

# Dating Business Cycle Turning Points\*

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## Abstract

This paper discusses formal quantitative algorithms that can be used to identify business cycle turning points. An intuitive, graphical derivation of these algorithms is presented along with a description of how they can be implemented making very minimal distributional assumptions. We also provide the intuition and detailed description of these algorithms for both simple parametric univariate inference as well as latent-variable multiple-indicator inference using a state-space Markov-switching approach.

We illustrate the promise of this approach by reconstructing the inferences that would have been generated if parameters had to be estimated and inferences drawn based on data as they were originally released at each historical date. Our recommendation is that one should wait until one extra quarter of GDP growth is reported or one extra month of the monthly indicators released before making a call of a business cycle turning point. We introduce two new measures for dating business cycle turning points, which we call the “quarterly real-time GDP-based recession probability index” and the “monthly real-time multiple-indicator recession probability index” that incorporate these principles. Both indexes perform quite well in simulation with real-time data bases. We also discuss some of the potential complicating factors one might want to consider for such an analysis, such as the reduced volatility of output growth rates since 1984 and the changing cyclical behavior of employment. Although such refinements can improve the inference, we nevertheless recommend the simpler specifications which perform very well historically and may be more robust for recognizing future business cycle turning points of unknown character.

JEL classification: E32

# 1 Introduction

The National Bureau of Economic Research (NBER) is a private research organization that, among other activities, identifies dates at which the U.S. would be said to be experiencing an economic recession. These dates, reported at <http://www.nber.org/cycles/cyclesmain.html>, are regarded as authoritative by both academic researchers and the public at large.

For example, in July, 2003, the NBER announced that the most recent recession had finally ended. Remarkably, what the NBER announced in July, 2003 was that the recession had actually ended in November, 2001. There had been a similar two-year delay in the previous recession, for which the NBER announced in December, 1992 that the recession had ended in March, 1991.

These quasi-official dates are the outcome of discussions of the NBER's Business Cycle Dating Committee, a group of highly respected academics who review a variety of economic indicators to form a qualitative judgment about the state of the economy. The delays are explained by the fact that the Committee wants to be quite confident about its assessment before making a public declaration. There is nevertheless a cost to this accuracy, in that many members of the public can continue to believe that the economy is in a recession long after a solid recovery is under way. For example, in the 1992 election, the opposition party declared that the U.S. was experiencing the worst economic downturn since the Great Depression. A look at most of the facts would lead one to dismiss this claim as political hyperbole. However, if it had been the case that the recession beginning in July 1990 was still persisting as of November 1992, as one might have legitimately inferred from the failure of the NBER to announce the recession as over, it indeed would have qualified as the longest economic downturn since the Depression. More recently, the widespread belief by the American public that the U.S. was still in recession in 2003 may have played a role in tax cuts approved by the U.S. Congress, the outcome of a special election for the governor of California, and a host of other policy and planning decisions by government bodies, private firms, and individual households.

During the last decade, academic researchers have come to treat the question of whether the economy is experiencing a recession as a formal statistical issue rather than a subjective qualitative assessment. This approach started with Hamilton (1989) and has since been adopted in hundreds of academic studies.<sup>1</sup> Given the importance to the public at large of identifying where the economy is at any given point in time, it seems worthwhile to investigate whether these formal quantitative methods could be used to produce announcements that might be useful to the public in real time. The purpose of this chapter is to review the performance of several such methods. We begin in Section 2 with a background discussion of this approach in a very simple application that uses only data on U.S. real GDP growth and minimal distributional assumptions. In Section 3 we implement a parametric version of this approach to GDP data. Section 4 describes a method for combining the inference from a number of different economic indicators.<sup>2</sup> Section 5 presents results from such multivariate inference, while Section 6 explores the robustness of these multivariate inferences to several alternative specifications.<sup>3</sup>

## 2 What can we infer from U.S. GDP growth rates?

Figure 1 plots quarterly growth rates (quoted at an annual rate) of U.S. real GDP since 1947, with dates of economic recessions as determined by the NBER indicated with shaded regions. Consider what we can say from this GDP data alone about the broad properties of NBER’s classifications. Forty-five of the 229 quarters between 1947:II and 2004:II were classified as “recession” and the remaining 184 as “expansion.” First consider the 45 recession quarters as representatives of a certain population, namely, what GDP growth looks like when the economy is in recession. The average quarterly growth rate in recession is -1.23% (expressed at an annual rate), with a standard

<sup>1</sup> For some alternatives see Lundbergh and Terasvirta (2002), van Dijk, Terasvirta and Franses (2002), Harding and Pagan (2002) and Artis, Marcelino and Proietti (2004).

<sup>2</sup> More specifically, we use a dynamic factor model with regime switching, as in Chauvet (1998), which is a nonlinear state space model. This class of models is very popular in several fields. Some of the important work in this area includes Gordon and Smith (1990), Carlin, Polson, and Stoffer (1992), Kitagawa (1987), Fridman and Harris (1998), Kim and Nelson (1999a), Durbin and Koopman (1997), among others.

<sup>3</sup> A companion paper by Chauvet and Piger (2005) compares the results from the method described in Section 4 with mechanical business cycle dating rules proposed by Harding and Pagan (2002).

deviation of 3.55. The top panel of Figure 2 plots a nonparametric kernel estimate of the density of these 45 quarters.<sup>4</sup> One is more likely to see GDP falling than rising during a recession, but this is by no means certain; in fact, 15 of the 45 recession quarters are associated with positive GDP growth.

[ insert Figure 1 about here ]

The bottom panel of Figure 2 plots the corresponding density for the 184 postwar quarters classified as economic expansion. These are characterized by a mean annualized growth rate of 4.49% with a standard deviation of 3.24. This distribution is overwhelmingly dominated by positive growth rates, though there again is some small probability of observing a negative growth rate during what is considered to be an economic expansion.

[ insert Figure 2 about here ]

If one simply selects a postwar quarterly growth rate at random, there's a 20% probability it would be one of the 45 quarters classified as a recession and an 80% probability of falling in an expansion. The unconditional distribution of GDP growth rates can be viewed as a mixture of the two distributions in Figure 2. This mixture is represented in the top panel of Figure 3, in which the height of the long-dashed line is found by multiplying the height of the top panel of Figure 2 by 0.2. The short-dashed line represents 0.8 times the bottom curve of Figure 2. The sum of these two curves (the solid line in the top panel of Figure 3) represents the unconditional density of one quarter's growth rate without knowing whether or not the quarter would be classified as recession.

[ insert Figure 3 about here ]

From the top panel of Figure 3, one could make an intelligent prediction as to what classification NBER will eventually arrive at (expansion or recession) as soon as the GDP figures are released. If GDP falls by more than 6%, most of the height of the solid line is coming from the long-dashed

<sup>4</sup> This was calculated using the "density" command in RATS with a Gaussian kernel and bandwidth set equal to 3.

density, suggesting that it is overwhelmingly likely that the quarter will be classified as recession. If GDP rises by more than 6%, almost none of the density comes from the short-dashed line, leading us to expect NBER to classify that quarter as expansion. Intuitively, we might use the ratio of the height of the long-dashed line to the height of the solid line as a measure of the likelihood that NBER would classify a quarter with GDP growth of an amount specified on the horizontal axis as being part of a recession. This ratio is plotted in the bottom panel of Figure 3.

Using this ratio in this way is more than intuitively appealing. It turns out to be precisely an application of Bayes Law for this setting. Specifically, let  $S_t = 1$  if the NBER ends up classifying quarter  $t$  as an expansion and  $S_t = 2$  if recession. Let  $y_t$  denote the quarter  $t$  GDP growth rate. Then  $f(y_t|S_t = 2)$  is the density of GDP growth rates in recession, a nonparametric estimate of which is given by the top panel of Figure 2, while the expansion density  $f(y_t|S_t = 1)$  corresponds to the bottom panel. Let  $\Pr(S_t = 2) = 0.20$  be the probability that any given quarter is classified as recession. Bayes Law states that the probability that NBER will declare a recession given that the GDP growth for the quarter is known to be  $y_t$  can be calculated from

$$\Pr(S_t = 2|y_t) = \frac{f(y_t|S_t = 2) \Pr(S_t = 2)}{f(y_t|S_t = 1) \Pr(S_t = 1) + f(y_t|S_t = 2) \Pr(S_t = 2)}. \quad (1)$$

But  $f(y_t|S_t = 2) \Pr(S_t = 2)$  is simply the height of the long-dashed line in Figure 3, while  $f(y_t|S_t = 1) \Pr(S_t = 1)$  is the height of the short-dashed line. Hence the ratio plotted in the bottom panel of Figure 3,

$$\Pr(S_t = 2|y_t) = \frac{0.2 f(y_t|S_t = 2)}{0.8 f(y_t|S_t = 1) + 0.2 f(y_t|S_t = 2)},$$

is indeed the optimal prediction  $\Pr(S_t = 2|y_t)$  about what NBER will declare if the quarter's GDP growth is  $y_t$ .

Predicting NBER's declaration if we get growth rates as extreme as  $\pm 6\%$  is obviously quite robust and sensible. Unfortunately, it is not particularly useful, since the vast majority of GDP growth rates are not this extreme, and for typical data the prediction about what NBER will declare in the bottom panel of Figure 3 is not very precise. Fortunately, there is another piece

of information about the NBER's classifications that can be extremely helpful here, which is the fact that the Committee usually makes the same declaration in  $t + 1$  that it made in  $t$ . Of the 45 quarters characterized as recession, 35 or 78% were followed by another quarter of recession. Of the 183 expansion quarters between 1947:II and 2004:I, 173 or 95% were followed by another quarter of expansion.

Suppose we observe a particular GDP growth rate for quarter  $t$  of  $y_t$ ; perhaps this is a value like  $y_t = -6$ , which we are reasonably confident will be described as a recession. Given this information, the probability that next quarter  $t + 1$  will also be classified as a recession is no longer 0.20 but is much higher. Specifically,

$$\begin{aligned} \Pr(S_{t+1} = 2|y_t) &= \Pr(S_{t+1} = 2|S_t = 2, y_t) \Pr(S_t = 2|y_t) + \Pr(S_{t+1} = 2|S_t = 1, y_t) \Pr(S_t = 1|y_t) \\ &= 0.78 \Pr(S_t = 2|y_t) + (1 - 0.95) \Pr(S_t = 1|y_t) \end{aligned}$$

where we've assumed that  $\Pr(S_{t+1} = 2|S_t = 2, y_t) = \Pr(S_{t+1} = 2|S_t = 2) = 0.78$ . For example, if there was convincing evidence of a recession in period  $t$  (say,  $\Pr(S_t = 2|y_t) = 0.9$ ), then the probability that we will still be in recession in  $t + 1$  would be  $(0.78)(0.9) + (1 - 0.95)(1 - 0.9) = 0.71$ . If we then learn the quarter  $t + 1$  growth rate  $y_{t+1}$  as well, the inference about  $S_{t+1}$  is found not from the height of the bottom panel of Figure 3, but instead from a mixture whose recession probability is 0.71 rather than 0.20, that is, equation (1) would be replaced with

$$\begin{aligned} \Pr(S_{t+1} = 2|y_{t+1}, y_t) &= \frac{f(y_{t+1}|S_{t+1} = 2, y_t) \Pr(S_{t+1} = 2|y_t)}{\sum_{j=1}^2 f(y_{t+1}|S_{t+1} = j, y_t) \Pr(S_{t+1} = j|y_t)} \\ &= \frac{0.71 f(y_{t+1}|S_{t+1} = 2, y_t)}{0.29 f(y_{t+1}|S_{t+1} = 1, y_t) + 0.71 f(y_{t+1}|S_{t+1} = 2, y_t)}. \end{aligned} \quad (2)$$

If we assume that recessions are the only source of GDP dynamics, so that  $f(y_{t+1}|s_{t+1}, y_t) = f(y_{t+1}|s_{t+1})$ , we could again use the height of the top panel of Figure 2 at the given value of  $y_{t+1}$  as our estimate of  $f(y_{t+1}|S_{t+1} = 2, y_t)$ , in which case we just replace the mixture in the top panel of Figure 3 (which assumed a 20% weight on the recession density and 80% on the expansion density), with a mixture that puts 71% weight on the recession density and 29% on the expansion

density, as in the top panel of Figure 4. The ratio of the height of the long-dashed curve to the solid curve in the top panel of Figure 4 gives the inference (2), plotted in the bottom panel of Figure 4. If we were reasonably confident that quarter  $t$  was a recession, we are much more prone to call  $t + 1$  a recession as well.

