Game Theory C *

Paul J. Feldman, Isla R. Globus-Harris and Erin L. Wolcott

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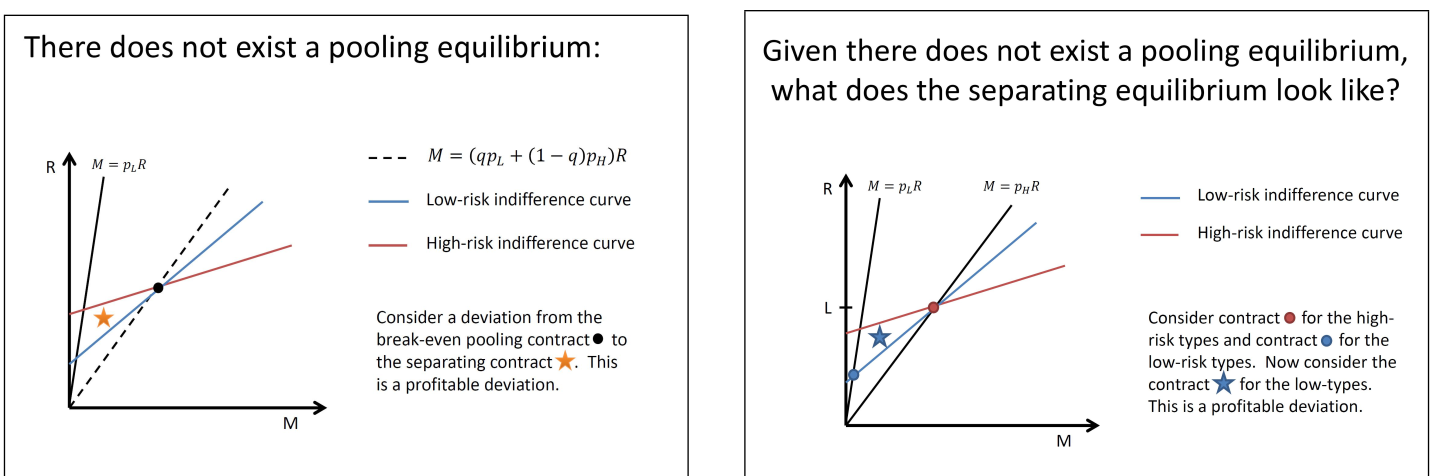
1 Screening Insurance Problem Continued

Recall, what we know, from the problem with full information:

- All contracts in equilibrium guarantee full insurance for agents.
- All firms make zero profits if there are at least two of them.

In order to identify the set of Perfect Bayesian Equilibria (PBE) we argue for a specific separating equilibrium. We will proceed as before by eliminating strategies that would not constitute an equilibrium, recall pooling equilibria have already been eliminated. First note that from our previous result, positive profits are unsustainable in any equilibria. Before we continue it is important to note that screening implies that the uninformed part of the market will use contracts to elicit information about agent types from the informed party, this commonly comes at a cost which normally falls on the good/low risk types. At the end, we will also argue why this might not be a feasible equilibrium.

