Econ 200C - April 2, 2012

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INTRODUCTION


Handouts:

1. The outline
2. One problem set.

The Webpage: www.econ.ucsd.edu/ jsobel/200C12:

1. Link on my web page.
2. Will contain announcements, links to handouts, notes, and slides.
I intend to post these slides (for as long as I create them). I urge you to use this as a reason to take fewer notes (instead, think and listen in class).

Replacement classes: Friday April 6 and Friday 27 (9:40) instead of Wednesday April 11 and Wednesday April 25.
OUTLINE

Subject: Continuation of Repeated Games, Incomplete information games and applications.

Unlike 200B Part 1: Cannot summarize class with one model and two basic results.

Big picture: subtle definitions (learn what a strategy is), techniques, and a few basic models.

Paternalism: Work as many problems as you can, carefully, and seriously. Try to create interesting problems.

Requirements: Homework, Midterm

Miller and I will aggregate grades by averaging them.
Warning

I don’t like to use slides.

Advantages:
- It will look like I am prepared.
- You’ll have access to the slides.
- I won’t need to stand for two hours.
- Maybe you get an advance overview of lecture material.

Disadvantages:
- I might not sustain the energy to prepare notes.
- Typos.
- Tendency to go too fast.
Repeated Games

Relevant Reading: Nageeb’s Notes, Chapter 1 Ingredients

1. A game $G$.
2. A discount factor $\delta \in (0, 1)$.
3. $n$ (either and a positive integer or infinity).

Loosely:
players play $G$, observe the outcome, play $G$ again, and so on.
Payoffs are the discounted sum of the payoffs from each “stage.”
Formal Definition

1. The repeated game has the same player set as $G$.
2. Strategies:
   - Let $H_0 = \emptyset$ and $H_t = S^t$ for $t > 0$.
   - The strategy set for player $i$: all sequences of functions $f_i^t : H_{t-1} \rightarrow S_i$.
3. The payoff function for the repeated game given strategy profile $f$ is:

   $$U_i(f) = (1 - \delta^n)^{-1}(1 - \delta) \sum_{t=1}^{n} \delta^{t-1} u_i(f^t(h_{t-1}))$$

   where $f^t = (f_1^t, \ldots, f_I^t)$ and $h_t$ is the sequence of histories induced by $f$ ($h_t = h_{t-1} \times f^t(h_{t-1})$).

   The interpretation is that $f_i^t(h_{t-1})$ is the action that $i$ takes in the $t$th repetition following history $h_{t-1}$. Some literature on “time average” criterion: roughly $\delta = 1$ case (which causes technical problems in infinite horizon games).
1. Strategy Space is HUGE.

2. \((1 - \delta^n)^{-1}(1 - \delta) \sum_{t=1}^{n} \delta^{t-1} = 1\).

3. Hence \((1 - \delta^n)^{-1}(1 - \delta)\) is a normalization factor (designed to make feasible payoff set independent of \(n\)).
Definitions

1. Feasible Set $F(G)$: convex hull of $\{u(s) : s \in S\}$.

2. Minmax: $v_i^j = \min_{\sigma_{-i} \in \Pi_{j \neq i}} \Delta(s_j) \max_{s_i \in S_i} u_i(s_i, \sigma_{-i})$. (Let $v_j^i$ be the payoff that $j$ obtains for strategies that lead to the payoff $v_i^j$ for $i$.)

3. Individually Rational Set of Payoffs:
   $IR(G) = \{ v : v_i \geq v_i^i \text{ for all } i \}.$

The set of strictly individually rational payoffs is:

$SIR(G) = \{ v : v_i > v_i^i \text{ for all } i \}.$

The feasible set consists of all possible payoffs that one could obtain from correlated strategies in the given game.

$v_i^j$ is a lower bound to the payoff that player $i$ can obtain in a Nash equilibrium. (The definition allows $i$ to best respond to the strategy of the other players, but for the other players to pick the strategy that makes $i$ worse off.)
Largest Possible Equilibrium Payoff Set

The set of equilibrium payoffs must be contained in the set of feasible and individually rational payoffs. This is true for $G$ and for any repeated version of $G$ (with proper normalization).
Subgames

Given any history, $h_t$, there is a subgame. The subgame is formally identical to the original game (same player set, same strategy sets, same payoff function). Strategies for the original game induce strategies for the subgame. Given a strategy $f$ for the original game, $f'$ defined by $f'(k_s) = f(h_t, k_s)$ is the strategy induced by $f$. $(h_t, k_s)$ is a $t + s$ period history. Similarly, subgame payoffs are induced. Denote these by $U_i(\cdot \mid h_t)$.