[ insert Figure 4 about here ]

Another perspective on this form of inference is obtained as follows. Suppose that GDP growth for quarter  $t$  is given by  $y_t = y$ , from which we calculate  $\Pr(S_t = 2|y_t = y)$  as in the bottom panel of Figure 3. We can then use this magnitude  $\Pr(S_t = 2|y_t = y)$  in place of the constant 0.20 to weight the recession distribution. The ratio of the heights of the recession curve to the combined distribution would then correspond to  $\Pr(S_{t+1} = 2|y_{t+1} = y, y_t = y)$ , that is, it is the probability of recession if we happened to observe GDP growth equal to  $y$  for two quarters in a row. This quantity is plotted in the bottom panel of Figure 5, which is substantially steeper than the plot of  $\Pr(S_{t+1} = 2|y_{t+1} = y)$  shown in the top panel. For example, if we had only a single quarter's observation of GDP, we would not have 50% confidence in predicting a recession unless GDP growth was below  $-3.4\%$ . By contrast, two consecutive quarters GDP growth of  $-1.8\%$  would also give us 50% confidence that the economy had entered a recession.

[ insert Figure 5 about here ]

We could use the same principle to get a better picture of whether the economy was in a recession in quarter  $t$  once we know the economic growth rate in quarter  $t + 1$ . Specifically, we first make a prediction about both  $S_t$  and  $S_{t+1}$  based on  $y_t$  alone,

$$\Pr(S_{t+1} = j, S_t = i|y_t) = \Pr(S_{t+1} = j|S_t = i, y_t) \Pr(S_t = i|y_t).$$



This magnitude can be calculated by multiplying  $\Pr(S_t = i|y_t)$  by the appropriate constant:

$$\Pr(S_{t+1} = j|S_t = i, y_t) = \begin{cases} 0.95 & \text{if } i = 1, j = 1 \\ 0.05 & \text{if } i = 1, j = 2 \\ 0.22 & \text{if } i = 2, j = 1 \\ 0.78 & \text{if } i = 2, j = 2 \end{cases}.$$

We then use Bayes Law to update this joint inference based on observation of  $y_{t+1}$ :

$$\begin{aligned} & \Pr(S_{t+1} = j, S_t = i|y_{t+1}, y_t) \\ = & \frac{\Pr(S_{t+1} = j, S_t = i|y_t)f(y_{t+1}|S_{t+1} = j, S_t = i, y_t)}{\sum_{i=1}^2 \sum_{j=1}^2 \Pr(S_{t+1} = j, S_t = i|y_t)f(y_{t+1}|S_{t+1} = j, S_t = i, y_t)}. \end{aligned} \quad (3)$$

We can again estimate  $f(y_{t+1}|S_{t+1} = j, S_t = i, y_t)$  by  $f(y_{t+1}|S_{t+1} = j)$ , that is, by the top panel of Figure 2 when  $j = 2$  and the bottom panel when  $j = 1$ . The desired inference about the economy at date  $t$  based on information observed at date  $t + 1$  is then

$$\Pr(S_t = i|y_{t+1}, y_t) = \sum_{j=1}^2 \Pr(S_{t+1} = j, S_t = i|y_{t+1}, y_t). \quad (4)$$

We have thus seen how, given nonparametric knowledge of how the distribution of GDP growth is different between expansions and contractions,

$$f(y_t|S_t = i) \quad \text{for } i = 1, 2,$$

of how frequently the economy stays in the same regime,

$$\Pr(S_{t+1} = j|S_t = i) \quad \text{for } i, j = 1, 2,$$

and the approximation that the state of the economy (recession or expansion) is the only probabilistic link between one quarter and the next,<sup>5</sup>

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$$\Pr(S_{t+1} = j|S_t = i) = \Pr(S_{t+1} = j|S_t = i, S_{t-1} = k, \dots, y_t, y_{t-1}, \dots)$$

<sup>5</sup> In the parametric application of this approach described in the next section, we tested this assumption by using several alternative specifications of the Markov switching model, including higher autoregressive processes or allowing the variance and mean to follow the same or distinct Markov processes. We find that the simplest representation describes the data quite well and is most robust on a recursive sample of real-time data.

$$f(y_{t+1}|S_{t+1} = j) = f(y_{t+1}|S_{t+1} = j, S_t = i, S_{t-1} = k, \dots, y_t, y_{t-1}, \dots), \quad (5)$$

one can use knowledge of GDP growth rates through date  $t$  to make a prediction about whether the economy is in recession at any date  $\tau$ ,

$$\Pr(S_\tau = i | y_1, y_2, \dots, y_t).$$

If  $t = \tau$ , these are referred to as the “filter probabilities”, whereas when  $t > \tau$  they are described as “smoothed probabilities.”

### 3 Parametric representation.

Although it is interesting to know how to perform these calculations nonparametrically, this degree of generality is really not needed for the problem at hand, since it appears from Figure 2 that a Gaussian distribution works quite well to describe these densities. The fact that the recession distribution has a standard deviation very similar to that for the expansion distribution implies that we would also lose little by assuming that the two distributions differ only in their means and share the same standard deviation  $\sigma$ . The suggestion is then that we replace the arbitrary density  $f(y_t|S_t = 2)$  in the top panel of Figure 2 with the  $N(\mu_2, \sigma^2)$  distribution,

$$f(y_t|S_t = 2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_t - \mu_2)^2}{2\sigma^2}\right), \quad (6)$$

where  $\mu_2$ , the mean growth rate in contractions, should be about -1.2 with  $\sigma$  around 3.5. Likewise we could easily parameterize the bottom panel of Figure 2,  $f(y_t|S_t = 1)$ , with the  $N(\mu_1, \sigma^2)$  density for  $\mu_1 = 4.5$ . Let  $p_{11}$  denote the probability that the economy remains in expansion from one quarter to the next,

$$p_{11} = \Pr(S_{t+1} = 1|S_t = 1),$$

and  $p_{22}$  the analogous probability for recessions:

$$p_{22} = \Pr(S_{t+1} = 2|S_t = 2).$$

Again the historical experience would lead us to expect that  $p_{11} = 0.95$  and  $p_{22} = 0.78$ . Let  $\boldsymbol{\theta} = (\mu_1, \mu_2, \sigma, p_{11}, p_{22})'$  denote the various unknown parameters.

A two-state Markov chain with transition probabilities  $p_{ii}$  has unconditional distribution given by<sup>6</sup>

$$\Pr(S_t = 2) = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} = \pi_2.$$

The likelihood of the first observation in the sample ( $y_t$  for  $t = 1$ ) is then given by the mixture

$$f(y_1; \boldsymbol{\theta}) = \sum_{i=1}^2 \frac{\pi_i}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_1 - \mu_i)^2}{2\sigma^2}\right),$$

which is simply a parametric expression for the calculations that produced the solid curve in the top panel of Figure 3. The filtered probability for the first observation is

$$\Pr(S_1 = i|y_1; \boldsymbol{\theta}) = [f(y_1; \boldsymbol{\theta})]^{-1} \frac{\pi_i}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_1 - \mu_i)^2}{2\sigma^2}\right), \quad (7)$$

as in the bottom panel of Figure 3.

These probabilities in turn imply a predicted probability for the second observation of

$$\Pr(S_2 = j|y_1; \boldsymbol{\theta}) = \sum_{i=1}^2 p_{ij} \Pr(S_1 = i|y_1; \boldsymbol{\theta}). \quad (8)$$

The conditional likelihood of the second observation is given by the mixture whose weights are the predicted probabilities from (8),

$$f(y_2|y_1; \boldsymbol{\theta}) = \sum_{j=1}^2 \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_2 - \mu_j)^2}{2\sigma^2}\right) \Pr(S_2 = j|y_1; \boldsymbol{\theta}), \quad (9)$$

or the kind of calculation that produced the solid curve in the top panel of Figure 4. From this we obtain as in the bottom panel of Figure 4 the filtered probabilities for the second observation,

$$\Pr(S_2 = i|y_2, y_1; \boldsymbol{\theta}) = [f(y_2|y_1; \boldsymbol{\theta})]^{-1} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_2 - \mu_i)^2}{2\sigma^2}\right) \Pr(S_2 = i|y_1; \boldsymbol{\theta}), \quad (10)$$

and predicted probabilities for the third:

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$$\Pr(S_3 = j|y_2, y_1; \boldsymbol{\theta}) = p_{ij} \Pr(S_2 = i|y_2, y_1; \boldsymbol{\theta}).$$

<sup>6</sup> See for example Hamilton (1994, p. 683).

Iterating in this fashion we obtain the log likelihood for the complete sample of observed GDP growth rates,  $y_1, y_2, \dots, y_T$ , as a function of the parameter vector  $\theta$ :

$$\log f(y_1; \theta) + \sum_{t=2}^T \log f(y_t | y_{t-1}, y_{t-2}, \dots, y_1; \theta). \quad (11)$$

We motivated this way of thinking about the data by taking the NBER’s conclusions as given and trying to characterize what the NBER has done.<sup>7</sup> However, no aspect of the NBER’s dating appears in the final result (11), which is solely a function of observed GDP growth rates and the unknown parameters  $\theta$ . One could accordingly choose as an estimate of  $\theta$  the value that maximizes the sample log likelihood of GDP growth rates (11). This maximum likelihood estimate is compared with the values we would have expected on the basis of the NBER inferences in Table 1.<sup>8</sup> The two sets of parameter values, although arrived at by different methods, are remarkably similar. This similarity is very encouraging, for two different reasons. First, it enhances the intellectual legitimacy of the perspective that the economy can be classified as being in an expansion or recession at any point in time, and that whether or not the economy is in recession can account for much of the variability and serial dependence of GDP growth rates. We did not impose any kind of conditions on the two means  $\mu_1$  and  $\mu_2$ , and one could imagine the data being better described by all sorts of choices, such as “very rapid growth” versus “normal growth”, or “normal growth” versus “slow growth.” Table 1 implies that, using just GDP data alone without any reference to what NBER may have said, we would come up with a very similar conceptual scheme to the one that economists and the NBER have traditionally relied on.

[ insert Table 1 about here ]

A second reason that the correspondence between the two columns in Table 1 is encouraging

<sup>7</sup> An alternative approach developed by Bry and Boschan (1971) attempts to formalize and elaborate on the rule of thumb that two quarters of falling GDP constitute a recession. However, this rule of thumb does not describe the decisions of the NBER Business Cycle Dating Committee, which defines a recession as “a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales” (<http://www.nber.org/cycles.html/>). We view our approach, unlike Bry and Boschan, as a direct statistical formalization of the NBER’s stated method for qualitative evaluation.

<sup>8</sup> Maximum likelihood estimates were found using the EM algorithm described in Hamilton (1990).

is that it raises the promise that we might be able to use GDP growth rates alone to arrive at classifications in a more timely and objective fashion than the NBER. The top panel of Figure 6 plots the filtered recession probabilities  $\Pr(S_t = 2|y_t, y_{t-1}, \dots, y_1; \hat{\theta})$  implied by the maximum likelihood estimate of the parameter vector  $\theta$ . For any date  $t$  this is the probability that the economy is in recession based on observations of GDP growth rates at the time. The dates of economic recessions as determined after the fact by NBER are indicated by shaded regions on the graph. It seems clear that the two methodologies are identifying the same series of events over the postwar period, with the filter probabilities rising above 75% at some point during every postwar recession and typically remaining below 30% in times of expansions. There are some minor differences, with the two consecutive quarters of falling GDP in 1947:II-III and the -1.9% growth in 1956:I temporarily pushing the filter probabilities a little over 50% in episodes that the NBER did not characterize as recessions. Also, in the 1990-91 recession, the filter probabilities did not come back below 50% until 1991:IV, although the NBER says that the recession ended in 1991:I. Overall, though, the correspondence seems quite strong.

[ insert Figure 6 about here ]

The bottom panel of Figure 6 plots the smoothed probabilities, for which the full sample of observations through 2004:II was used to form an inference about the state of the economy at any given date. Using the full sample substantially smooths out a number of the minor temporary blips evident in the filter estimates, and brings the 1947 and 1956 inferences just under 50%, ever so slightly favoring the NBER final call. Dates at which recessions began and ended according to the NBER are compared with the dates for which the smoothed probabilities are above 50% in Table 2. The smoothed probabilities date the 1980 recession as beginning 3 quarters earlier than the date assigned by the NBER. The two methods never differ by more than a quarter for either the starting date or ending date for any other recession.