Figure 1: Types of Equilibria



(a) Non-existence of Pooling Equilibria

(b) Separating Equilibria: Part I

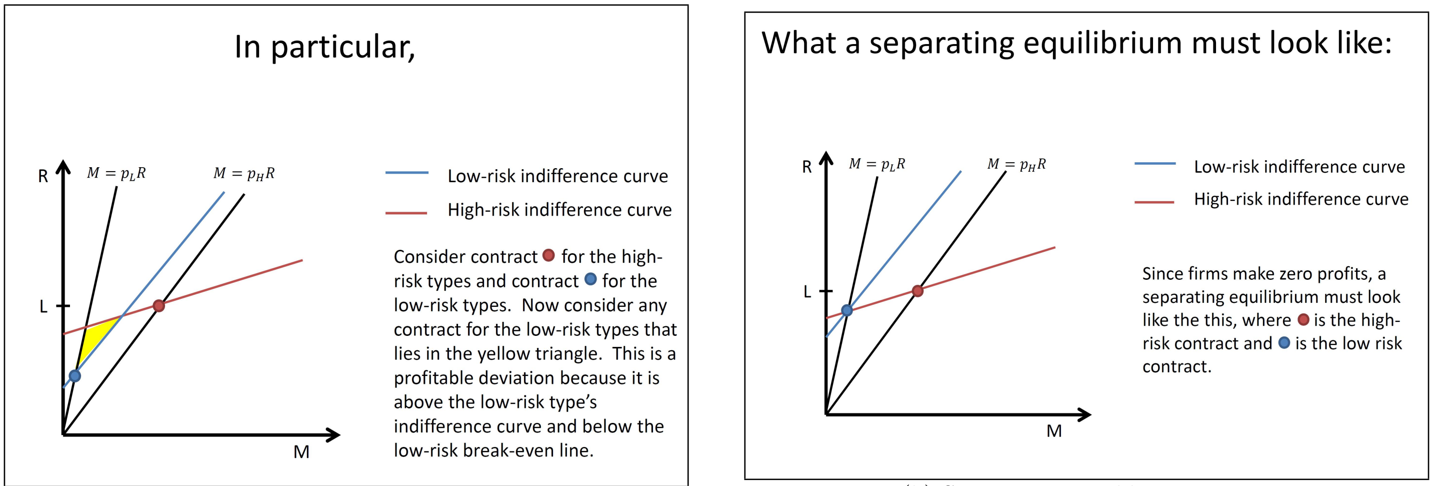
1.1 Separating Equilibria

This case is illustrated in the Figure 1(a). Since $P_l < P_h$, the probability of each type of being in an accident, we have that the low risk types indifference curves are steeper than the high risk types. That is, for the low risk types, changes in the cost of the insurance contract M will necessitate a greater change in the insurance payoff R, under the bad state, to keep them indifferent. This is because we have assumed agents are risk adverse. We are now ready to construct the set of separating equilibria by the following observations:

1. Since profits are zero, firms must offer actuarially fair contracts. Note that these contracts are a parametric family of the form $\{p_i R = M\}$.

*Course taught by Joel Sobel Dept. of Economics, Univ. of California, San Diego, 9500 Gilman Dr., La Jolla, CA 92093

Figure 2: Separating Equilibria



(a) Separating Equilibria: Part II

(b) Separating Equilibria: Part III

2. From Figure 1(a) we ruled out the case where a firm offers $\{M, R\} = \{[(q)p_l + (1-q)p_h]R, R\}$. This is because low risk agents could be offered a more enticing contract, that would yield positive profits for a firm.

3. Firms offer full insurance to the riskier types, i.e. $\{M, R\} = \{p_h L, L\}$. The reason is as follows: suppose that one firm offers contract $\{M, R\}$ such that $R \neq L$ and $M = p_h R$ and high risk agents buy the contract. If follows that the high risk types would strictly prefer $\{M', R'\}$ where $\{M', R'\} = .5\{M, R\} + .5\{p_h L, L\}$ because $\{M', R'\}$ leads to less variation in final wealth than $\{M, R\}$ and is still actuarially fair. Hence high risk types will buy $\{M' + \epsilon, R'\}$ for sufficiently small epsilon and this contract will make positive profits (low risk types might buy this contract too, but that will only increase the deviant's profits further).

4. Note that if a firm offers a contract to the low risk types that is actuarially fair to these types and is below the riskier type's indifference curve then the other firm can deviate and offer a profitable contract to the low risk types. If the contract is higher than the riskier type's indifference curves then it will attract them and the firm will make a loss. This is illustrated in Figure 1(b)and 2(a).

5. Therefore, at the separating equilibrium both firms offer two contracts $\{M, R\} = \{L, p_h L\}$ and for the low risk types they offer an actuarial fair contract, below full insurance, that low types consider indifferent to $\{M, R\} = \{p_h L, L\}$. This case is represented in Figure 2(b). Note that in this equilibrium profits are zero.

6. Since high risk types are risk adverse it is individually rational for them to accept either contract. High types strictly prefer the less than full insurance contract since their indifference curves are steeper. Thus, at this equilibrium high risk types accept the full insurance contract and the low risk types accept the less than full insurance contract.

Note that we have described how an equilibrium must be, but we have not shown that it exists. In fact, it may not always be the case that it exists. As illustrated in Figure 3, in some instances there is a profitable deviation to a pooling contract. If such a deviation exists, then there will be no separating equilibrium. However, if the pooling deviation is not possible, then a separating equilibrium exists and is as described above.

Finally, what's the difference between the signaling and the screening versions of this problem?

- Signaling has many more equilibria: it has many pooling equilibria and many separating equilibria, and many of the separating equilibria require an inefficient amount of effort/sacrifice to separate.

- The only possible equilibrium in a screening model corresponds to the pareto efficient separating equilibrium from signaling models (the point is that signaling and screening models do focus on the same outcome).

- However, there is a sense in which they are related. Asymmetric information leads to the good types, the less risky types in this example, getting the short end of the stick. In this problem they got less than full insurance. In the signaling game they had to invest resources and get educated.

