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BREEDING AND RAIDING

A Theory of Strategic Production of Skills

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Some of the skills that firms require are obtained only through on-the-job-training. This paper concentrates on the strategic production of skills within the firm. Firms obtain high-quality workers either by training their own (breeding) or by using the open market to bid away workers trained by other firms (raiding). Even when all firms have access to the same technology of production, training, and breeding, it will typically be the case that both breeding and raiding will be pursued, with equal profitability, in equilibrium. Thus, we explain raiding behavior as part of an equilibrium theory. This paper also studies the effect of the decision to train workers on the optimal firm size and the distribution of income.

1. Introduction

The theory of hierarchical organizations, pioneered by Simon (1957), Lydall (1968) and Mayer (1960), seeks to provide explanation for existing income distributions, firm-size distributions, and growth patterns through the internal arrangements used by firms. More recently, the contributions of Lucas (1978) on the size distribution of firms, Williamson (1967) and Calvo and Wellisz (1978) on the optimal size and span of control of the firm, Rosen (1982) and Calvo and Wellisz (1979) on the wage distribution over abilities, and Stiglitz (1975) and Mirrlees (1976) on incentives, supervision schemes, and their motivation, have provided further rationalizations for the observed internal organization of firms, as well as its implications. These papers, as well as others in the field, assume the existence of different types of workers, so that a distribution of ability levels is an exogenously given part of the description of the labor force. However, some of the skills that firms require are obtained only through on-the-job training. [Mincer (1971) provides empirical evidence that on-the-job training accounts for the skills of a large fraction of the work force.] Most readers have seen the familiar request for a 'management consultant wanted with at least five years experience'. It is not

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possible to become qualified for this job solely on the basis of innate skill or from training obtained at a professional school. Thus, if the benefits of these skills are sufficiently high, then some firms will find it in their interest to train workers. In this way, one sees that the production (or acquisition) of skills is as fundamental to the efficient internal organization of the firm as the allocation of skills.

This paper concentrates on the production of skills within the firm. We present a model in which equilibrium behavior involves some firms providing costly training to their workers. Moreover, we are able to give conditions under which training is provided by only some of the firms in the industry even though the firms have access to identical technologies. Thus, we can give a justification for the observation that some firms in an industry have a policy of filling positions by promoting personnel from within, while some firms fill positions by hiring from outside.

Just as the supervision models require high-level workers to spend time observing low-level workers and away from direct output-producing activities, we assume that training is provided by high-level workers, who could otherwise be used as direct inputs in production. We then give conditions under which a unique equilibrium exists with two types of active firm: breeding firms, who train their own workers, and raiding firms, who acquire all their trained workers from breeding firms. This type of equilibrium can exist for the following reason. Workers who begin at a raiding firm have no opportunity to obtain training. Thus, untrained workers are willing to work at breeding firms for a lower salary. Indeed, the higher the wage paid to trained workers, and the higher the probability that they will get trained, the lower the wage needed to attract entry-level applicants at a breeding firm. If the wage paid to trained workers is sufficiently high, the breeding firm will train more than it needs for its own purposes in order to save on wages to unskilled workers. The surplus of trained workers is enough to provide an opportunity for raiding firms to enter the industry. The training is not firm specific so that the lower wages paid to untrained workers by breeders is consistent with the standard human-capital theories [see, for example, Becker (1975)] which suggest that the cost of training should be borne by the workers.

Our model allows us to analyze the effect of training on income distribution and equilibrium firm size as well as the effects of minimum wage legislation on training levels. In particular, we show that the distribution of income becomes more skewed as training costs increase and give conditions under which breeding firms are larger than raiding firms.

Rosen (1972) presents a model in which training is considered explicitly. In it firms are able to offer a collection of work activities; some of them involve pure production and no training opportunities while others offer some training opportunities. The relationship between the marginal rate of transformation of those jobs and market prices determines a firm's hiring (or offer) policy. Our model differs from Rosen's because we introduce training costs in terms of output foregone when skilled labor has to spend time training unskilled labor. Also, we allow firms to compete for skilled labor by choosing wages and training intensities. This lets us characterize market equilibria and exhibit conditions that guarantee the existence of raiding. Raiding does not occur in Rosen's model since all firms follow identical policies.

2. The model

This section describes the framework in which the analysis of the next section is carried out.

Workers: Workers live for two periods. They are wage takers and seek to maximize the undiscounted sum of expected wages. There are two types of workers, h (for high) and l (for low). In the first period of life all workers are type l; they become type h workers in the second period if and only if they have been trained in the first period. Workers are perfectly mobile between firms, training is not firm specific, and trained workers can be identified without cost. There is an unlimited supply of l-workers.

Firms: There is an unlimited supply of potential firms. Each has access to the same production technology $f(H,L)$ and training technology $g(H)$. $f(H,L)$ is the amount of output that can be produced by H h-workers and L l-workers. An h-worker can be used as a direct input in production or in training. $g(H)$ is the number of l-workers that can be trained if H h-workers are devoted to training and at least $g(H)$ l-workers are available to be trained. That is, if a firm hires H h-workers and L l-workers, and devotes a fraction $(1-a)\in(0,1]$ of the h-workers to training, it produces output $f(aH,L)$ as well as training $\min[L,g((1-a)H)]$ l-workers. The trained l-workers will be next period's h-workers. A firm that trains workers will be called a *breeder*. Alternatively, a firm may decide to devote all of its h-workers directly to production, and thereby produce no h-workers itself. Such a firm will be called a *raider* since it must raid breeding firms in order to obtain trained workers. It is assumed that f is twice continuously differentiable, strictly increasing, strictly concave, and that $f(0,L)$, $f(H,0)\leqq 0$, $f(0,0)<0$, $\lim_{H\to\infty} f_{11}(H,L)=\lim_{L\to\infty} f_{22}(H,L)=0$ and $f(H_0,L_0)>0$ for some H_0 and L_0. The subscripts denote partial differentiation in the usual way. No training is possible without a positive input, so we assume $g(H)=0$ for $0\leqq H\leqq H_0$. For $H>H_0$, g is twice continuously differentiable, strictly increasing, strictly concave, and $g'(H)>1$. These standard assumptions guarantee that firms typically have a positive efficient scale of production; the assumption that $g'(H)>1$ guarantees that training is effective.