[ insert Table 2 about here ]

This suggests that using a mechanical algorithm to identify business cycle turning points holds

considerable promise. However, even the filter probabilities in the top panel of Figure 6 do not accurately capture the predictions that one could actually make with this framework in real time, for two reasons. First, the complete sample of data through 2004 was used to estimate the values of the parameter vector  $\theta$ . This perhaps is not an overwhelming concern, since, as we saw in Table 1, one would have arrived at very similar magnitudes for  $\theta$  just based on the properties that one expects expansions and recessions should have. The second, more serious, problem is that the GDP figures as originally released by the Bureau of Economic Analysis can differ substantially from the historical series now available.

Croushore and Stark (2003) have established that the second issue can be extremely important in practice, and have helped develop an extensive data set archived at the Federal Reserve Bank of Philadelphia (available at <http://www.phil.frb.org/econ/forecast/reaindex.html>). This data set includes the history of GDP values that would have actually been available to a researcher or forecaster at any given point in time. The database consists of one set of GDP levels for 1947:I-1965:III that would have been reported as of the middle of 1965:IV, a second set of GDP levels for 1947:I-1965:IV reported as of the middle of 1966:I, and so on, ending with a data set of GDP levels from 1947:I-2004:II as reported in the middle of 2004:III, with the latter data set being the one on which Figure 6 was based. There are a few gaps in this series, such as resulted from the benchmark GDP revision released in 1992:I. As originally released this revision only went back to 1959:I rather than all the way to 1947:I. To construct the inferences reported below, we assume that a researcher in 1992:I had available the GDP figures for 1947:I-1958:IV that technically were not published until 1993:I.

For each date  $T$  between 1968:II and 2004:II, we constructed the values for GDP growth for quarter  $t$  that a researcher would have had available as of date  $T + 1$ , denoted  $y_t^{[T]}$ , for  $t = 1947:II$  through  $T$ . We estimated the value  $\hat{\theta}^{[T]}$  that maximized the log likelihood of  $\{y_1^{[T]}, y_2^{[T]}, \dots, y_T^{[T]}\}$  and used this estimate to form inferences about the economy for each date  $t$  between 1947:II and  $T$ . The last value for GDP growth in this sample,  $y_T^{[T]}$ , (for example, the value of GDP for

2004:II as reported in 2004:III), is apt to be particularly noisy. Furthermore, there is a substantial gain in accuracy from using the one-quarter smoothed probability rather than the current filtered probability. For these reasons, our recommendation is that one should wait to make a real-time assessment of the state of the economy in 2004:I until the first estimate of 2004:II growth (and revised estimate of 2004:I growth) is released in August 2004.

The top panel of Figure 7 plots these real-time one-quarter-smoothed inferences  $\Pr(S_{T-1} = 2 | y_1^{[T]}, y_2^{[T]}, \dots, y_T^{[T]}; \hat{\theta}^{[T]})$  as a function of  $T - 1$ . The quality of the inference degrades a bit using real-time released data in place of the full revised data set as now available. In particular, successfully calling the end of the 1990-1991 recession would have been quite difficult with the data as actually released in 1992. Notwithstanding, the inference in each of the other recessions based on using real-time GDP estimates with one-quarter of smoothing seems to produce quite a satisfactory result overall.

[ insert Figure 7 about here ]

We will refer to the magnitude  $q_t^{(q)} = 100 \times \Pr(S_t = 2 | y_1^{[t+1]}, y_2^{[t+1]}, \dots, y_{t+1}^{[t+1]}; \hat{\theta}^{[t+1]})$  as our “quarterly real-time GDP-based recession probability index,” whose value represents an inferred probability (in percent) as to whether the economy was in a recession at date  $t$  using the first-reported GDP growth for quarter  $t + 1$ . The “(q)” superscript indicates that the index is based on quarterly data, in contrast to the monthly index that is developed in Section 5 below. We are also interested in the possibility of rendering quasi-official pronouncements based on this index. For this purpose, it seems prudent to build in a bit of conservatism into any announced changes in the economy. Let  $D_t^{(q)} = \text{“expansion”}$  if we are declaring the economy to have been in an expansion in quarter  $t$  and  $D_t^{(q)} = \text{“recession”}$  otherwise, where this declaration is intended as a qualitative summary of the information in  $q_t^{(q)}$ . If last quarter we had declared the economy to be in an expansion ( $D_{t-1}^{(q)} = \text{“expansion”}$ ), then this quarter we propose to declare the same thing

as long as the one-quarter-smoothed probability of expansion remains above 35%:

$$D_t^{(q)} = \begin{cases} \text{“expansion”} & \text{if } D_{t-1}^{(q)} = \text{“expansion”} \text{ and } q_t^{(q)} \leq 65 \\ \text{“recession”} & \text{if } D_{t-1}^{(q)} = \text{“expansion”} \text{ and } q_t^{(q)} > 65 \end{cases} .$$

Likewise, if last quarter we had declared the economy to be in a recession, then this quarter we will declare the same thing as long as the one-quarter-smoothed probability of recession remains above 35%:

$$D_t^{(q)} = \begin{cases} \text{“recession”} & \text{if } D_{t-1}^{(q)} = \text{“recession”} \text{ and } q_t^{(q)} \geq 35 \\ \text{“expansion”} & \text{if } D_{t-1}^{(q)} = \text{“recession”} \text{ and } q_t^{(q)} < 35 \end{cases} .$$

Table 3 reports values for our real-time GDP-based recession probability index  $q_t^{(q)}$  along with the proposed announcement  $D_t^{(q)}$  for each quarter. The algorithm does quite a satisfactory job of identifying the dates at which recessions began and ended. Its performance is compared with NBER news releases in Table 4. NBER would have beaten our mechanical algorithm by one quarter on two occasions, declaring the start of the 2001 recession and the end of the 1991 recession one quarter earlier than we would have. On two other occasions (the start of the 1990-91 recession and end of the 1979-1980 recession), the mechanical rule beat NBER by one quarter. Our algorithm also would have declared the start of the 1979-80 recession two quarters earlier, and end of the 2001 recession four quarters earlier than did NBER. In all the other episodes, the two approaches would have made the same announcement in the same historical quarter.

[ insert Table 3 about here ]

These calculations suggest that an algorithmically-based inference could do quite a satisfactory job of calling business cycle turning points in real time. Not only does its quantitative performance seem to be a little better than NBER’s, but there is an added benefit of objectivity. Given the potential of recession pronouncements to influence elections and policy decisions, there is always a possibility that there could be pressure to delay or accelerate making a subjective declaration in order to try to influence these outcomes. Our approach, by contrast, is completely objective and its mechanical operation transparent and reproducible.



[ insert Table 4 about here ]

Our approach does have an ambiguity that the NBER announcements lack, however, in that it highlights the uncertainty inherent in the enterprise and calls direct attention to the fact that sometimes the situation is very difficult to call one way or another (for example, when the recession probability index is near 50%). We would suggest, however, that this is inherent in the nature of the question being asked, and that openly recognizing this ambiguity is intellectually more honest and accurate than trying to conceal it. As long as we take the view that an economic recession is a real, objective event that may or may not have accounted for the observed data, there will always be some uncertainty in determining when and if one actually occurred. For better or worse, an objective assessment of the state of the economy of necessity must communicate not just a judgment (expansion or recession), but also some information about how compelling that conclusion is, given the data. The combined information conveyed by our proposed measures  $q_t$  and  $D_t$  seems a very promising way to communicate this information.

#### **4 Using multiple indicators to identify turning points.**

One drawback of the GDP-based measure is that it is only available quarterly. Given the lags in data collection and revision, this introduces an inherent 5-month delay in reporting of the index. A variety of measures available on a monthly basis might be used to produce much better inferences. By modeling the behavior of a number of different variables simultaneously, we can capture pervasive cyclical fluctuations in various sectors of the economy. As recessions and expansions are caused by different shocks over time, the inclusion of different variables increases the ability of the model to represent and signal phases of the business cycle in the monthly frequency. In addition, the combination of variables reduces measurement errors in the individual series and, consequently, the likelihood of false turning point signals, which is particularly important when monthly data are used.

Certainly the NBER dating committee does not base its conclusions just on the behavior of

quarterly GDP. Inspired by the seminal work of Burns and Mitchell (1946), the NBER Business Cycle Dating Committee today primarily looks at four key monthly indicators<sup>9</sup>, including the growth rates of manufacturing and trade sales (sales), total personal income less transfer payments (income), civilian labor force employed in nonagricultural industries (employment), and industrial production (IP). Let  $\mathbf{y}_t$  denote the  $(4 \times 1)$  vector whose first element  $y_{1t}$  is sales growth,  $y_{2t}$  is income growth,  $y_{3t}$  is employment growth, and  $y_{4t}$  is IP growth. In this section, we show how one can adapt the method of the previous section to use all four variables to infer the state of the business cycle.

A simple vector generalization of the approach in the preceding section would be quite straightforward. We could simply posit that the vector  $\mathbf{y}_t$  has one mean  $\boldsymbol{\mu}^{(1)}$  in expansions and a second mean  $\boldsymbol{\mu}^{(2)}$  in recessions, where we indicate the economic regime with a superscript, reserving subscripts in this section to denote individual elements of a vector or to indicate the value of a variable for a particular date  $t$ . For example, the first element of the vector  $\boldsymbol{\mu}^{(2)}$  would denote the average growth rate of sales during a recession. If  $\mathbf{H}$  denotes the variance-covariance matrix of these growth rates in either expansion or recession, then we could simply replace the scalar  $N(\mu_2, \sigma^2)$  distribution in (6) with the vector  $N(\boldsymbol{\mu}^{(2)}, \mathbf{H})$  distribution,

$$f(\mathbf{y}_t | S_t = 2) = \frac{1}{(2\pi)^{n/2} |\mathbf{H}|^{-1/2}} \exp \left\{ -(1/2) [\mathbf{y}_t - \boldsymbol{\mu}^{(2)}]' \mathbf{H}^{-1} [\mathbf{y}_t - \boldsymbol{\mu}^{(2)}] \right\}, \quad (12)$$

where  $n = 4$  denotes the number of elements in the vector  $\mathbf{y}_t$ . In every formula where we previously had the scalar  $f(y_t | S_t = j)$  we would now have the scalar  $f(\mathbf{y}_t | S_t = j)$ . For example, to calculate the probability of a recession given only GDP growth  $y_t$  in Figure 3 we took the ratio of the height of two lines. In the vector case we would be taking the ratio of the height of

<sup>9</sup> In NBER's FAQ page on business cycle dating at <http://www.nber.org/cycles/recessions.html#faq>, it is stated that "The committee places particular emphasis on two monthly measures of activity across the entire economy: (1) personal income less transfer payments, in real terms and (2) employment. In addition, the committee refers to two indicators with coverage primarily of manufacturing and goods: (3) industrial production and (4) the volume of sales of the manufacturing and wholesale-retail sectors adjusted for price changes. The committee also looks at monthly estimates of real GDP such as those prepared by Macroeconomic Advisers (see <http://www.macroadvisers.com>). Although these indicators are the most important measures considered by the NBER in developing its business cycle chronology, there is no fixed rule about which other measures contribute information to the process." We follow Chauvet (1998) in using civilian labor force in nonagricultural industries rather than employees on nonagricultural payrolls as used by NBER, for reasons detailed in Section 6 below.

two multidimensional surfaces, where the ratio of  $f(\mathbf{y}_t|S_t = 2) \Pr(S_t = 2)$  to the sum  $[f(\mathbf{y}_t|S_t = 1) \Pr(S_t = 1) + f(\mathbf{y}_t|S_t = 2) \Pr(S_t = 2)]$  would tell us the probability of a recession given that the vector of growth rates is observed to equal  $\mathbf{y}_t$ , a calculation that could be performed for any possible  $\mathbf{y}_t$ . In essence, we would be judging the probability of a recession by whether, taken as a group, the elements of  $\mathbf{y}_t$  are closer to the values we typically associate with expansions,  $\boldsymbol{\mu}^{(1)}$ , or closer to the values we typically associate with recessions,  $\boldsymbol{\mu}^{(2)}$ , with closeness based on the respective values of  $[\mathbf{y}_t - \boldsymbol{\mu}^{(j)}]'\mathbf{H}^{-1}[\mathbf{y}_t - \boldsymbol{\mu}^{(j)}]$  for  $j = 1$  or  $2$ , but also taking into account how likely we expected an expansion or recession to be  $\Pr(S_t = j)$  before seeing the data  $\mathbf{y}_t$ .

