1. Write down the extensive form for the following three-player game. Player I moves first. He can move either down or right. If Player I moves right, then Player II moves (knowing Player I’s move) either down or right. If Player II moves right, the game ends and the payoffs are (1, 1, 0). If either of the first two players move down, then Player III moves. Player III is not told which player moved down. Player III can move either left or right. This is where I change things. The payoffs are (3, 0, 3) if Player I moves down and Player III moves left; (0, 3, 0) if Player I moves down and Player III moves right; (0, 3, 0) if Player I moves right, Player II moves down, and Player III moves left; and (3, 0, 3) if Player I moves right, Player II moves down, and Player III moves right.

2. Show that there is no Nash equilibrium of the game in Question 1 in which both Player I and Player II move right with positive probability.

3. Find the Nash equilibria of the game in Question 1.

4. Are there any Nash Equilibria of the game in Question 1 that fail to be subgame perfect? Any that fail to be Bayesian Nash Equilibria? Explain.

5. Consider a bargaining problem with two parties, a buyer and a seller. The seller can make an item at a cost of $c$. The item is worth $V$ to the buyer but nothing to the seller. If the seller doesn’t make the item, the game ends without trade (players obtain zero payoffs). Otherwise, the buyer takes possession of the item and can either pay or a price $p$ or refuse to pay. Assume $V > p > c > 0$ and that the players maximize expected net value.

   (a) Find the equilibria of the game (distinguish, if necessary, between subgame perfect and Nash equilibria).

   (b) Modify the game. Suppose at a cost $c'$ the seller can build the item in such a way that she can make it useless to the buyer (“breakable” item). In the new game, first the seller decides whether to make nothing, make the item as in the previous part, or make the item either breakable or unbreakable. The buyer, not knowing whether the item is breakable or not, decides whether to buy. The seller then decides (if the item is breakable) whether to break it. Suppose that

}\footnote{Observe that if Player I thinks that Player III will go right and Player II thinks that Player III will go left, then it is a best response for both Player I and III to move right. Furthermore, if both Player I and Player III move right, then neither player learns what Player II actually does. This is what makes the example interesting.}
\[ c' - c = \epsilon > 0 \] and that the seller gets payoff \( \epsilon/2 \) from breaking the item when the buyer refuses to pay, but payoff \(-\epsilon/2\) from breaking the item when the buyer agrees to pay. Find the equilibria.

(c) Repeat the previous part. This time assume that the buyer believes that there is positive probability that the seller’s cost of making a breakable item is less than \( c \). (More precisely, in this game, first nature moves and determines the cost of making a breakable item. The seller, knowing the cost, decides what to do. The buyer, not knowing the cost or whether the item is breakable or not, decides whether to pay.)

6. The MWG problems are interesting, but you need not hand them in.