Equilibrium: The paper characterizes the steady-state Nash equilibrium in the factor market. All firms take the output price, which is normalized to one, as given. Strategies consist of, for each firm i, wages $w^H(i)$, $w_1^L(i)$, and $w_2^L(i)$ paid to h-workers, first-period l-workers, and second-period l-workers, respectively, input levels of raided h-workers, $H^R(i)$, bred h-workers, $H^B(i)$, first-period l-workers, $L_1(i)$, and second-period l-workers, $L_2(i)$, and a training level $a(i)$. Strategies for the workers consist of rules that tell them where to work. For the strategies to be in equilibrium it is required that:

(a) Workers select firms to maximize their expected income, taking wages as given. That is,

(i) an h-worker works for firm j only if

$$w^H(j)=\max_i\,[w^H(i), w_2^L(i)], \tag{1}$$

(ii) a second-period l-worker works for firm j only if

$$w_2^L(j)=\max_i\, w_2^L(i), \tag{2}$$

(iii) a first-period l-worker works for firm j only if

$$\begin{aligned} &w_1^L(j)+p(j)\tilde{w}^H+(1-p(j))\min\,[1, L_2(j)/L_1(j)(1-p(j))]w_2^L(j) \\ &=\max_i\,(w_1^L(i)+p(i)\tilde{w}^H+(1-p(i))\min\,[1, L_2(i)/L_1(i)(1-p(i))]w_2^L(i)), \end{aligned} \tag{3}$$

where $\tilde{w}^H$ is the maximum value in (1), and $p(i)$ is the probability of being trained at firm i,

$$p(i)=\min\,[1, g((1-a(i))(H^R(i)+H^B(i)))/L_1(i)].$$

(b) Firms, taking the behavior of the other firms as given, maximize profits subject to attracting workers. That is, firm j picks $w^H(j)$, $w_1^L(j)$, $w_2^L(j)$, $H^R(j)$, $H^B(j)$, $L_1(j)$, $L_2(j)$, and $a(j)$ to solve:

(A) $\qquad \max f(a(H^R+H^B), L_1+L_2)-w^H(H^R+H^B)-w_1^L L_1-w_2^L L_2,$

subject to

$$L_1 \geqq g((1-a)(H^R+H^B)) \geqq H^B,$$

$$H^R=H^B=0 \quad \text{unless} \quad w^H=\tilde{w}^H,$$

$$L_2=0 \qquad\qquad \text{unless} \quad w_2^L=\tilde{w}_2^L,$$

$$L_1=0 \qquad\qquad \text{unless} \quad w_1^L+p\tilde{w}^H+(1-p)\min\,[1, L_2/L_1(1-p)]w_2^L=2\tilde{w},$$

where $\tilde{w}_2^L$ and $2\tilde{w}$ are the maximum values in (2) and (3), respectively, and $p=\min[1, g((1-a)(H^R+H^B))/L_1]$.

(c) Supply and demand for labor are equal. That is,

$$\sum_{i \in \{i: a(i)>0\}} g((1-a(i))(H^R(i)+H^B(i))) = \sum_i (H^R(i)+H^B(i)),$$

$$\sum_i L_1(i) = \sum_i (L_2(i)+H^R(i)+H^B(i)).$$

The interpretation of condition (a) is straightforward. Workers, taking wages as given, select a firm that maximizes their expected wage. Notice that condition (a.iii) assumes that untrained workers believe that they will only be able to work for the firm they started with. If $L_2 < L_1(1-p)$ so that the firm is not hiring enough second-period l-workers to offer jobs to all of the untrained workers, then this is taken into account; $\min[1, L_2/L_1(1-p)]w_2^L$ should be interpreted as an untrained worker's expected second-period wage. Condition (b) is the maximization problem of the firm. In principle, firms are allowed to breed and raid h-workers, and fire or hire untrained l-workers. Condition (c) simply balances the labor market.

A few observations simplfy the search for equilibria. Since trained workers are a necessary input, all active firms must be able to attract them, so there is a uniform high wage, w^H. Similarly, there is a uniform wage, w_2^L, paid to untrained second-year workers. Condition (a.i) implies that h-workers can work in unskilled jobs; it follows that $w^H \geqq w_2^L$ is needed to attract h-workers to skilled jobs. In order to fulfil a worker's expectations, untrained workers must be retained by their employer; otherwise they could be hired for wages below w_2^L. Thus, in equilibrium l-workers will not be mobile. To attract entry-level workers, all active firms must offer the same expected wage, $2w$, to entry-level workers. Finally, $w = w_2^L$ in equilibrium. If $w > w_2^L$, then it would be profitable for firms to train no workers, and hire only second-period untrained workers or workers trained by other firms. On the other hand, $w \geqq w_2^L$ because firms solve:

$$\text{(B)} \qquad \max f(a(H^R+H^B), L_1+L_2) + g((1-a)(H^R+H^B)) - H^R - H^B)w^H$$
$$-2L_1 w - \max[L_2 - L_1 + g((1-a)(H^R+H^B)), 0]w_2^L,$$

which is (A) after the constraint $2w = w_1^L + pw^H + (1-p)\min[L_2/(1-p)L_1, 1]w_2^L$ has been substituted. From (B), differentiation with respect to L_1 and L_2 imply that

$$f_2 \leqq 2w - w_2^L \quad \text{and} \quad f_2 \geqq w_2^L, \tag{4}$$

where the inequalities are not necessarily equalities due to the kink in the last term of (B). (4) implies that $w \geqq w_2^L$ so, by our earlier observation $w = w_2^L$ in equilibrium.