Though this would be one possibility, it is not the best way to approach monthly data, since our simplifying assumption in equation (5) that recessions account for all of the observed dynamic behavior of  $\mathbf{y}_t$  is no longer a very good one when we get to these higher frequency, more detailed data. We therefore adopt a generalization of the above method which has the basic effect of allowing  $\boldsymbol{\mu}^{(j)}$ , the vector of growth rates that we expect when the economy is in regime  $j$  at date  $t$ , to depend not just on the current regime  $j$  but also on the previous economic regime  $S_{t-1} = i$  as well as the whole history of previous values for  $\mathbf{y}_{t-m}$ . The same is potentially true for the variance-covariance matrix  $\mathbf{H}$ . Thus the general approach is based on a specification of

$$\begin{aligned} & f(\mathbf{y}_t|S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}) \\ &= \frac{1}{(2\pi)^{n/2}} \left| \mathbf{H}_t^{(i,j)} \right|^{-1/2} \exp \left\{ -(1/2) \left[ \mathbf{y}_t - \boldsymbol{\mu}_t^{(i,j)} \right]' \left[ \mathbf{H}_t^{(i,j)} \right]^{-1} \left[ \mathbf{y}_t - \boldsymbol{\mu}_t^{(i,j)} \right] \right\} \end{aligned} \quad (13)$$

where  $\mathbf{Y}_{t-1}$  denotes the history of observations obtained through date  $t-1$ :

$$\mathbf{Y}_{t-1} = (\mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \dots, \mathbf{y}'_1)'$$

The dependence on both  $S_t$  and  $S_{t-1}$  presents no real problems. Rather than forming an inference in the form of a probability that the current regime  $S_t = j$ , we will be calculating a joint probability that  $S_t = j$  and  $S_{t-1} = i$ ,

$$\Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_t).$$

Indeed, we already saw exactly how to do this in equation (3). Here we are basically calculating how close the various elements of  $\mathbf{y}_t$  are to the corresponding elements of  $\boldsymbol{\mu}_t^{(i,j)}$ , that is, how close they are to what we would have predicted given that  $S_t = j$  and  $S_{t-1} = i$  and the past observations  $\mathbf{y}_{t-m}$ . The inference then favors those combinations  $i, j$  with the best fit to  $\mathbf{y}_t$ , taking into account also how likely the combination  $i, j$  was regarded to be before seeing  $\mathbf{y}_t$ .

The question then is what growth rates  $\boldsymbol{\mu}_t^{(i,j)}$  we expect for  $\mathbf{y}_t$  in different phases of the business cycle. We follow Chauvet (1998) and Kim and Nelson (1999a) in their specification of how a recession may affect different economic indicators at the same time.

Our basic assumption is that there exists an aggregate cyclical factor  $F_t$  that evolves according to

$$F_t = \alpha^{(S_t)} + \phi F_{t-1} + \eta_t \quad S_t = 1, 2, \quad (14)$$

where  $\eta_t \sim N(0, \sigma_\eta^2)$  and  $\alpha^{(S_t)} = \alpha^{(1)}$  when the economy overall is in an expansion ( $S_t = 1$ ) and  $\alpha^{(S_t)} = \alpha^{(2)}$  in contraction. Note that if  $F_t$  corresponded to GDP growth, equation (14) would include the dynamic process assumed for quarterly recession dynamics in the previous section as a special case when  $\phi = 0$ , with  $\alpha^{(1)}$  then corresponding to  $\mu_1$  (the mean growth rate in expansions) and  $\alpha^{(2)}$  corresponding to  $\mu_2$ . When  $\phi$  is a number greater than zero (but presumably less than unity), expression (14) also allows for serial correlation in growth rates even without a business cycle turning point, and implies that in an expansion, the aggregate factor eventually trends toward a growth rate of  $\alpha^{(1)}/(1 - \phi)$ .

We assume that the growth rate of the  $r$ th monthly indicator  $y_{rt}$  is determined by the aggregate factor  $F_t$  and an idiosyncratic factor  $v_{rt}$ ,

$$y_{rt} = \lambda_r F_t + v_{rt} \quad \text{for } r = 1, 2, 3, 4 \quad (15)$$

with  $v_{rt}$  itself exhibiting  $AR(1)$  dynamics:<sup>10</sup>

$$v_{rt} = \theta_r v_{r,t-1} + \varepsilon_{rt}. \quad (16)$$

---

<sup>10</sup> Residual diagnostics and likelihood ratio tests favor first-order autoregressive processes for both the disturbance terms and the dynamic factor.

When the aggregate factor  $F_t$  changes, this induces a change in each variable in  $\mathbf{y}_t$ , with the  $r$ th series changing by  $\lambda_r \Delta$  when the aggregate factor changes by  $\Delta$ ; the bigger  $\lambda_r$ , the more series  $r$  responds to these aggregate fluctuations. The  $r$ th series also experiences shocks  $v_{rt}$  that have no consequences for the variables in  $\mathbf{y}_t$  other than  $y_{rt}$ .

We will continue to assume as in the preceding section that business cycle transitions are the outcome of a Markov chain that is independent of previous realizations:<sup>11</sup>

$$\Pr(S_t = j | S_{t-1} = i, S_{t-2} = k, \dots, \mathbf{Y}_{t-1}) = p_{ij}.$$

The above system can be cast as a Markov-switching state space representation such as those analyzed by Chauvet (1998) and Kim and Nelson (1999a). The key to such a representation is a state vector  $\mathbf{f}_t$  which contains (along with the regime  $S_t$ ) all the information needed to forecast any of the individual series in  $\mathbf{y}_t$ . For this set-up, the state vector is a  $(5 \times 1)$  vector,

$$\mathbf{f}_t = (F_t, v_{1t}, v_{2t}, v_{3t}, v_{4t})'$$

whose dynamics are characterized by

$$\begin{bmatrix} F_t \\ v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix} = \begin{bmatrix} \alpha^{(S_t)} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi & 0 & 0 & 0 & 0 \\ 0 & \theta_1 & 0 & 0 & 0 \\ 0 & 0 & \theta_2 & 0 & 0 \\ 0 & 0 & 0 & \theta_3 & 0 \\ 0 & 0 & 0 & 0 & \theta_4 \end{bmatrix} \begin{bmatrix} F_{t-1} \\ v_{1,t-1} \\ v_{2,t-1} \\ v_{3,t-1} \\ v_{4,t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

or in matrix notation,

$$\mathbf{f}_t = \alpha^{(S_t)} \mathbf{e}_5 + \Phi \mathbf{f}_{t-1} + \mathbf{a}_t \tag{17}$$

where  $\mathbf{e}_5 = (1, 0, 0, 0, 0)'$ . We assume that the disturbances in  $\mathbf{a}_t$  are uncorrelated with each other

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<sup>11</sup> We test for the number of states versus a linear version of the model using the approach described in Garcia (1998). Garcia uses the results from Hansen (1993, 1996), treating the transition probabilities as nuisance parameters to test regime switching models. We construct Garcia's test statistic and compare with the the critical values reported in his paper. The critical values are significantly smaller than the likelihood ratio test for the dynamic factor with Markov regime switching yielding some evidence in rejecting the one state null hypothesis.

and uncorrelated across time:

$$\mathbf{a}_t | \mathbf{a}_{t-1}, \mathbf{a}_{t-2}, \dots, \mathbf{a}_1, S_t, S_{t-1}, \dots \sim N(\mathbf{0}, \Sigma)$$

where  $\Sigma$  is a diagonal matrix.

The observed variables  $\mathbf{y}_t$  are related to the state vector through the observation equation,

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 & 0 \\ \lambda_2 & 0 & 1 & 0 & 0 \\ \lambda_3 & 0 & 0 & 1 & 0 \\ \lambda_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_t \\ v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix}. \quad (18)$$

The  $r$ th row of (18) just reproduces (15). Again (18) can be conveniently written in matrix form as

$$\mathbf{y}_t = \mathbf{\Lambda} \mathbf{f}_t. \quad (19)$$

The model also requires a normalization condition, because if we doubled the standard deviation of each element of  $\mathbf{a}_t$  and halved the value of each  $\lambda_r$ , the implied observed behavior of  $\mathbf{y}_t$  would be identical. Our benchmark model resolves this normalization by setting  $\sigma_{\eta}^2$ , the first element of  $\Sigma$ , equal to unity.

Note that equations (14) through (16) imply

$$y_{rt} = \lambda_r \left[ \alpha^{(S_t)} + \phi F_{t-1} + \eta_t \right] + \theta_r v_{r,t-1} + \varepsilon_{rt}$$

or

$$y_{rt} = \mu_{rt}^{(S_t)} + \lambda_r \eta_t + \varepsilon_{rt} \quad (20)$$

where

$$\mu_{rt}^{(S_t)} = \lambda_r \left[ \alpha^{(S_t)} + \phi F_{t-1} \right] + \theta_r v_{r,t-1}.$$

Equation (20) can be stacked into a vector for  $r = 1, 2, 3, 4$  using the notation of (17) and (19),

$$\mathbf{y}_t = \mathbf{\Lambda} \alpha^{(S_t)} \mathbf{e}_5 + \mathbf{\Lambda} \Phi \mathbf{f}_{t-1} + \mathbf{\Lambda} \mathbf{a}_t$$

$$= \boldsymbol{\mu}_t^{(S_t)} + \boldsymbol{\Lambda} \mathbf{a}_t \quad (21)$$

for

$$\boldsymbol{\mu}_t^{(S_t)} = \alpha^{(S_t)} \boldsymbol{\Lambda} \mathbf{e}_5 + \boldsymbol{\Lambda} \boldsymbol{\Phi} \mathbf{f}_{t-1}.$$

In other words,

$$\mathbf{y}_t | S_t = j, \mathbf{f}_{t-1} \sim N \left( \boldsymbol{\mu}_t^{(j)}, \boldsymbol{\Lambda} \boldsymbol{\Lambda}' \right). \quad (22)$$

If we observed  $\mathbf{f}_{t-1}$ , this distribution would play the role of the  $N(\boldsymbol{\mu}_t^{(i,j)}, \mathbf{H}_t^{(i,j)})$  distribution in (13), and indeed, would be a little simpler than the general case in that  $\boldsymbol{\mu}_t^{(i,j)}$  would not depend on  $i$  and  $\mathbf{H}_t^{(i,j)}$  would not depend on  $i, j$ , or  $t$ . In this simple case, we see from (20) that  $\mu_{rt}^{(S_t)}$ , the growth rate we expect for  $y_{rt}$  when  $S_t = 2$ , would be the sum of: (a)  $\lambda_r \alpha^{(2)}$  (the product of  $\lambda_r$ , the response of series  $r$  to the aggregate factor, with  $\alpha^{(2)}$ , the contribution of a recession to the aggregate factor); (b)  $\lambda_r \phi F_{t-1}$  (the product of  $\lambda_r$  with  $\phi F_{t-1}$ , where  $\phi F_{t-1}$  is our forecast of the non-recession component of the aggregate factor  $F_t$ ); and (c)  $\theta_r v_{r,t-1}$  (our expectation of  $v_{rt}$ , the factor that is unique to series  $r$ ).<sup>12</sup>

Unfortunately, using this framework is a little more complicated than this, because even if we knew for certain that  $S_{t-1} = i$ , and had observed the values of  $\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{y}_1$ , we still would not know the value  $\mathbf{f}_{t-1}$ . We could, however, use methods described below to form an estimate of it, denoted  $\mathbf{f}_{t-1|t-1}^{(i)}$ :

$$\mathbf{f}_{t-1|t-1}^{(i)} = E(\mathbf{f}_{t-1} | S_{t-1} = i, \mathbf{Y}_{t-1}).$$

The true value  $\mathbf{f}_{t-1}$  differs from this estimate by some error  $\mathbf{h}_{t-1|t-1}^{(i)}$ :

$$\mathbf{f}_{t-1} = \mathbf{f}_{t-1|t-1}^{(i)} + \mathbf{h}_{t-1|t-1}^{(i)}. \quad (23)$$

Suppose we approximate the distribution of this error with the Normal distribution:

$$\mathbf{h}_{t-1|t-1}^{(i)} \sim N \left( \mathbf{0}, \mathbf{P}_{t-1|t-1}^{(i)} \right). \quad (24)$$

---

<sup>12</sup> Extensions of the model such as allowing for more than two regimes, time-varying transition probabilities, and different lags for the factors are straightforward extensions of the specification described here.

