Cheap Talk

Joel Sobel

UCSD and UAB

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- Cut off rule qualitatively as before.
- Randomization.
- Agents think that they are decisive with sufficient probability.
PROPERTIES

- No information sharing.
- No committees.
WHAT IS MISSING

- No joint production or complementarity.
- No differences of opinion.
NEW TOPIC

- Focus on differences of opinion.
- Minimal need for information exchange.
- Single decision maker

Significant change of direction.
I hope to develop some ideas about communication and then go back and combine them with decision making.
Two agents.

One (Sender) has private information. The other (Receiver) takes action.

Nature picks state $\theta \in [0, 1]$ from prior.

Sender learns $\theta$. Receiver does not.

Sender sends message $m \in M$ to Receiver.

Receiver takes action $y \in [0, 1]$. 
Preferences

\[ U^i(\theta, y), \ i = R, S. \]

Note (important): \( U^i \) does not depend on \( m \).

Talk is cheap.
ASIDE: OTHER POSSIBLE ASSUMPTIONS

- Standard “Spence" signaling: $U^i(\cdot)$ depends on $m$. Normally assume single crossing.
- Verifiable information. $M(\theta)$ set of messages available to $\theta$. ($M(\theta) = \{\theta\}$, truth required. $M(\theta) = M$, cheap talk.)
ASSUMPTIONS

Leading example:
\( U^R(\theta, y) = -(y - \theta)^2 \) and \( U^S(\theta, y) = -(y - \theta - b)^2, \ b > 0. \)
prior uniform.
yi solves: max \( U^i(\theta, y) \)
Note: \( y^S(\theta) > y^R(\theta) \).
(In quadratic example, \( y^S(\theta) = \theta + b \) and \( y^R(\theta) = \theta \).)
Three elements:
\( \sigma(m \mid \theta) \), probability that \( \theta \) sends \( m \).
\( y(m) \) is \( R \)'s response to \( m \).
\( \mu(\theta \mid m) \) is \( R \)'s beliefs about \( \theta \) given \( m \).

Three conditions
\( S \) best responds: \( m \) used means it solves:
\[
\max U^S(\theta, y(m')).
\]
\( R \) best responds: \( y(m) \) solves \( \max EU^R(\theta, m)d\mu(\theta \mid m) \).
\( \mu(\theta \mid m) \) derived from prior and \( \sigma(m \mid \theta) \).
Why does $R$ use a pure strategy?
Concavity.

What about $S$? It turns out we can take $M$ to be finite and $S$’s strategy to be pure.
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What about $S$? It turns out we can take $M$ to be finite and $S$’s strategy to be pure.
There is always a “babbling" (non informative) equilibrium.
Induced action:
\{ y : y = y(m) \text{ and } m \text{ sent with positive probability} \}.
Observation: Only finite number of actions induced in equilibrium.
Intuition: If too many actions, then both $R$ and $S$ can approximate their favorite action.
Unit interval partitioned. Types in each partition element send the same message. $R$ best responds. Incentive constraints determine edges of partition.
PROPERTIES

There is always an equilibrium with 1 action induced. If there is an equilibrium with $N$ actions, then there is one with $N - 1$. The equilibrium with most actions induced is “nicer.” The maximal number of actions induced decreases with $b$. If $\theta = 0$ prefers pooling to separating, then there is a unique equilibrium.