Turning to firm behavior in equilibrium, observe that firms will retain all of their untrained workers, that is, for each firm i,

$$L_2(i) = L_1(i) + g((1-a(i))(H^R(i) + H^B i))).$$

This follows because the partial derivative of the objective function in (B) with respect to L_2 is strictly positive (equal to f_2) whenever

$$L_2(i) < L_1(i) + g((1-a(i))(H^R(i) + H^B(i)).$$

Also, it is never necessary for a firm to both breed and raid h-workers in equilibrium. To see this note that H^R and H^B enter the objective function in (A) only through the sum $H = H^R + H^B$. Therefore, a firm that sets $H^R > 0$ will do no worse by letting $H^R = H$ and $H^B = 0$. This change might improve profits because it weakens the $g((1-a)(H^R + H^B)) \geqq H^B$ constraint. Finally, free entry guarantees that all firms make zero profits in equilibrium.

The remarks made above allow us to restrict attention to an equilibrium characterized by two types of firm, raiders who train no workers, and breeders who produce all of the h-workers that they need. These firms offer wages w^H to h-workers, w to second-period l-workers, and an entry-level wage that brings the average wage to w. Therefore, since the probability of receiving training is zero at raiding firms, all workers at these firms earn w in both periods, but the breeding firms pay w^L such that

$$2w = w^L + pw^H + (1-p)w, \tag{5}$$

where p is the probability of acquiring training. In equilibrium, therefore, firms either solve:

(C) $\quad \max f(H, 2L) - 2wL - w^H H,$

if they are raiders, or

(D) $\quad \max f(aH, 2L - g((1-a)H)) - 2wL + w^H(g((1-a)H) - H),$

subject to

$$L \geqq g((1-a)H) \geqq H,$$

if they are breeders. Problem (D) is obtained from Problem (A) using (5).

It follows that firms face a problem of joint production. They can engage solely in the production of output or can produce skilled labor as well. When skilled labor is essential for production, some firms must train workers. Whether to engage in joint production depends on its profitability relative to production of output alone. In equilibrium, free entry guarantees that both activities earn normal profits.

The next section will construct an equilibrium by analyzing the problems (C) and (D). However, some of the features of equilibrium should be clear. If both breeding firms and raiding firms are active then raiders pay l-workers more than breeders do. This is because entry-level workers at raiding firms give up all opportunity to earn w^H. Breeding firms are willing to train more h-workers than they need because a policy of training leads to the ability to lower entry level wages. In the second period of a worker's life his wage is w if he has not been trained. This is true even for untrained workers at breeding firms because the possibility of gaining training is no longer of value to them. In this equilibrium untrained workers are not mobile, but some trained workers move to raiding firms. Worker's expectations are realized. Notice that it is not necessary for raiding firms to observe the training process or to be able to identify trained workers directly. Instead, the raiders could hire any worker offered w^H by a breeding firm.

3. Main results

In this section the equilibrium of the labor market is characterized. The characterization is done in a series of steps. The idea of the argument is as follows. There is a one-parameter family of wages that yields zero profits for the breeding firms. We parameterize this family by w^H, the wage paid to high-ability workers, and show that profit to the raiding firms is decreasing in w^H. Combined with the observations that breeding firms train more workers than they use themselves whenever w^H exceeds some number $\bar{A}$, and, in certain circumstances, that raider's profits are positive at $w^H=\bar{A}$, we can conclude that there is a unique set of wages that lead to zero profits for both types of firm. A verification shows that these wages form an equilibrium.

Lemma 1. There is an $\underline{A}>0$ such that for each $w^H>\underline{A}$ there exists unique values w, w^L, a, H, and L that satisfy (5) and such that a, H, and L solve (D) given w^H, w, and w^L. That is, given w^H, there is a unique solution to a breeder's profit-maximization problem that yields zero profits.

The proofs of Lemmas 1–5 are in the appendix.

Notice that the parameter values w, w^L, a, H, and L that yield zero profits in the solution to (D) for a given w^H vary continuously with w^H. We will let

$\hat{w}(w^H)$, $\hat{w}^L(w^H)$, $\hat{a}(w^H)$, $\hat{H}(w^H)$, and $\hat{L}(w^H)$ denote the functional relationships. Since the objective function in (D) is non-decreasing in w^H and decreasing in w, it follows that $\hat{w}(w^H)$ is non-increasing; $\hat{w}(w^H)$ will be strictly increasing whenever the $g((1-a)H)\geqq H$ constraint in (D) is not binding.

Lemma 2. There exists an $\bar{A}\geqq \underline{A}$ *such that* $g((1-\hat{a}(w^H))\hat{H}(w^H))=\hat{H}(w^H)$ *if and only if* $w^H\in[\underline{A},\bar{A}]$. *That is, for* w^H *sufficiently high, a breeding firm trains more workers than it uses.*