The  $r$ th diagonal element of  $\mathbf{P}_{t-1|t-1}^{(i)}$  would be small if we had a good inference about the value of  $f_{r,t-1}$ . Treating  $\mathbf{f}_{t-1}$  as known corresponds to the special case when  $\mathbf{P}_{t-1|t-1}^{(i)} = \mathbf{0}$ .

Imperfect inference about  $\mathbf{f}_{t-1}$  affects our ability to forecast  $\mathbf{f}_t$ . Substituting (23) into (17),

$$\begin{aligned}\mathbf{f}_t &= \alpha^{(S_t)} \mathbf{e}_5 + \Phi \left[ \mathbf{f}_{t-1|t-1}^{(i)} + \mathbf{h}_{t-1|t-1}^{(i)} \right] + \mathbf{a}_t \\ &= \alpha^{(S_t)} \mathbf{e}_5 + \Phi \mathbf{f}_{t-1|t-1}^{(i)} + \mathbf{q}_{t|t-1}^{(i)}\end{aligned}\quad (25)$$

where

$$\begin{aligned}\mathbf{q}_{t|t-1}^{(i)} &= \Phi \mathbf{h}_{t-1|t-1}^{(i)} + \mathbf{a}_t \sim N(\mathbf{0}, \mathbf{Q}_{t|t-1}^{(i)}) \\ \mathbf{Q}_{t|t-1}^{(i)} &= \Phi \mathbf{P}_{t-1|t-1}^{(i)} \Phi' +\end{aligned}\quad (26)$$

with the last expression following from the definition of  $\mathbf{P}_{t-1|t-1}^{(i)}$  in (24) and the fact that  $\mathbf{a}_t$  is independent of anything dated  $t-1$  or earlier. Substituting (25) into (19),

$$\mathbf{y}_t = \alpha^{(S_t)} \Lambda \mathbf{e}_5 + \Lambda \Phi \mathbf{f}_{t-1|t-1}^{(i)} + \Lambda \mathbf{q}_{t|t-1}^{(i)}.\quad (27)$$

Considering the case when  $S_{t-1} = i$  and  $S_t = j$ , expression (27) implies that

$$\mathbf{y}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1} \sim N\left(\boldsymbol{\mu}_{t|t-1}^{(i,j)}, \mathbf{H}_{t|t-1}^{(i)}\right)\quad (28)$$

where

$$\boldsymbol{\mu}_{t|t-1}^{(i,j)} = \alpha^{(j)} \Lambda \mathbf{e}_5 + \Lambda \Phi \mathbf{f}_{t-1|t-1}^{(i)}\quad (29)$$

$$\mathbf{H}_{t|t-1}^{(i)} = \Lambda \mathbf{Q}_{t|t-1}^{(i)} \Lambda'.$$

Expression (28) is the generalization we sought in (13). In this case, the value we expect for  $y_{rt}$  when  $S_{t-1} = i$  and  $S_t = 2$  is the sum of: (a)  $\lambda_r \alpha^{(2)}$ , just as in the case when we regarded  $\mathbf{f}_{t-1}$  as if known; (b)  $\lambda_r \phi F_{t-1|t-1}^{(i)}$  (the product of  $\lambda_r$  with  $\phi F_{t-1|t-1}^{(i)}$ , where  $\phi F_{t-1|t-1}^{(i)}$  is our expectation of the non-recession component of the aggregate factor  $F_t$ , with this expectation based on  $F_{t-1|t-1}^{(i)}$ , which is where we thought the factor was at date  $t-1$ , given that  $S_{t-1} = i$ ); and (c)  $\theta_r v_{r,t-1|t-1}^{(i)}$  (what we expect for the dynamic factor  $v_{rt}$  that is unique to series  $r$  based on where we thought the idiosyncratic factor was at  $t-1$ ). The variance of our error in forecasting  $\mathbf{y}_t$ , denoted  $\mathbf{H}_{t|t-1}^{(i)}$ ,



depends on the date because having a larger number of observations  $\{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{t-1}\}$  can help us to improve the accuracy of the inference  $\mathbf{f}_{t-1|t-1}^{(i)}$ .

The one additional step necessary before proceeding on to observation  $t + 1$  is to update the inference  $\mathbf{f}_{t-1|t-1}^{(i)}$  to incorporate date  $t$ 's information. This is accomplished through a device known as the Kalman filter. The basic idea is to use the known correlation between the new observation  $\mathbf{y}_t$  and the unobserved magnitude  $\mathbf{f}_t$  to revise the prediction of  $\mathbf{f}_t$  that we would have made using  $\mathbf{f}_{t-1|t-1}^{(i)}$  alone. One could imagine doing this with a regression of  $\mathbf{f}_t$  on  $\mathbf{y}_t$  and  $\mathbf{f}_{t-1|t-1}^{(i)}$ . Although we don't have any observations on  $\mathbf{f}_t$  with which to perform such a regression, we know from the structure of the model what the regression coefficients would turn out to be if we had an infinite number of such observations. In the appendix we show that these ideal regression coefficients are given by

$$\begin{aligned}\mathbf{f}_{t|t}^{(i,j)} &= E(\mathbf{f}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_t) \\ &= \alpha^{(j)} \mathbf{e}_5 + \Phi \mathbf{f}_{t-t|t-1}^{(i)} + \mathbf{Q}_{t|t-1}^{(i)} \Lambda' [\mathbf{H}_{t|t-1}^{(i)}]^{-1} [\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}].\end{aligned}\quad (30)$$

Expression (30) gives the inference about  $\mathbf{f}_t$  given both  $S_{t-1} = i$  and  $S_t = j$  in addition to the observed data  $\mathbf{y}_t, \mathbf{y}_{t-1}, \dots, \mathbf{y}_1$ . The inference conditioning only on the current regime  $S_t = j$  is found from

$$\begin{aligned}\mathbf{f}_{t|t}^{(j)} &= E(\mathbf{f}_t | S_t = j, \mathbf{Y}_t) \\ &= \sum_{i=1}^2 E(\mathbf{f}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_t) \Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t) \\ &= \sum_{i=1}^2 \mathbf{f}_{t|t}^{(i,j)} \Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t).\end{aligned}\quad (31)$$

The probability necessary to calculate this last magnitude can again be found from Bayes Law:

$$\Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t) = \frac{\Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_t)}{\Pr(S_t = j | \mathbf{Y}_t)}.$$

The appendix also shows that the population mean squared error of the inference (31) is given

by

$$\begin{aligned}
\mathbf{P}_{t|t}^{(i,j)} &= E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_t \right\} \\
&= \mathbf{Q}_{t|t-1}^{(i)} - \mathbf{Q}_{t|t-1}^{(i)} \mathbf{\Lambda}' \left[ \mathbf{H}_{t|t-1}^{(i)} \right]^{-1} \mathbf{\Lambda} \mathbf{Q}_{t|t-1}^{(i)} + \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right]'. \quad (32)
\end{aligned}$$

Again this is converted to a magnitude that only depends on  $j$  from

$$\begin{aligned}
\mathbf{P}_{t|t}^{(j)} &= E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right]' \middle| S_t = j, \mathbf{Y}_t \right\} \\
&= \sum_{i=1}^2 \mathbf{P}_{t|t}^{(i,j)} \Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t).
\end{aligned}$$

There is just one problem with this algorithm. We assumed in (24) that the date  $t - 1$  inference had an error with a Normal distribution, conditional on  $S_{t-1} = i$ . But when we sum the inferences over the two values of  $i$  as in the last line of (31), this would produce not a Normal distribution but a mixture of Normals. The mean and variance of this distribution are correctly given by  $\mathbf{f}_{t|t}^{(j)}$  and  $\mathbf{P}_{t|t}^{(j)}$ , and the updating rule in (30) can still be motivated as the population regression. But when  $\mathbf{h}_{t-1|t-1}^{(i)}$  is not Normal, the distribution in (28) is no longer exact but only an approximation. This approximation, suggested by Kim (1994), is certainly necessary, because without the summation in (31), the number of possibilities would end up cascading, with the inference about  $\mathbf{f}_T$  depending on  $S_T, S_{T-1}, \dots, S_1$ . Fortunately, experience has shown that approximating the mixture distribution with a Normal distribution works very well in practice and we seem to lose little when we adopt it.<sup>13</sup>

To summarize, our inference for the vector case is based on an iterative algorithm, calculated sequentially for  $t = 1, 2, \dots, T$ . As a result of step  $t - 1$  of these calculations, we would have calculated the following three magnitudes:

$$\Pr(S_{t-1} = i | \mathbf{Y}_{t-1}) \quad (33)$$

$$\mathbf{f}_{t-1|t-1}^{(i)} \quad (34)$$

---

<sup>13</sup> For example, Chauvet and Piger (2005) estimate the dynamic factor model with regime switching in real time using both Kim's algorithm and Bayesian estimation methods (see Shepard 1994, Albert and Chib 1993, or Kim and Nelson 1999a). The results obtained using these two methods were found to be very similar.

$$\mathbf{P}_{t-1|t-1}^{(i)}. \quad (35)$$

At step  $t$  we then calculate

$$\Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_{t-1}) = p_{ij} \Pr(S_{t-1} = i | \mathbf{Y}_{t-1})$$

$$\boldsymbol{\mu}_{t|t-1}^{(i,j)} = \alpha^{(j)} \boldsymbol{\Lambda} \mathbf{e}_5 + \boldsymbol{\Lambda} \boldsymbol{\Phi} \mathbf{f}_{t-1|t-1}^{(i)}$$

$$\mathbf{Q}_{t|t-1}^{(i)} = \boldsymbol{\Phi} \mathbf{P}_{t-1|t-1}^{(i)} \boldsymbol{\Phi}' +$$

$$\mathbf{H}_{t|t-1}^{(i)} = \boldsymbol{\Lambda} \mathbf{Q}_{t|t-1}^{(i)} \boldsymbol{\Lambda}'.$$

These magnitudes are then all we need to construct the density of the  $t$ th observation given

$$S_{t-1} = i, S_t = j,$$

$$\begin{aligned} f(\mathbf{y}_t | S_{t-1} = i, S_t = j, \mathbf{Y}_{t-1}) = \\ \frac{1}{(2\pi)^{n/2}} \left| \mathbf{H}_{t|t-1}^{(i)} \right|^{-1/2} \exp \left\{ -(1/2) \left[ \mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)} \right]' \left[ \mathbf{H}_{t|t-1}^{(i)} \right]^{-1} \left[ \mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)} \right] \right\}, \end{aligned}$$

the density not conditioning on  $S_{t-1}$  or  $S_t$ ,

$$f(\mathbf{y}_t | \mathbf{Y}_{t-1}) = \sum_{i=1}^2 \sum_{j=1}^2 f(\mathbf{y}_t | S_{t-1} = i, S_t = j, \mathbf{Y}_{t-1}) \Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_{t-1}), \quad (36)$$

and the filter probability that  $S_t = j$ :

$$\Pr(S_t = j | \mathbf{Y}_t) = \frac{\sum_{i=1}^2 f(\mathbf{y}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}) \Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_{t-1})}{f(\mathbf{y}_t | \mathbf{Y}_{t-1})}. \quad (37)$$

This last calculation gives us the input (33) that we will need to proceed with the iteration for  $t + 1$ . We update (34) by calculating

$$\Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t) = \frac{f(\mathbf{y}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}) \Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_{t-1})}{\sum_{i=1}^2 f(\mathbf{y}_t | S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}) \Pr(S_t = j, S_{t-1} = i | \mathbf{Y}_{t-1})} \quad (38)$$

$$\mathbf{f}_{t|t}^{(i,j)} = \alpha^{(j)} \mathbf{e}_5 + \boldsymbol{\Phi} \mathbf{f}_{t-t|t-1}^{(i)} + \mathbf{Q}_{t|t-1}^{(i)} \boldsymbol{\Lambda}' \left[ \mathbf{H}_{t|t-1}^{(i)} \right]^{-1} \left[ \mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)} \right]$$

$$\mathbf{f}_{t|t}^{(j)} = \sum_{i=1}^2 \mathbf{f}_{t|t}^{(i,j)} \Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t).$$

Finally, we update the third input (35) from

$$\mathbf{P}_{t|t}^{(i,j)} = \mathbf{Q}_{t|t-1}^{(i)} - \mathbf{Q}_{t|t-1}^{(i)} \boldsymbol{\Lambda}' \left[ \mathbf{H}_{t|t-1}^{(i)} \right]^{-1} \boldsymbol{\Lambda} \mathbf{Q}_{t|t-1}^{(i)} + \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right]'$$

$$\mathbf{P}_{t|t}^{(j)} = \sum_{i=1}^2 \mathbf{P}_{t|t}^{(i,j)} \Pr(S_{t-1} = i | S_t = j, \mathbf{Y}_t).$$

Note that as a consequence of performing this iteration for  $t = 1, 2, \dots, T$ , we have calculated the filter probabilities (37), one-month smoothed probabilities (38), and conditional density of the  $t$ th observation (36). The latter can be used to construct the log likelihood for the entire sample,

$$\ln f(\mathbf{Y}_T) = \ln f(\mathbf{y}_1) + \sum_{t=2}^T \ln f(\mathbf{y}_t | \mathbf{Y}_{t-1}). \quad (39)$$

The value obtained from (39) will depend on the values of the population parameters that were used to perform the above calculations. These consist of  $\boldsymbol{\theta} = (\alpha^{(1)}, \alpha^{(2)}, p_{11}, p_{22}, \phi, \theta_1, \theta_2, \theta_3, \theta_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2, \sigma_{\varepsilon_3}^2, \sigma_{\varepsilon_4}^2)'$ . We then choose values of these parameters so as to maximize the log likelihood (39).

