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Let $G$ be given. Let $V = F(G) \cap SIR(G)$. If $G$ satisfies a regularity condition (sufficient: $F(G)$ is 1 dimensional), then for each $v \in V$ and $\varepsilon > 0$ there is a $\delta_0 > 0$ such that the infinitely repeated version of $G$ with discount factor $\delta > \delta_0$ has a subgame perfect equilibrium payoff $v^*$ such that $|v - v^*| < \varepsilon$.

Informally: in an infinitely repeated game between patient players any individually rational and feasible payoff can be a subgame perfect equilibrium payoff.
Assumptions

- Patience
- Solution concept
- Regularity
Normal phase: play to get target.
Punishment phase: min max deviator.
Problem: Punishment may not be “credible” (not an equilibrium in a subgame)
Easy Folk Theorem, II: Nash Threats

Let $V' = \{ v \in F(G) : \text{for each } i \text{ there exists a } NE \text{ of } G \text{ with payoffs } \tilde{v}_i, \text{ such that } v_i > \tilde{v}_i \}$. $V'$ is smaller than $V$ (it may be empty).
Proof

Normal phase: play to get target.
Punishment phase: play worst NE for deviator.
Definition: Strategy $g_i$ is a one-shot deviation of $f_i$ if there exists a history $h'_t$ such that

1. $g^s_i(h_s) = f^s_i(h_s)$ for all $s \neq t$ and for $s = t$ and $h_s \neq h'_t$
2. $g^t_i(h'_t) \neq f^t_i(h'_t)$

That is, $g_i$ assigns a different action than $f_i$ for exactly one history.
Who Cares?

1. Repeated games have huge strategy sets.
2. Testing for equilibrium requires testing whether any non-equilibrium strategy does better than the equilibrium strategy.
3. One-shot deviations are a tractable, small subset of all strategies.
4. We’ll show that in order to test an equilibrium, it suffices to look in the smaller set of one-shot deviations.
The One-Shot Deviation Principle

Given a strategy profile \( f \), the one-shot deviation \( g_i \) of \( f_i \) is profitable at \( h'_t \) if

\[
U_i(g_i(\cdot \mid h'_t), f_{-i}(\cdot \mid h'_t)) > U_i(f(\cdot \mid h'_t)).
\]

Claim: A strategy profile \( f \) is a subgame perfect equilibrium if and only if there are no profitable one-shot deviations.
1. Only if part is obvious.
2. If part has content superficially because not all deviations are one-shot deviations.
3. If part requires proof. (It is not true without discounting and it is not true for Nash equilibrium in place of subgame perfect.)
4. The claim is the key to using dynamic-programming methods in repeated game theory.
Proof in Words

1. Need to show that if there is a profitable deviation, there is a profitable one-shot deviation.
2. If there is a profitable deviation, then there is a profitable deviation that agrees with original strategy for all histories greater than some finite $M$.
3. Working backwards from $M$, find the “first” time that the deviant strategy is better.
Let $\tilde{v}^i$ be the lowest subgame perfect equilibrium payoff to player $i$ in the infinitely repeated discounted version of $G$. You can check to see whether something is a subgame perfect equilibrium payoff by checking whether it is attractive to deviate if punishment is to worse equilibrium. This requires the existence of worst equilibrium for each player. It does not depend on the discount factor being close to one, although in practice it is hard to compute worst payoff.