Lemmas 1 and 2 combine to describe how the wages generating zero profits for a breeder depend on w^H. For relatively low values of w^H, a breeder will train only enough workers to meet its own demands. In these cases, a breeder's profit does not depend on w^H and neither do the other wages and input values. These values, denoted by $\underline{w}$, $\underline{H}$, $\underline{L}$, and $\underline{a}$, are solutions to:

$$\text{(E)} \qquad \max f(aH, 2L\text{-}g((1-a)H))+w^H(g((1-a)H)-H)-2wL,$$

subject to

$$L\geqq g((1-a)H)\geqq H.$$

Therefore, for $w^H\in[\underline{A},\bar{A}]$, $\underline{w}$, $\underline{H}$, $\underline{L}$, and $\underline{a}$ satisfy

$$(1-G'(\underline{H}))f_1(\underline{H}-G(\underline{H}),2\underline{L}-\underline{H})-\underline{w}-\underline{\lambda}=0, \tag{6}$$

$$f_2(\underline{H}-G(\underline{H}),2\underline{L}-\underline{H})-\underline{w}+\underline{\lambda}=0, \tag{7}$$

$$\underline{\lambda}(\underline{L}-\underline{H})=0, \tag{8}$$

$$\underline{a}=(\underline{H}-G(\underline{H}))/\underline{H}, \tag{9}$$

$$f(\underline{H}-G(\underline{H}),2\underline{L}-\underline{H})=\underline{w}(L+\underline{H}), \tag{10}$$

for some $\underline{\lambda}\geqq 0$. Here G is the inverse of g. Conditions (6), (7), and (8) are derived from the Lagrangian expression

$$f(H-G(H),2L-H)-2wL+2\lambda(L-H), \tag{11}$$

in the usual way; (6) and (7) imply that the partial derivatives of (11) with respect to H and L are zero, and (8) is the complementary-slackness condition. Condition (9) defines $\underline{a}$ since $g((1-\underline{a})\underline{H})=\underline{H}$, and (10) is the zero-profit condition. Since $\underline{A}$ was defined to be the lowest wage at which positive profits could be made, $\underline{w}=\underline{A}$.

In order to compute a raider's profit at wages that yield zero profits for the breeder it is necessary to characterize $\bar{A}$, the value of w^H at which the $g((1-a)H) \geqq H$ constraint loosens. At $\bar{A}$, it must be the case that

$$\underline{H}\{f_1(\underline{a}\underline{H}, \underline{L}) + g'((1-\underline{a})\underline{H})[\underline{A} - \bar{A} + \underline{\lambda}]\} = 0, \tag{12}$$

$$\underline{a} f_1(\underline{a}\underline{H}, \underline{L}) - g'((1-\underline{a})\underline{H})(1-\underline{a})[\underline{A} - \bar{A} + \underline{\lambda}] - \bar{A} = 0, \tag{13}$$

the first-order conditions of the unconstrained problem (E), are satisfied. Combining (12) and (13), it follows that $\bar{A}$ is determined from $\underline{A}$, $\underline{H}$, $\underline{L}$, and $\underline{a}$ by

$$f_1(\underline{a}\underline{H}, \underline{L}) - \bar{A} = 0. \tag{14}$$

The next two results verify that a raiding firm's profit depends on input prices in the usual way. This leads to a characterization of equilibrium wages.

Lemma 3. If a raiding firm takes the wages w^H and $\hat{w}(w^H)$ as given, then its profits are strictly decreasing in w^H whenever these profits are positive.

Lemma 4. If a raiding firm can make positive profits when $w^H = \bar{A}$ and $w = \underline{A}$, then there exists a unique $\bar{w}^H$ such that a profit-maximizing raiding firm that takes $\bar{w}^H$ and $\bar{w} = \hat{w}(\bar{w}^H)$ as given operates at positive levels and makes zero profits.

Provided a raiding firm can make positive profits at $w^H = \bar{A}$ and $w = \hat{w}(\bar{A}) = \underline{A}$, Lemmas 1 through 4 characterize a single potential equilibrium given by the wages $\bar{w}^H$, $\bar{w}$, and $\bar{w}^L = \hat{w}^L(\bar{w}^H)$. At these wages, both breeders and raiders operate at positive levels of production and, if they select inputs optimally, they earn zero profits. Moreover, breeders produce a surplus of high-quality workers, so raiding is possible without limiting the availability of trained workers for breeders. Furthermore, no other wages that satisfy (5) have these properties. In order to show that the wages $\bar{w}^H$, $\bar{w}$, and $\bar{w}^L$ actually support an equilibrium, it remains to show that it does not pay a firm to offer workers a different wage contract.

Given the wages $\bar{w}^H$, $\bar{w}$, and $\bar{w}^L$ a firm considering raiding must offer a high wage at least as great as $\bar{w}^H$ to attract high workers, and a wage at least as great as $\bar{w}$ to attract low workers, who could earn $\bar{w}$ by working for another firm. Therefore, since a raider's profits are decreasing in w and w^H, it will never pay a raiding firm to offer wages that differ from $\bar{w}$ and $\bar{w}^H$.

Breeding firms can do no better either. First observe that breeders do not want to raid since the $g((1-a)H) \geqq H$ constraint is not restrictive at $\bar{w}^H$. Also, they must offer h-workers at least $\bar{w}^H$. In addition, offering l-workers wages below $\bar{w}^L$, in exchange for a higher wage w^H if trained, is not effective. By the definition of equilibrium, all workers — whether they work for a defecting firm or not — will expect to earn w^H if trained. Therefore, untrained workers will not be attracted to the defecting firm at wages below $\bar{w}^L$. Thus, since all breeders must offer wages $(w^H, w^L) \geqq (\bar{w}^H, \bar{w}^L)$ to attract workers, these wages support an equilibrium.