All that is needed to implement the above procedure is the starting values of (33) through (35) for observation  $t = 1$ , given initial values for  $\boldsymbol{\theta}$ . For the probabilities we use as initial condition the probabilities associated with the ergodic distribution of the Markov chain,  $\Pr(S_{t-2} = h, S_{t-1} = i | \mathbf{Y}_{t-1}) = \Pr(S_0 = i) = \pi_i = (1 - p_{jj}) / (2 - p_{ii} - p_{jj})$ ,  $i = 1, 2$ , where  $\pi_i$  is the ergodic probability. For the state vector, its unconditional mean and unconditional covariance matrix are used as initial values, that is,  $\mathbf{f}_{0|0}^{(i)} = E(\mathbf{f}_t)$  and  $\mathbf{P}_{0|0}^{(i)} = \boldsymbol{\Phi} \mathbf{P}_{0|0}^{(i)} \boldsymbol{\Phi}' + \boldsymbol{\Sigma}$ .<sup>14</sup>

## 5 Empirical performance of the monthly recession probability index.

In this section we investigate the ability of the multivariate version of the Markov switching model in dating business cycle turning points at the monthly frequency. We used numerical search algorithms (e.g., Hamilton, 1994, Section 5.7) to find the value of the parameter vector  $\hat{\boldsymbol{\theta}}$  that maximizes the log likelihood (39) of the observed historical sample of growth rates of sales, income, employment, and IP. These maximum likelihood estimates are reported in Table 5. For any date  $t$  we can evaluate current filtered probabilities of expansions,  $\Pr(S_t = 1 | \mathbf{Y}_t; \hat{\boldsymbol{\theta}})$ , and recessions,

<sup>14</sup> Since  $\mathbf{f}_t$  is unobserved, we use the average of the unconditional mean of the four series in  $\mathbf{Y}_t$ .

$\Pr(S_t = 2|\mathbf{Y}_t; \hat{\theta})$ , as calculated in equation (37) now based on the maximum likelihood estimate  $\hat{\theta}$ . We can also construct a smoothed inference that uses both current and future observations of the series  $\mathbf{y}_t$ . For example, the conditional probability that the economy is in a recession at date  $t$  based on all future observations of the series  $\mathbf{y}_t$  is  $\Pr(S_t = 2|\mathbf{Y}_T; \hat{\theta})$ .

[ insert Table 5 about here ]

As a first step in evaluating the ability of the model to reproduce the NBER dates, consider Figure 8, which plots the estimated full sample smoothed probabilities of recessions. The shaded areas represent periods dated as recessions by the NBER. The probabilities indicate that our model reproduces the NBER chronology very closely. During periods that the NBER classifies as expansions, the probabilities of recession are usually close to zero. At around the beginning of the NBER-dated recessions the probabilities rise and remain high until around the time the NBER dates the end of the recession. In particular, every time the probability of recession increases above 50%, a recession follows. Conversely, the recession probabilities decrease below 50% at the recession trough.

[ insert Figure 8 about here ]

The model-based inferences about recession dates are compared with the dates determined by the NBER in Table 6. The first column reports the month in which the recession started according to the NBER dates. The second column shows the first month in which the full sample smoothed probability of a recession rose above 50%. The NBER recession dates and the model-based dates are very close, either exactly coinciding or differing by only one month. The one exception is the 2001 recession, in which the estimated probabilities started increasing in 2000, six months before the recession began as declared by the NBER. Our quarterly GDP-based full-sample inferences reported in Table 2 also suggested that this recession actually began in the fourth quarter of 2000. Some special features of this recession will be discussed in more detail below in connection with data that would have actually been available in real time.

[ insert Table 6 about here ]

The third column of Table 6 indicates the NBER date for the end of the recession, and the fourth column reports the last month for which the smoothed probability of a recession was above 50%. Once again the model-based inference and the NBER dating for troughs are strikingly similar, even more so than for business cycle peaks.

These full sample smoothed probabilities are an important tool that can be used to revise historically the model assessment of business cycle phases. However, since these smoothed probabilities rely on future information  $T - t$  steps ahead, they can not be used to evaluate the state of the business cycle on a current basis. In order to investigate the real-time performance of the multivariate Markov switching model for dating business cycles, two features should be taken into account that not even the use of current filtered probabilities would accomplish. First, only information available at the time the forecast is formed should be used. Thus, recursive estimation is applied to estimate the parameters of the model and infer the probabilities. Second, the real-time exercise needs to be implemented using only the same knowledge of data revisions that would have been available at the time. Thus, for each end of sample date in the recursive estimation the first release of the data that was available is used.

For each month between January 1978 and January 2004, we obtained values for the complete history of each of the four monthly variables in  $\mathbf{y}_t$  going back to January 1959, as that history would have been reported as of the indicated date. These data were assembled by hand from various issues of *Business Conditions Digest* and the *Survey of Current Business, Employment and Earnings* (both published monthly by the Bureau of Economic Analysis), and *Economic Indicators* (published monthly by the Council of Economic Advisers). As with our real-time GDP series described in Section 3, there were gaps in the full series for some vintages that were filled in with the next available observation. There were also occasionally large outliers, which were also replaced with the next release.

Using these data, we ran recursive estimations of the model starting with the sample from January 1959 to November 1977. The filter probability for the terminal date of this first data set,

$\Pr(S_t = 2 | \mathbf{y}_1^{[t]}, \mathbf{y}_2^{[t]}, \dots, \mathbf{y}_t^{[t]}; \hat{\boldsymbol{\theta}}^{[t]})$  where  $t$  corresponds to November 1977, is the first data point of the real-time filter probabilities corresponding to the single date  $t$ . We will refer below to 100 times this magnitude,

$$p_t^{(m)} = 100 \times \Pr(S_t = 2 | \mathbf{y}_1^{[t]}, \mathbf{y}_2^{[t]}, \dots, \mathbf{y}_t^{[t]}; \hat{\boldsymbol{\theta}}^{[t]}),$$

as our “preliminary monthly real-time recession probability index”. The sample is then extended by one month, to December 1977, using now a completely new set of observations  $\mathbf{y}_1^{[t+1]}, \mathbf{y}_2^{[t+1]}, \dots, \mathbf{y}_{t+1}^{[t+1]}$  to come up with a new maximum likelihood estimate  $\hat{\boldsymbol{\theta}}^{[t+1]}$  and a new terminal filter probability  $\Pr(S_{t+1} = 2 | \mathbf{y}_1^{[t+1]}, \mathbf{y}_2^{[t+1]}, \dots, \mathbf{y}_t^{[t+1]}; \hat{\boldsymbol{\theta}}^{[t+1]})$  which will produce the preliminary index  $p_{t+1}^{(m)}$  for date  $t + 1$ . This procedure is repeated for each of the 315 recursive estimations until the final sample is reached, which extends from January 1959 to January 2004.

Notice that for each end of sample date in the recursive estimation procedure we use the first release of the data that was available for *all* four variables. The series employment and industrial production are more timely - they are released with only one month delay, whereas personal income and manufacturing and trade sales are released with a delay of two months.<sup>15</sup> In order for the four real-time variables to enter the model estimation, we use the data vintage that contains the latest information on sales and personal income. For example, for the second sample from January 1959 to December 1977, we use the first release of data that included information on all four series for December 1977, which is February 1978.

Figure 9 plots the real-time recursive probability of a recession. Each point in the graph corresponds to a recursive estimation of real-time unrevised data,  $p_t^{(m)}/100$ , plotted as a function of  $t$ .<sup>16</sup> The probabilities match closely the NBER recessions, rising around the beginning of recessions and decreasing around their end. Once again, the probabilities remain below 50% during

<sup>15</sup> The first releases of employment and industrial production for a given month are available, respectively, around the first and third weeks of the subsequent month, whereas the first releases of personal income and manufacturing and trade sales are available in the last week of the second month.

<sup>16</sup> The values plotted in Figure 10 for dates  $t$  before November 1977 are the filter probabilities from the sample of the first vintage,  $\Pr(S_t = 2 | \mathbf{y}_1^{(1977:11)}, \mathbf{y}_2^{(1977:11)}, \dots, \mathbf{y}_t^{(1977:11)}; \hat{\boldsymbol{\theta}}^{(1977:11)})$ .

expansions, usually only rising beyond this threshold during recessions as dated by the NBER.

[ insert Figure 9 about here ]

The real-time recursive filtered probabilities are spikier than the filtered or smoothed probabilities obtained using revised data, which is expected given that unrevised data are generally noisier than revised releases. The real-time filtered probabilities are also intrinsically more noisy than their smoothed counterparts. We could immediately call a business cycle turning point if the real-time filtered probabilities move from below 50% to above 50% or vice versa. This rule maximizes the speed at which a turning point might be identified, but increases the chances of declaring a false positive. It seems more prudent to require confirmation of the turning point, by verifying it with more information. As in Section 3, we investigate the gain in accuracy from using a low-order smoothed probability in addition to the current filtered probability. We combine the information on the readily available filtered probabilities with the more precise information obtained from  $h$ -step ahead (where  $h$  is a low number) smoothed probabilities in real-time assessment of the business cycle phases. For example, the one-month ahead smoothed probabilities are used to create what we call our “revised monthly real-time recession probability index”:

$$q_t^{(m)} = 100 \times \Pr(S_t = 2 | \mathbf{Y}_{t+1}^{[t+1]}; \hat{\boldsymbol{\theta}}^{[t+1]}) = \sum_{i=1}^2 \Pr(S_t = 2, S_{t+1} = i | \mathbf{Y}_{t+1}^{[t+1]}; \hat{\boldsymbol{\theta}}^{[t+1]}). \quad (40)$$

Figure 10 displays real-time  $h$ -month-smoothed inferences for  $h = 1, 2, 3$ . The shaded areas correspond to recessions as dated by the NBER. The quality of the inference in terms of accuracy improves as more information is used to form the smoothed probabilities. Figure 11 shows the real-time current filtered probabilities and the  $h$ -month-smoothed probabilities recession by recession. A distinct common pattern across the probabilities for the 1980, 1981, and 1990 recessions is that the *current filtered probabilities* declare the beginning of recessions a couple of months after the NBER says that a recession began, while they call the end of recessions at about the same time as the NBER dating. This is less accentuated for the 1980 and 1981 recessions than for the 1990 recession. The smoothed probabilities, however, increasingly adjust the date of recession peaks to



earlier months, converging to a match to the NBER date. Regarding the end of recessions, the dates called by the current filtered probabilities for these recessions are timely with the NBER, and the smoothed probabilities obtained 1, 2, and 3 months later simply confirm these dates. Thus, there seems to be a gain in combining information from the current filtered probability and the smoothed probabilities in tabulating a chronology of expansion peaks in real time.

[ insert Figure 10 about here ]

[ insert Figure 11 about here ]

The inference from the multivariate Markov switching model for the 2001 recession is a bit distinct from previous downturns. The current filtered probabilities declare the beginning of the recession to have occurred at about the same time as the NBER date. The smoothed probabilities, however, increasingly adjust the peak date to a couple of months *before* the NBER date. We earlier observed the same thing with inferences based on quarterly GDP growth rates. In the case of the monthly index, these dynamics of the estimated probabilities are associated with the behavior of the growth rates of industrial production and personal income, which showed a decline already in 2000, before the recession had begun. The end of the 2001 recession is in accord with the NBER dating even when only the current filtered probabilities are used, as it is the case for previous recessions. However, this result for the last recession is sensitive to the choice of the employment series used in the estimation of the multivariate Markov switching model, as discussed in the next section.

