Paternalism:

• You’ve heard me (and others) say that doing problems is really important. There, I said it again.
• Try to make your answers as complete as possible. (Hand waving is for lectures!)
• Clear intuitive statements are better than nothing, but don’t forget the previous point.
• If the question is too hard, make stronger assumptions.
• By all means work together. Please write down answers independently (because I think that you’ll learn something writing the arguments down).
• You may be able to find answers to these questions if you look for them. Please try to solve them yourself first.

1. Give an example of a set $X$ an a negatively transitive and irreflexive binary relation $\succ$ on $X$ such that there exists no $u(\cdot)$ such that $x \succ y$ if and only if $u(x) > u(y)$ for all $x, y \in X$. (This is easy.)

2. Let $X = \Pi_{i=1}^{N} Y$ (that is, $X$ is the cartesian product of $N$ copies of the set $Y$). Assume that there exists a preference relation $\succ$ and a function $u : Y \to \mathbb{R}$ such that $x = (y_1, \ldots, y_N) \succ x' = (y'_1, \ldots, y'_N)$ if and only if $\sum_{i=1}^{N} u(y_i) > \sum_{i=1}^{N} u(y'_i)$. State as many properties as you can that $\succ$ must satisfy. (There are representation theorems that identify necessary and sufficient conditions for the existence of an additively separable representation. The hard direction is showing that if $\succ$ satisfies certain properties, then an additively separable representation exists. I am asking the easy question.)

3. Let $X$ be a convex set. Prove that if a preference relation $\succ$ on $X$ can be represented by a concave function, then $\{y : y \succ x\}$ is a convex subset of $X$.
   Is $\{(x, y) : y \succ x\}$ a convex subset of $X \times X$?

4. Can you give conditions on a preference relationship that guarantee that the utility function that represents it is a concave function? Explain.