It may be more realistic to modify the definition of equilibrium so that a trained worker expects to work for either a raiding firm or the firm that trains him. Under this assumption, it is possible for a breeding firm to attract l-workers by offering a wage $z^L < \bar{w}^L$ in exchange for a high wage $z^H > w^H$ because the only way for a worker to receive z^H is by working for a period at z^L. However, it turns out that even in this situation it does not pay a breeder to offer wages different from $\bar{w}^L$ and $\bar{w}^H$. To see this, suppose that a breeding firm offers h-workers the wage $z^H \geqq \bar{w}^H$. If the firm uses inputs H and L, training parameter a, and offers a low wage z^L and an average wage to untrained workers z, then an entry-level worker expects to earn

$$z = z^L + (1 - g((1-a)H)/L)\bar{w} + (H/L)z^H + ((g((1-a)H) - H)/L)\bar{w}^H, \quad (15)$$

where $1 - g((1-a)H)/L$ is the probability that the worker does not get trained and therefore earns $\bar{w}$ in period two, H/L is the probability that the worker receives a high-level job at the firm he entered, earning z^H, and $(g((1-a)H) - H)/L$ is the probability that the worker is trained, but must find a job elsewhere, earning w^H. Therefore, a breeding firm can attract workers at wages z^L and z^H provided that $z^H \geqq \bar{w}^H$ and $z \geqq \bar{w}$. It follows that a breeding firm considering defection faces the problem, pick z, z^H, z^L, a, H, and L to solve

$$\text{(F)} \qquad \max f(aH, 2L - g((1-a)H)) - z^H H - z^L L - z(L - g((1-a)H)),$$

subject to

$$L \geqq g((1-a)H) \geqq H, \quad z \geqq \bar{w}, \quad z^H \geqq \bar{w}^H,$$

where z is defined in (15). Since the objective function in (F) is decreasing in z^L and z is increasing in z^L, the $z \geqq \bar{w}$ constraint will be binding in the solution to (F). Substituting this constraint into the objective function yields

$$f(aH, 2L - g((1-a)H)) + w^H(g((1-a)H) - H) - 2\bar{w}L,$$

independent of z^H. Thus (F) reduces to the problem that a breeder would face if it took wages as given. It follows from the definitions of $\bar{w}$ and $\bar{w}^H$ that no breeder can make positive profits. Notice that this discussion also shows that breeders will not choose to vary the level of untrained workers that they employ, either by hiring other l-workers or reducing the average wage of the l-workers that they did not train. This follows because profit does not depend on second-period wages. We are now able to conclude that $\bar{w}^L$, $\bar{w}$, and $\bar{w}^H$ determine the unique equilibrium provided that the raider can make positive profits at $w=\hat{w}(\bar{A})=\underline{A}$ and $w^H=\bar{A}$. At equilibrium wages both types of firms make zero profits and keep the workers that they hire and do not train. Moreover, breeding firms produce more trained workers than they need. However, our free-entry condition guarantees that there will be enough raiding firms to absorb the excess supply of h-workers.

If the raiding firm cannot make positive profits at the wages $w^H=\bar{A}$ and $w=\underline{A}$, then at these wages, there is an equilibrium in which no raiding occurs. To see this, notice first that raiding firms are not viable by assumption. Next, at these wages breeding firms train exactly the number of h-workers they need and earn zero profits. The arguments presented earlier imply that breeders do not benefit from offering different wages. This completes the construction of equilibrium. We summarize the results in Proposition 1.

Proposition 1. If raiding firms can make positive profits at wages that induce the breeding firms to train more workers than they need, then there exists an equilibrium in which both breeders and raiders are active. Otherwise, only breeders are active.

The nature of equilibrium can be clarified. We can provide conditions on the technology that guarantee that equilibrium involves active raiders. The next two results will be stated and proved for the case where the $L\geqq g((1-a)H)$ constraint is not binding. The results hold, with only a minor reinterpretation, when the constraint binds.

Lemma 5. A raiding firm can make positive profits at wages $w^H=\bar{A}$, $w=\hat{w}(\bar{A})$, and $w^L=\hat{w}^L(\bar{A})$ if $1-G'(\underline{H})>(\underline{H}-G(\underline{H}))/\underline{H}$.

Lemma 5 can be used to derive meaningful restrictions that imply the existence of raiding. The idea is to define H^* to satisfy

$$1-G'(H^*)=(H^*-G(H^*))/H^*. \tag{16}$$

H^* exists and is unique since $G(0)>0$ and is strictly convex. Lemma 5 implies that equilibrium involves raiding if $\underline{H}<H^*$. To find out when $\underline{H}<H^*$, we add a shift parameter to the technology f and see how $\underline{H}$ and H^* change with changes in the parameter. One condition of this form is given below.

Proposition 2. Consider a family of technologies $f(cH,L)$, indexed by c. There exists a $\bar{c}$ such that for $c>\bar{c}$ raiding firms are active in equilibrium.

Proof. By Lemma 5, it suffices to show that $\underline{H} < H^*$ for c sufficiently large. Since H^* depends only on G, changes in c influence only $\underline{H}$, and it is enough to show that $\lim_{c \to \infty}(\underline{H}(c) - G(\underline{H}(c))) = 0$. To see this, recall that $\underline{H}$, $\underline{L}$, and $\underline{w}$ are defined from the system

$$f(c(\underline{H} - G(\underline{H})), 2\underline{L} - \underline{H}) - 2\underline{w}\underline{L} = 0,$$

$$f_2(c(\underline{H} - G(\underline{H})), 2\underline{L} - \underline{H}) = \underline{w},$$

$$(1 - G'(\underline{H})) f_1(c(\underline{H} - G(\underline{H})), 2\underline{L} - \underline{H}) = \underline{w}. \tag{17}$$

If $\underline{H} - G(\underline{H})$ does not approach zero, then f_1 approaches zero, so that $\underline{w}$ approaches zero. However, in that case the optimizing choice of L would lead to positive profits, contradicting (17).

Proposition 2 says that raiding firms will be active in equilibrium if trained workers are productive enough. The result is sensible. As the marginal productivity of h-workers increases, their equilibrium wage will increase, and increasing w^H makes it profitable for breeding firms to train more workers than they need themselves. A similar proposition — that increasing the differential productivity between h-workers and l-workers tends to make raiding viable — can be established by making the parameter c influence an l-worker's productivity.

It there are no raiding firms in equilibrium, then all firms train exactly as many h-workers as they need. It is possible that this involves training all of the entry-level workers, but this need not be the case. In their second period, trained workers earn w^H. If raiding is profitable, then breeding firms train more workers than they actually need. Some of the trained workers stay with the firm, while others move to other jobs, but all earn w^H. The workers who do not get trained also earn a higher wage. This is because they sacrificed earning potential in the first period in order to have an opportunity to obtain a skill. Since workers live for only two periods, untrained workers will not forfeit wages for a chance to be trained in their second period.