While visual inspection of the probabilities yields some insight, it is difficult to ascertain how close the turning points determined by the multivariate model are to the NBER dates without compiling specific dates. In order to do this a formal definition is needed to convert the estimated probabilities into business cycle dates. We use a combination of the current filtered probabilities  $p_t^{(m)}$  and one-month-smoothed probabilities  $q_t^{(m)}$  to evaluate the performance of the multivariate Markov switching model in signalling business cycle turning points. We follow a similar rule to the one adopted for the univariate inference using real-time quarterly GDP, though there we only made

use of the one-quarter smoothed probabilities  $q_t^{(q)}$ . Note that, just as we waited until one extra quarter's data on GDP growth ( $y_{t+1}^{[t+1]}$ ) becomes available before announcing the quarterly index  $q_t^{(q)}$  for quarter  $t$ , we will require one extra month's data on sales, income, employment, and IP ( $y_{t+1}^{[t+1]}$ ) before announcing the revised monthly index  $q_t^{(m)}$  for month  $t$ . Let  $D_t^{(m)} = \text{"recession"}$  if we declare the economy to have been in a recession in month  $t$  and  $D_t^{(m)} = \text{"expansion"}$  otherwise. If we had declared that the economy was in an expansion in month  $t-1$ , ( $D_{t-1}^{(m)} = \text{"expansion"}$ ), then we would declare that a recession began in month  $t$  only if (1) the filtered probability of recession at  $t$  had risen above 65% (the preliminary index  $p_t^{(m)} > 65$ ) and (2) this result is confirmed by the one-month ahead smoothed probability of expansion for assessment of the economy for that same month  $t$  (the revised index  $q_t^{(m)} > 65$ ). Otherwise, we would declare the expansion to have continued through month  $t$ . Formally,

$$D_t^{(m)} = \begin{cases} \text{"expansion"} & \text{if } D_{t-1}^{(m)} = \text{"expansion"}, \text{ and either } p_t^{(m)} \leq 65 \text{ or } q_t^{(m)} \leq 65 \\ \text{"recession"} & \text{if } D_{t-1}^{(m)} = \text{"expansion"}, \text{ and both } p_t^{(m)} > 65 \text{ and } q_t^{(m)} > 65 \end{cases}.$$

Similarly, if we had declared that the economy was in a recession in month  $t-1$ , then we would declare that a recovery began in month  $t$  only if both the filtered and the one-month smoothed probabilities of recession for month  $t$  are less than 35%:

$$D_t^{(m)} = \begin{cases} \text{"recession"} & \text{if } D_{t-1}^{(m)} = \text{"recession"}, \text{ and either } p_t^{(m)} \geq 35 \text{ or } q_{t+1|t}^{(m)} \geq 35 \\ \text{"expansion"} & \text{if } D_{t-1}^{(m)} = \text{"recession"}, \text{ and both } p_t^{(m)} < 35 \text{ and } q_{t+1|t}^{(m)} < 35 \end{cases}.$$

The preliminary index  $p_t^{(m)}$ , revised index  $q_t^{(m)}$ , and announcement  $D_t^{(m)}$  are reported in Table 7.

[ insert Table 7 about here ]

Note that a more precise turning point signal comes at the expense of how quickly we would call it, since the timing when we would be able to make the announcement in real time would be delayed by one extra month. For example, for assessment of the current state of the economy at  $t = 1990:7$ , the first release of the real-time data for all four variables would be available in 1990:9. By using the one-month smoothed probability, we would have to wait until data released in 1990:10 to make a decision. Thus, there is a three-month delay in announcing turning points.

We find that the gain in precision by using  $q_t^{(m)}$  in addition to  $p_t^{(m)}$  more than compensates the small loss in timing by one month.

Table 8 compares NBER news releases with the performance of the multivariate Markov switching model in dating and announcing business cycle chronology. Regarding dating the phases, the model would have made the identical declaration of the date of the 2001 business cycle peak as did the NBER, but lags the NBER dates by two or three months for the other three recessions. The difference between the model-based dates and the NBER's is smaller for troughs, coinciding in two occasions and differing by one or two months in the other two recessions.

[ insert Table 8 about here ]

The great advantage of the objective method regards the timely announcement of turning points. The algorithm does very well in announcing the beginning and end of downturns compared with statements released by the NBER. The model would have beaten the NBER in calling the beginning of a recession in two out of four occasions (the start of the 1990 and 2001 recessions, respectively) and would have coincided in two cases (the start of the 1980 and 1982 recessions). The advantage of the dates inferred from the multivariate model is even more significant for dating the end of recessions. The model beats the NBER announcements in all occasions, with leads from three to seventeen months. The model would have announced the end of the 1980 recession 8 months before the NBER's announcement, the end of the 1982 recession three months earlier than the NBER, the 1990 recession 17 months earlier, and the more recent recession in 2001 would have been declared to have ended 14 months before the announcement by the NBER.

Comparing the quarterly and monthly results, the multivariate Markov switching model and the univariate one applied to GDP usually convey similar information, but complement each other on some occasions. This indicates that there are clear gains in combining information from our quarterly real-time GDP-based recession probability index ( $D_t^{(q)}$  and  $q_t^{(q)}$ ) and our monthly real-time multivariate-based recession probability indicators ( $D_t^{(m)}$ ,  $p_t^{(m)}$ , and  $q_t^{(m)}$ ) in dating business cycle and announcing these dates in real time. For example, the quarterly real-time index dates

the end of the 1990 recession only in the second quarter of 1992, and the announcement of this date would have been available in February 1993, three months after the NBER announcement. The monthly index, on the other hand, dates the end of this recession as March 1991, coinciding with the trough declared by the NBER. This date would have been available from the monthly index in July 1991, 17 months before the announcement by the NBER in December 1992. Regarding the 2001 recession, the monthly index dates the end of the 2001 recession in January 2002, two months after the trough in November 2001 declared by the NBER. The quarterly index, on the other hand, declares the end of this recession in the fourth quarter of 2001, coinciding with the NBER date. The monthly index would have announced this trough 14 months before the NBER declared the end of this recession, and the quarterly index would have announced it 12 months before.

In general, there is a gain in speed of announcement by using the monthly-based recession index, given that the monthly data are available more quickly than quarterly GDP mainly with respect to business cycle troughs. While the NBER's announcements sometimes beat the quarterly index, the monthly index consistently anticipates the recession end before the NBER's decisions. On the other hand, the monthly index (particularly if one relied only on  $p_t^{(m)}$  or  $q_t^{(m)}$  alone) shows more short-run volatility than does the quarterly index. Although combined inference is best, either index alone would have overall delivered more timely indications than did NBER in declaring the start or the end of the recessions in the real time sample, and the business cycle chronology obtained would have matched closely the NBER dating.

These results suggest that the algorithm-based inference contributes to the assessment of business cycle phases in real time, and offers quantitative improvements compared to the NBER methods. In addition, our approach is more objective and mechanical, which makes its potential use widespread.

## 6 Alternative approaches to monthly inference.

In this section we report briefly on our investigations of some alternative specifications for the monthly index. We explored different autoregressive processes for the components of the state equation and tried specifications with one or two of the elements of  $\mathbf{y}_t$  deleted or one or two other monthly series added. None of these changes seemed to make much difference for the inference.

One feature that does modify the results somewhat is the changing cyclical behavior of employment. In particular, the employment series used by the NBER, employees on non-agricultural payrolls (ENAP), displayed a very slow recovery in the last recession. In fact, real-time assessment of the recent economic recession using this series would have indicated that the downturn did not end until 2003. The real-time probabilities of recession obtained when this measure of employment is included in the estimation suggested that there was a slight recovery in economic activity from October 2001 to July 2002, but this was followed by a weakening of the economy in the subsequent months until early 2003. The use of this employment series also yields delays in signaling turning points for previous recessions. This is in agreement with Chauvet (1998), who found that this employment series lags the business cycles and documented the improvement in using alternative employment variables. Stock and Watson (1991) also found that payroll employment is a lagging indicator rather than a coincident variable of business cycle since its estimated residuals are serially correlated. For this reason, both Chauvet and Stock and Watson included lagged values for the factor in the measurement equation for payroll employment. On the other hand, this correction is not necessary when using other employment measurements.

Our analysis in Section 5 was instead based on an alternative employment series, Total Civilian Employment (TCE). This variable coincides with business cycle phases and delivers a much faster call of turning points in real time, as described in the previous section. The inclusion of this series allows us to keep the specification simple and yet robust to the use of real time data.

There are several reasons why these two series diverge sometimes, and a lot of controversy has

emerged in the last few years on the best measure of employment. ENAP is based on a survey of business establishments, whereas TCE is based on a survey among households. These two employment series have generally moved together, with some minor differences around business cycle turning points until very recently. In particular, ENAP tends to overestimate employment around the beginning of recessions and underestimate around their end. As the results of estimation of our model based on the two different measures has reflected, these dynamics became very accentuated in the last recession in 2001. The main differences between these two series are that ENAP does not count agricultural and self-employed workers. More important, ENAP counts an individual twice if he or she works two jobs or changes jobs during the pay period. As a result of a debate regarding the sources of the differences, the Bureau of Labor and Statistics has produced some studies and concluded that a correction in population trend and addition of non-farmer workers in the TCE series would bring the two closer together in level and ex-post for the recent period (Di Natale, 2003; U.S. Department of Labor, 2004). This is also discussed in Juhn and Potter (1999). A comprehensive summary of these results and the debate can be found in Kane (2004).

However, the adjustment by BLS does not deal with the reliability and differences between these two series in real time, which is the focus of our analysis. The ENAP series only includes job destruction and creation with a lag, it does not include self-employment and contractors or off-the-books employment, and it double counts jobs if a person changes jobs within a payroll survey reference period. These can be very important cyclical factors around business cycle turning points. In particular, the first three differences can lead ENAP to signal a more severe recession and delay detection of a recovery, while the fourth one can overestimate employment around peaks. In addition, the first release of ENAP is preliminary and undergoes substantial revisions in subsequent months. There is also a significant revision of this series once a year when the smaller initial sample collected is adjusted by using as a benchmark the universe count of employment derived from Unemployment Insurance tax records that almost all employers are

required to file. These corrections make real-time data on ENAP very different from the revised versions. Thus, although the revised ENAP may reflect better labor conditions ex-post, its performance in capturing real time cyclical changes in the economy is meager compared to the household survey (TCE).

In addition, we have also examined the performance of the model when a break in volatility in 1984 is taken into account. Kim and Nelson (1999b), McConnell and Perez-Quiros (2000), and Chauvet and Potter (2001) have found that the US economy became more stable since this date, particularly the quarterly GDP series. When this feature is incorporated in the model the results improve substantially with respect to the last two recessions, which took place after the structural break in volatility. We have nevertheless chosen not to correct for the decrease in volatility in the US economy in order to keep the analysis simple and robust.

Different rules were also investigated to declare the beginning and end of recessions. The one chosen, as described in the previous section, was not the one that necessarily maximizes the precision or speed of business cycle signals, but the one that worked as well with both simple and more complicated specifications. That is, we have chosen the rule that gives us the most confidence that it will be robust in future applications. We are less interested in fine-tuning the improvement of the algorithm than in obtaining a specification and rules that have a better chance to work well in the future. Thus, we recommend the simpler specification, which does not make any allowance for changes in the variance of economic fluctuations over time.

Overall, most of the options we investigated would result in quite reasonable estimates. Our conclusion is nevertheless that the benchmark model and inference rules presented in Section 5 appear to be the most robust with respect to changes in specification and data revision, and therefore recommend them as likely to prove most reliable for analyzing data and recognizing the business cycle trends in an ever-changing economy.