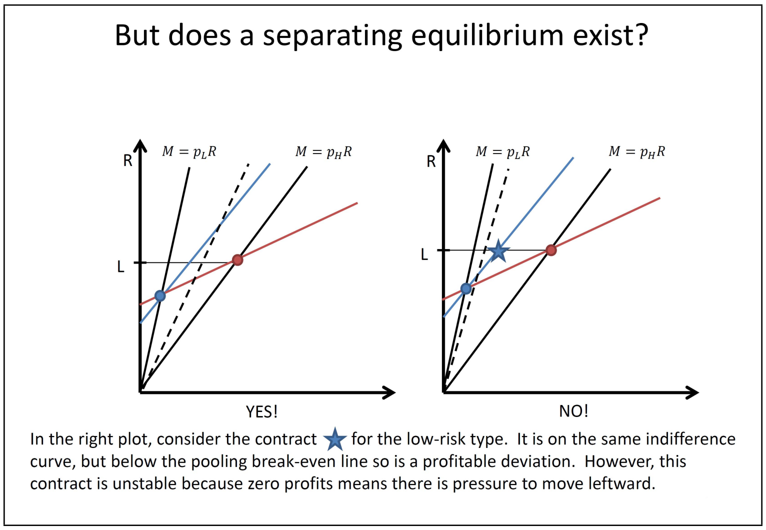


Figure 3: Separating Equilibria: Part III

2 Agency

2.1 Labor Market Problem

This is a game between an agent and a manager. The manager offers a contract to the agent, then the agent chooses how much effort to put in. The manager cannot directly observe the effort level, but can observe output. Output is stochastic and is correlated with effort level.

One interpretation of this problem is a land owner (the manager) and a farmer (the agent) who is paid to work the land. The manager owns the land and wants to maximize profits. The output depends on both the farmer's effort, as well as stochastic factors such as soil conditions.

Specifically:
The agent chooses an effort level: $E = \{e_1, e_2, e_3\}$.
There are two levels of potential profit: $\pi_H = 10$ and $\pi_L = 0$.
Profits are affected by effort levels as follows:
$f(\pi_H|e_1) = \frac{2}{3}$, $f(\pi_H|e_2) = \frac{1}{2}$, $f(\pi_H|e_3) = \frac{1}{3}$
$f(\pi_L|e_1) = \frac{1}{3}$, $f(\pi_L|e_2) = \frac{1}{2}$, $f(\pi_L|e_3) = \frac{2}{3}$
The costs of the effort levels are:
$g(e_1) = \frac{5}{3}$, $g(e_2) = \frac{8}{5}$, $g(e_3) = \frac{4}{3}$
The agent is risk averse and has Bernoulli utility function $u(w) = \sqrt{w}$.
If the agent rejects all contracts, he gets $\bar{u} = 0$.

Note that there's a trade off between incentives and insurance. The worker wants to smooth consumption (due to risk aversion), but perfectly smooth consumption may not provide the proper incentives to induce the manager's desired effort level.

The manager can pick two wages, w_H and w_L.
Manager gets: $\pi_H f(\pi_H|e^*) + \pi_L f(\pi_L|e^*) - w_H f(\pi_H|e^*) - w_L f(\pi_L|e^*)$
where e^* solves agent's problem:

$$\begin{aligned} &\text{Max}_{\{e\}} u(w_H) f(\pi_H|e) + u(w_L) f(\pi_L|e) - g(e) \\ \text{subject to} \quad &u(w_H) f(\pi_H|e^*) + u(w_L) f(\pi_L|e^*) - g(e^*) \geq \bar{u}. \end{aligned}$$

2.2 Insurance Problem

Reformulation of the insurance problem with variable effort levels:
We can reformulate our previous insurance problem as a problem where outcome (probability of an accident) is correlated with the insured's effort. For instance, the insured may need to expend effort (such as paying close attention to the road) to decrease the chance of a car wreck.
Note that we only have one type of agent in this model, and the risk profile is affected by effort, not type.

There are various effort levels, $E = \{e_1, e_2, ..., e_k\}$.
The agent picks an effort level.
The possible outcomes are $\Pi = \{\pi_1, \pi_2, ..., \pi_n\}$.
Effort affects the probability of getting into an accident: $f(\pi_i | e_j)$.
The insurer offers (M, R) as before.
Questions:

- What do preference constraints look like?
- How do we interpret f?

For the following, assume there are two possible outcomes, π_A (accident) and π_N (no accident). Note that $f(\pi_A|e) + f(\pi_N|e) = 1$.
The agent picks e to solve: $max_{\{e\}} f(\pi_A|e)u(W - M - L + R) + f(\pi_N|e)u(W - M) - g(e)$.
Or better: $max_{\{e\}} f(\pi_A|e)u(W - M - L + R - g(e)) + f(\pi_N|e)u(W - M - g(e))$.
In insurance setting: one can continue to write the objective function for the insurer and the choice problem of the insured. Please refer to the notes that Prof. Sobel distributed on this subject.