The next result establishes a sense in which equilibrium is efficient. Let H^0 be defined by

$$g'(H^0) = g(H^0)/H^0, \tag{18}$$

so that $H^0 = G(H^*)$, where H^* is defined in (16). When H^0 h-workers are used for training, $g(H)/H$, the number of workers trained per unit of training is maximized. Thus, breeders would use H^0 h-workers for training if the industry equilibrium is efficient. Proposition 3 shows that this is the case.

Proposition 3. If the breeding firms hire more l-workers than they train and there is raiding in equilibrium, then $g'((1-\bar{a})\bar{H}) = g((1-\bar{a})\bar{H})/((1-\bar{a})\bar{H})$.

Proof. The first-order conditions associated with a breeder's profit-maximization problem (E) imply that

$$\bar{w}^H = (\bar{w}^H - \bar{w})g'((1-\bar{a})\bar{H}), \tag{19}$$

$$f_1(\bar{a}\bar{H}, 2\bar{L} - g((1-\bar{a})\bar{H})) = \bar{w}^H, \tag{20}$$

$$f_2(\bar{a}\bar{H}, 2\bar{L} - g((1-\bar{a})\bar{H})) = \bar{w}. \tag{21}$$

On the other hand, both breeders and raiders make zero profits in equilibrium. Moreover, (20) and (21) imply that the optimizing inputs of the raider are $\bar{a}\bar{H}$ h-workers and $2\bar{L} - g((1-\bar{a})\bar{H})$ l-workers. Hence,

$$f(\bar{a}\bar{H}, 2\bar{L} - g((1-\bar{a})\bar{H})) + \bar{w}^H(g((1-\bar{a})\bar{H}) - \bar{H}) - 2\bar{w}\bar{L} = 0, \tag{22}$$

$$f(\bar{a}\bar{H}, 2\bar{L} - g((1-\bar{a})\bar{H})) - \bar{w}^H(\bar{a}\bar{H}) - \bar{w}(2\bar{L} - g((1-\bar{a})\bar{H})) = 0, \tag{23}$$

where (22) and (23) are zero-profit conditions for breeders and raiders, respectively. Combining (22) and (23) yields

$$(1-\bar{a})\bar{H}\bar{w}^H = (\bar{w}^H - \bar{w})g((1-\bar{a})\bar{H}). \tag{24}$$

The proposition follows from (19) and (24).

The intuition behind Proposition 3 is that breeding firms acting alone will pick training levels to satisfy (19). This is because the left-hand side of (19) gives the cost of devoting another h-worker to training, and the right-hand side gives the gains associated with an increase in the probability of training that result from the reduced first-period salary needed to attract l-workers. On the other hand, the ability to raid causes the savings from internal training, $p(w^H - w)$ per worker or $g(w^H - w)$ in total, to increase. The value of $w^H - w$ will be increased until the additional cost of l-workers to raiders balances off the savings available to firms that can use all of their h-workers as productive inputs. Hence, (24) is needed to equalize profit. Proposition 3 need not hold if breeders train all of their entry-level employees; in this case the number of h-workers devoted to training will be no greater than H^0.

Proposition 3 can also be understood in the following way. Because l-workers do not lose productivity when they are being trained, the training process can be thought of as separate 'night school' operated in conjunction with some firms. Only l-workers hired by breeders are allowed to go to these schools; the wage differential $w - w^L$ can be thought of as tuition. By our assumptions on the breeding technology, the efficient number of training schools is independent of the efficient number of firms. When there is room for more firms that just produce output than for training firms, some firms are raiders and training is provided at the efficient level. However, since training can take place only in connection with production, more than the efficient number of workers will be trained when the efficient number of training outlets exceeds the efficient number of firms. In this case raiders are not active in equilibrium.

Proposition 3 can be used to see how the technology influences the distribution of wages. A natural measure of the wage distribution in the industry is the ratio w/w^H, since w is the expected income of a worker and w^H is the wage of an h-worker.

Proposition 4. Consider a family of technologies $f(H, L, c)$ indexed by c. If raiders are active in equilibrium, and breeding firms do not train all of their l-workers, then w^H/w is independent of c.

Proof. Proposition 3, (18) and (19) show that $w^H/(w^H - w) = g'(H^0)$, so that the optimal wage ratio depends only on g.

Thus, changes in f change the distribution of wages only when raiders are not viable or if the breeders train all of the l-workers. Otherwise, wages adjust to keep the level of training at its optimal level. Since this level depends only on g in our model, Proposition 4 follows.

The next result says that the distribution of income becomes more skewed the more costly training is. Since workers must bear the cost of training, non-firm-specific human-capital theory predicts the result.

Proposition 5. Consider a family of training technologies $g(H, c)$ indexed by c. If raiders are active in equilibrium and breeding firms do not train all of their l-workers, then w^H/w is increasing in c if and only if $g_c < 0$.

Proof. Using the Envelope Theorem on (20) and (21), and recalling that $f_2 = w$, yields

$$w_c^H = -(w^H - w)g_c((1-a)H)/(g((1-a)H) - (1-a)H), \tag{25}$$

$$w_c = -aHw_c^H/2L, \tag{26}$$

where subscripts denote derivatives in the usual way. It follows from (26) that w^H/w is increasing if and only if $w_c^H > 0$. The proposition follows from $w^H > w$ and (25).

Thus uniform changes in the training technology makes wages move in opposite directions. Less expensive training makes w^H go down, decreasing the relative wage differential, while more expensive training makes h-workers more valuable, which is reflected in a higher wage differential.

