqslant \alpha V_1$.

Proof of Theorem 3 when $V_2 \geqslant \alpha V_1$: $\alpha V_1 > V_2$ is used in the construction of low-profit equilibria only to guarantee that the seller does not deviate if the size of the market is $G(C,c) \in (m, m+1]$. When $V_2 > \alpha V_1$, the equilibrium strategies must be modified. Replace $g(\cdot)$ by $\min\{g(\cdot), V_2\}$ and let (14) describe the buyers' response to $p \neq g(C,c)$ even when $G(C,c) \in (m, m+1]$. Set $U = V_2$ in the definition of $v(\cdot)$ so that

(A30) $$\text{if} \quad G(C,c) = m+1, \quad \text{then} \quad (1-\beta) v(C,c) = V_2.$$

Lemma 4 still holds. Moreover, when $G(C,c) \in (m, m+1]$, the seller can do no better than to charge $(1-\beta)V_1 + \beta V_2$; a mass $C - D + 1$ of high valuers buy at this price, where $G(D, c+1) = m+1$. In the continuation the seller earns the stationary equilibrium profits. This strategy yields a payoff equal to

(A31) $$\begin{aligned}&[(1-\beta)V_1 + \beta V_2]\alpha(C - D + 1) + \beta V_2(m+1) + \beta^2 V_2/(1-\beta)\\ &\quad \leqslant (1-\beta)V_1 + \beta V_2(m+2) + \beta^2 V_2/(1-\beta),\end{aligned}$$

where the upper bound in (A31) follows since $G(C,c) \leqslant m+1$ and $G(D, c+1) = m+1$. If the seller follows the equilibrium, then her earnings are bounded below by the right-hand side of (A16). Therefore, it is sufficient to show that if β is sufficiently close to one, then

(A32) $$V_2(m-\eta) + \beta v(C,c) > (1-\beta)V_1 + \beta V_2(m+2) + \beta^2 V_2/(1-\beta),$$

whenever $G(C,c) \geqslant m$. Inequality (A32) follows from (10) and (A30).

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