In our model breeders and raiders have access to the same technologies. Does the decision to engage in training have any implications on the optimum firm size?

Proposition 6. If raiding firms are active in equilibrium and breeding firms do not train all of their l-workers, then breeders and raiders have equal outputs. If breeders train all of their l-workers, then a breeder's output exceeds a raider's output if and only if $f_1 f_{12} > f_2 f_{11}$.

Proof. When some l-workers are not trained, the proposition follows directly from the observation that the first-order conditions characterizing the solution to a breeder's problem [(20) and (21)] also characterize the solution to a raider's problem. Therefore, the raider will hire $\bar{a}\bar{H}$ h-workers and have a total of $2\bar{L} - g((1-\bar{a})\bar{H})$ l-workers and the outputs of the two types of firm will be equal.

When breeding firms train all of their l-workers, then the problems of raiders and breeders differ. Formally, if H and L are the direct inputs to production, then

$$f_1(H, L) = \bar{w}^H \quad \text{and} \quad f_2(H, L) = \bar{w} - \lambda,$$

where $\lambda = 0$ for raiders and $\lambda > 0$ for breeders when their l-worker constraint is binding. A straightforward argument shows that increasing λ increases output if and only if $f_1 f_{12} > f_2 f_{11}$, establishing the proposition.

Proposition 6 does not say that the inputs of the two types of firm will be the same. Indeed, since breeders must use some of their h-workers to train l-workers, breeders need to hire more h-workers. If all of the l-workers hired by the breeder are trained, then Proposition 6 suggests that raiders will have smaller outputs. Because raiders are not constrained to hire l-workers to train them, a binding $L \geqq g((1-a)H)$ constraint makes the shadow wage of l-workers lower for breeders than for raiders. This will always cause breeders to use more l -workers than raiders. It will cause the output of breeders to exceed that of raiders whenever $f_1 f_{12} > f_2 f_{11}$; in particular, whenever $f_{12} > 0$ so that lowering the wage of one type of workers increases the demand for the other type.

In our model, there are no restrictions imposed on wages. If all wages had to be at least M, then equilibrium would not exist whenever M exceeded $\bar{w}^L$, the wage that breeders pay to untrained workers. Potential breeding firms would have no incentive to train workers because they would be unable to recover the cost of training. Lowering the wages of trained workers will not be sustainable since raiding firms will appropriate the trained workers. This result can be interpreted in several ways. First, it says that training programs or subsidies to firms engaging in training programs may be needed in conjunction with minimum-wage legislation in order to guarantee adequate supplies of high-quality labor. Alternatively, firms can be induced to train their own workers if there is legislation that prevents raiding. Second, in a model in which skill levels are determined endogenously, equilibrium might still exist in the model, but at lower equilibrium levels of training. Although the adverse effects of minimum-wage legislation on training have been demonstrated in empirical literature [for example, Fleischer (1981), Tauchen (1981), and Leighton and Mincer (1981)], their theoretical foundations are less well established.

4. Conclusions and extensions

This paper presents a model that describes how skills are developed within a firm. In order to obtain essential inputs, some firms must use resources to train workers. Since the skills acquired are not firm specific, workers are willing to pay for the training by taking lower wages in training period in exchange for the prospect of a greater earning potential in the future. Even with identical technologies, industry equilibrium may involve asymmetric behavior. If there is a fixed cost associated with training, then there are situations in which some firms engage in training while others do not.

Several aspects of the model could be extended without trouble. We could assume that breeding firms train all of their new workers and promote only those who have shown some degree of proficiency. Our formulation is conceptually equivalent to this, but the implementation is slightly different. For example, we could think of a labor force composed of workers who are ex ante identical (to themselves as well as to firms) but differ in an unobservable potential β such that the greater the β, the fewer efficiency units of training needed to acquire the high-ability skills. If $g(aH, L)$ gives the number of efficiency units per worker spent on training, then we can say that all workers with $\beta > g(aH, L)$ will successfully complete the training. Letting $K(\beta)$ be the fraction of ability levels less than or equal to β, it follows that the probability of successfully completing the training is $1 - K(g(aH, L))$. Our analysis applies directly to this formulation. However, in this setting, the time spent training low-ability workers is wasted, leading to a loss in welfare not present in the earlier formulation. If workers have better information about β than firms, then firms could implement contracts [see Salop and Salop (1976) or Guasch and Weiss (1982)] that sort workers into ability classes in order to reduce training costs.

As we have presented our model, the skill levels of the H's and L's are fixed exogenously. The model can be extended to allow firms to choose the optimal skill level. Normalize the skill level of l-workers and let r denote the skill level of the h-workers trained. The production function could be written $f(arH, L)$. In equilibrium, the skill level selected by the firm is the same as that of the current h-workers. Thus, wages and skills are determined endogenously. In this framework, it would be possible to examine the distribution of skill levels and the associated income distributions, and to measure the skewness of income distributions as a function of equilibrium skill levels.

In our formulation a worker's effort level is fixed. If workers could control their level of effort and effort is imperfectly observed by firms, then firms would need to construct incentive schemes that induce appropriate effort levels. Then it may be possible to combine the recent results of Lazear and Rosen (1981) and Nalebuff and Stiglitz (1982) with our approach to analyze the implications of effort supervision and training on the internal structure of the firm and income distribution.

Appendix

Proof of Lemma 1. Given w^H, our assumptions on $f(\cdot)$ and $g(\cdot)$ guarantee that (D) has a unique solution for each value of $w \in (0, w^H]$. Let $\underline{A}$ be defined so that the solution to

$$\max f(aH, 2L-g((1-a)H)) - \underline{A}\underline{H} - \underline{A}(2L-g((1-a)H)),$$

subject to

$$L \geqq g((1-a)H) \geqq H, \tag{A.1}$$

yields zero profits. If $w^H = w = \underline{A}$ in (D), then (A.1) results. Since the objective function in (A.1) is decreasing in w for every a, H, and L and since positive profits are available to a breeder for any w^H when $w=0$, $\underline{A}$ exists because of our assumptions on $f(\cdot)$. It follows from the definition of $\underline{A}$ that if $w^H < \underline{A}$, then a breeder can earn positive profits for any value of $w \in (0, w^H]$ and for $w^H \geqq \underline{A}$ there is a unique value of w such that a breeder can breakeven at its optimal input combination. This completes the proof.

Proof of Lemma 2. Since the objective function in (A.1) is strictly increasing in a, it follows that the constraint $g((1-a)H) \geqq H$ will be binding when $w^H = \underline{A}$. Letting G denote the inverse of g and assuming that the constraint $g((1-a)H) \geqq H$ is binding in (A.1), the objective function of (A.1) becomes

$$f(H - G(H), 2L - H) - 2wL. \tag{A.2}$$

Notice that (A.2) does not depend on w^H. Therefore, for all w^H such that the

$g((1-a)H) \geqq H$ constraint is binding, $\hat{w}(w^H) = \underline{A}$. Let $\bar{A}$ be the largest value of w^H such that the solution to

$$\max f(aH, 2L - g((1-a)H)) + w^H(g((1-a)H) - H) - 2wL, \tag{A.3}$$

subject to

$$L \geqq g((1-a)H) \geqq H,$$

involves $g((1-a)H) = H$. $\bar{A}$ exists because for any a, H, and L such that $L > g((1-a)H) > H$ the objective function in (A.3) is positive for all sufficiently large w^H. Since the objective function in (A.3) is non-decreasing in w^H, the constraint $g((1-a)H) \geqq H$ will be binding for all $w^H \in [\underline{A}, \bar{A}]$.

Proof of Lemma 3. Since a raider's profits are given by $f(H, 2L) - w^H H - 2wL$ and $\hat{w}(w^H)$ is increasing in w^H, an increase in w^H increases both of the input prices and therefore reduces a raider's profit whenever it is operating at a positive level.

Proof of Lemma 4. Uniqueness of $\bar{w}^H$ follows directly from Lemma 3. Existence follows since, by our assumptions on $f(H, L)$, there exists w such that $f(H, L) \leqq w(H + L)$ for all H and L.

Proof of Lemma 5. By hiring $\underline{aH}$ trained workers and employing a total of $2\underline{L} - \underline{H}$ untrained workers, a raiding firm can make $f(\underline{aH}, 2\underline{L} - \underline{H}) - \bar{A}\underline{aH} - \underline{A}(2\underline{L} - \underline{H})$ when $w^H = \bar{A}$ and $w = \hat{w}(\bar{A}) = \underline{A}$. Therefore, since $f(\underline{aH}, 2\underline{L} - \underline{H}) = 2\underline{A}\underline{L}$ by (10), a raiding firm can make positive profits whenever $\underline{A} > \underline{a}\bar{A}$. Conditions (6) and (14) imply that $(1 - G'(\underline{H}))\bar{A} = \underline{A}$. Therefore, the lemma follows from (9).

References

Alchian, A.A. and H. Demsetz, 1972, Production, information costs and economic organization, American Economic Review 62, no. 5, 777–795.

Becker, G.S., 1975, Human capital (Columbia University Press, New York).

Calvo, G.A. and S. Wellisz, 1978, Supervision, loss of control and the optimum size of the firm, Journal of Political Economy 80, no. 5, 943–952.

Calvo, G.A. and S. Wellisz, 1979, Hierarchy, ability and income distribution, Journal of Political Economy 87, no. 5, 991–1010.

Fleischer, B.M., 1981, Minimum wage regulation in retail trade (American Enterprise Institute, Washington, DC).

Guasch, J.L. and A. Weiss, 1982, An equilibrium analysis of wage–productivity gaps, Review of Economic Studies 49, no. 4, 485–498.

Lazear, E. and S. Rosen, 1981, Rank-order tournaments as optimal labor contracts, Journal of Political Economy 89, no. 5, 841–864.

Leighton, L. and J. Mincer, 1981, Effects of minimum wages on human capital formation, in: S. Rottenberg, ed., The economics of legal minimum wages (American Enterprise Institute, Washington, DC).

Lucas, R.E., Jr., 1978, On the size distribution of business firms, Bell Journal of Economics 9, Autumn, 508–523.
Lydall, H.F., 1960, The structure of earnings (Clarendon Press, Oxford).
Mayer, T., 1960, The distribution of ability and earnings, Review of Economics and Statistics 42, 189–195.
Mincer, J., 1971, Schooling, age and earnings (National Bureau of Economic Research, New York).
Mirrlees, J., 1976, The optimal structure of incentives and authority within an organization, Bell Journal of Economics 7, Spring, 105–131.
Nalebuff, B. and J.E. Stiglitz, 1983, Prizes and incentives: Towards a theory of compensation and competition, Bell Journal of Economics 14, Autumn.
Rosen, S., 1972, Learning and experience in the labor market, Journal of Human Resources 7, 326–342.
Rosen, S., 1982, Authority, control and the distribution of earnings, Bell Journal of Economics 13, Autumn, 311–323.
Salop, J.K. and S.C. Salop, 1976, Self-selection and turnover in the labor market, Quarterly Journal of Economics 90, no. 4, 619–628.
Simon, H.A., 1957, The compensation of executives, Sociometry 20, no. 1, 32–35.
Stiglitz, J.E., 1975, Incentives, risk and information: Notes towards a theory of hierarchy, Bell Journal of Economics 6, Autumn, 552–579.
Tauchen, G.E., 1981, Some evidence on cross-sector effects of the minimum wages, Journal of Political Economy 89, no. 3, 529–547.
Williamson, O.E., 1967, Hierarchical control and optimum firm size, Journal of Political Economy 75, no. 2, 123–138.