## Appendix

Here we derive equations (30) and (32). Suppose we have vectors  $\mathbf{z}$  and  $\mathbf{y}$  which have mean zero and a joint Normal distribution. Then the expectation of  $\mathbf{z}$  given  $\mathbf{y}$  turns out to be<sup>17</sup>

$$E(\mathbf{z}|\mathbf{x}) = E(\mathbf{z}\mathbf{x}') [E(\mathbf{x}\mathbf{x}')]^{-1} \mathbf{x} \quad (41)$$

which is just a population version of the familiar regression formula. The conditional variance is known to be

$$E[\mathbf{z} - E(\mathbf{z}|\mathbf{x})][\mathbf{z} - E(\mathbf{z}|\mathbf{x})]' = E(\mathbf{z}\mathbf{z}') - E(\mathbf{z}\mathbf{x}') [E(\mathbf{x}\mathbf{x}')]^{-1} E(\mathbf{x}\mathbf{z}'). \quad (42)$$

To apply these formulas here, let  $\mathbf{z} = \mathbf{f}_t - \alpha^{(j)}\mathbf{e}_5 - \Phi\mathbf{f}_{t-t|t-1}^{(i)}$  and  $\mathbf{x} = \mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}$ , which both have mean zero conditional on  $S_t = j, S_{t-1} = i, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{y}_1$ . The updated inference about  $\mathbf{f}_t$  is then given by

$$\begin{aligned} & E\left[\mathbf{f}_t - \alpha^{(j)}\mathbf{e}_5 - \Phi\mathbf{f}_{t-t|t-1}^{(i)} | \mathbf{y}_t; S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right] \\ = & E\left\{\left[\mathbf{f}_t - \alpha^{(j)}\mathbf{e}_5 - \Phi\mathbf{f}_{t-t|t-1}^{(i)}\right] \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right\} \times \\ & \left(E\left\{\left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right] \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right\}\right)^{-1} \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right]. \end{aligned} \quad (43)$$

But notice from (25) and (27) that

$$\begin{aligned} & E\left\{\left[\mathbf{f}_t - \alpha^{(j)}\mathbf{e}_5 - \Phi\mathbf{f}_{t-t|t-1}^{(i)}\right] \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right\} \\ = & E\left\{\mathbf{q}_{t|t-1}^{(i)} \left[\mathbf{q}_{t|t-1}^{(i)}\right]' \boldsymbol{\Lambda}'\right\} \\ = & \mathbf{Q}_{t|t-1}^{(i)} \boldsymbol{\Lambda}' \end{aligned} \quad (44)$$

for  $\mathbf{Q}_{t|t-1}^{(i)}$  the variance of  $\mathbf{q}_{t|t-1}^{(i)}$  defined in (26). Similarly from (28),

$$E\left\{\left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right] \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right\} = \mathbf{H}_{t|t-1}^{(i)}. \quad (45)$$

Substituting (44) and (45) into (43),

$$\underline{E\left[\mathbf{f}_t - \alpha^{(j)}\mathbf{e}_5 - \Phi\mathbf{f}_{t-t|t-1}^{(i)} | \mathbf{y}_t; S_t = j, S_{t-1} = i, \mathbf{Y}_{t-1}\right]} = \mathbf{Q}_{t|t-1}^{(i)} \boldsymbol{\Lambda}' \left(\mathbf{H}_{t|t-1}^{(i)}\right)^{-1} \left[\mathbf{y}_t - \boldsymbol{\mu}_{t|t-1}^{(i,j)}\right],$$

<sup>17</sup> See for example Hamilton (1994, p. 102).



which upon rearranging yields (30).

Similar calculations allow the updated variance to be found from (42):

$$\begin{aligned}
& E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} \right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_t \right\} \\
&= \mathbf{Q}_{i|t-1}^{(i)} - \mathbf{Q}_{t|t-1}^{(i)} \mathbf{\Lambda}' \left[ \mathbf{H}_{i|t-1}^{(i)} \right]^{-1} \mathbf{\Lambda} \mathbf{Q}_{i|t-1}^{(i)}.
\end{aligned} \tag{46}$$

To relate this magnitude to that given in (32), notice that

$$\begin{aligned}
\mathbf{P}_{i|t}^{(i,j)} &= E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(j)} \right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_t \right\} \\
&= E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} + \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} + \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_t \right\} \\
&= E \left\{ \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} \right] \left[ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} \right]' \middle| S_t = j, S_{t-1} = i, \mathbf{Y}_t \right\} \\
&\quad + \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right] \left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right]'
\end{aligned} \tag{47}$$

where cross-product terms disappear from the last expression because  $E \left\{ \mathbf{f}_t - \mathbf{f}_{t|t}^{(i,j)} \middle| S_t = j, \mathbf{Y}_t \right\} = \mathbf{0}$  and  $\left[ \mathbf{f}_{t|t}^{(i,j)} - \mathbf{f}_{t|t}^{(j)} \right]$  is a deterministic function of  $S_t = j, S_{t-1} = i, \mathbf{Y}_t$ . Substituting (46) into (47) results in (32).

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## Figure legends

Figure 1. U.S. real GDP growth rates, 1947-2004.

Figure 2. Top panel: nonparametric estimate of the density of U.S. annualized growth rates for those postwar U.S. quarters classified by NBER as falling in recessions. Bottom panel: density for expansions.

Figure 3. Determining the probability of a recession from GDP growth rate alone. Top panel: probability of observing given GDP growth during expansions or contractions. Bottom panel: probability of being in a contraction having observed specified value for GDP growth.

Figure 4. Determining the probability of a recession when previous quarter gave a strong recession signal. Top panel: probability of observing given GDP growth in quarter  $t + 1$  given probability of recession of 0.90 in quarter  $t$ . Bottom panel: probability of being in a contraction in quarter  $t + 1$  given previous quarter's probability of 0.90 and having observed specified value for GDP growth in quarter  $t + 1$ .

Figure 5. Probability of being in a recession based on one quarter's GDP data alone (top panel) and based on two quarter's GDP growth rates (bottom panel).

Figure 6. Probability that the U.S. economy is in recession based on revised quarterly data. Top panel: filter probabilities (inference using only GDP growth rates available at the time). Bottom panel: smoothed probabilities (inference using the full historical record). Shaded regions: dates of recessions as determined by NBER up to two years after the fact.

Figure 7. Probability that the U.S. economy is in recession. Top panel: one-quarter smoothed probabilities based on data as released historically as of two quarters after the indicated date (e.g., the entry for 2003:I is based on GDP growth rates for 2003:II as reported in August 2003). Bottom panel: one-quarter smoothed probabilities based on historically revised data as reported for all quarters in August 2004.

Figure 8. Full-sample smoothed probabilities and NBER recessions based on revised monthly

data.

Figure 9. Current filter probabilities and NBER recessions based on real-time monthly data after 1977:12 and revised monthly data before 1977:12.

Figure 10. Current filtered probabilities and h-month-smoothed probabilities of recession (h=1, 2, 3) based on real-time monthly data.

Figure 11. Filtered and smoothed probabilities with real-time monthly data over recursive samples. Top panel: 1980 and 1981-82 recessions. Middle panel: 1990-1991 recession. Bottom panel: 2001 recession.

## Tables

Table 1

Parameter estimates based on (1) characteristics of expansions and recessions as classified by NBER, and (2) values that maximize the observed sample log likelihood of postwar GDP growth rates

Parameter	Interpretation	Value from NBER classifications	Value from GDP alone
$\mu_1$	average growth in expansion	4.5	4.62
$\mu_2$	average growth in recession	-1.2	-0.48
$\sigma$	standard deviation of growth	3.5	3.34
$p_{11}$	prob. expansion continues	0.95	0.92
$p_{22}$	prob. recession continues	0.78	0.74



Table 2

Dates of recessions as determined by (1) NBER and (2) properties of GDP growth alone

Start of recession		End of recession	
NBER	Smoothed probabilities	NBER	Smoothed probabilities
1948:IV	1948:IV	1949:IV	1949:IV
1953:II	1953:III	1954:II	1954:II
1957:III	1957:II	1958:II	1958:I
1960:II	1960:II	1961:I	1960:IV
1969:IV	1969:III	1970:IV	1970:IV
1973:IV	1973:III	1975:I	1975:I
1980:I	1979:II	1980:III	1980:III
1981:III	1981:II	1982:IV	1982:IV
1990:III	1990:II	1991:I	1991:II
2001:I	2000:IV	2001:IV	2001:IV

Table 3

Real-time GDP-based recession probability index and proposed announcements. Note: index and announcements are not released until 5 months after the end of the indicated quarter.

Quarter ( $t$ )	$q_t$	$D_t$
1967:IV	4	expansion
1968:I	2	expansion
1968:II	1	expansion
1968:III	2	expansion
1968:IV	6	expansion
1969:I	13	expansion
1969:II	20	expansion
1969:III	46	expansion
1969:IV	89	recession
1970:I	98	recession
1970:II	92	recession
1970:III	95	recession
1970:IV	95	recession
1971:I	16	expansion
1971:II	10	expansion
1971:III	12	expansion
1971:IV	4	expansion
1972:I	1	expansion
1972:II	0	expansion
1972:III	1	expansion
1972:IV	0	expansion
1973:I	1	expansion
1973:II	10	expansion
1973:III	20	expansion
1973:IV	72	recession
1974:I	100	recession
1974:II	99	recession
1974:III	99	recession
1974:IV	100	recession
1975:I	100	recession
1975:II	43	recession
1975:III	0	expansion
1975:IV	0	expansion
1976:I	0	expansion
1976:II	1	expansion
1976:III	2	expansion
1976:IV	2	expansion
1977:I	0	expansion

1977:II	0	expansion
1977:III	1	expansion
1977:IV	7	expansion
1978:I	12	expansion
1978:II	0	expansion
1978:III	2	expansion
1978:IV	1	expansion
1979:I	49	expansion
1979:II	70	recession
1979:III	40	recession
1979:IV	39	recession
1980:I	82	recession
1980:II	100	recession
1980:III	37	recession
1980:IV	7	expansion
1981:I	2	expansion
1981:II	60	expansion
1981:III	79	recession
1981:IV	99	recession
1982:I	99	recession
1982:II	79	recession
1982:III	92	recession
1982:IV	85	recession
1983:I	32	expansion
1983:II	1	expansion
1983:III	0	expansion
1983:IV	1	expansion
1984:I	0	expansion
1984:II	1	expansion
1984:III	7	expansion
1984:IV	7	expansion
1985:I	30	expansion
1985:II	19	expansion
1985:III	17	expansion
1985:IV	27	expansion
1986:I	10	expansion
1986:II	20	expansion
1986:III	16	expansion
1986:IV	14	expansion
1987:I	7	expansion
1987:II	6	expansion
1987:III	2	expansion
1987:IV	2	expansion
1988:I	3	expansion
1988:II	5	expansion

1988:III	8	expansion
1988:IV	4	expansion
1989:I	5	expansion
1989:II	6	expansion
1989:III	9	expansion
1989:IV	12	expansion
1990:I	29	expansion
1990:II	41	expansion
1990:III	80	recession
1990:IV	95	recession
1991:I	97	recession
1991:II	90	recession
1991:III	81	recession
1991:IV	78	recession
1992:I	63	recession
1992:II	54	recession
1992:III	24	expansion
1992:IV	14	expansion
1993:I	35	expansion
1993:II	21	expansion
1993:III	8	expansion
1993:IV	3	expansion
1994:I	4	expansion
1994:II	4	expansion
1994:III	3	expansion
1994:IV	3	expansion
1995:I	12	expansion
1995:II	10	expansion
1995:III	33	expansion
1995:IV	33	expansion
1996:I	21	expansion
1996:II	11	expansion
1996:III	8	expansion
1996:IV	5	expansion

1997:I	4	expansion
1997:II	4	expansion
1997:III	4	expansion
1997:IV	4	expansion
1998:I	3	expansion
1998:II	6	expansion
1998:III	3	expansion
1998:IV	1	expansion
1999:I	2	expansion
1999:II	5	expansion
1999:III	1	expansion
1999:IV	0	expansion
2000:I	1	expansion
2000:II	1	expansion
2000:III	8	expansion
2000:IV	16	expansion
2001:I	27	expansion
2001:II	58	expansion
2001:III	84	recession
2001:IV	36	recession
2002:I	23	expansion
2002:II	20	expansion
2002:III	17	expansion
2002:IV	22	expansion
2003:I	24	expansion
2003:II	6	expansion
2003:III	1	expansion
2003:IV	2	expansion
2004:I	3	expansion

Table 4

Dates at which recession starts and ends were announced by NBER and would have been announced using the one-quarter smoothed inferences from real-time released data.

Start of recession			End of recession		
Start as determined by NBER	Date start announced by NBER	Date start announced by algorithm	End as determined by NBER	Date end announced by NBER	Date end announced by algorithm
1969:IV	N.A.	May 1970	1970:IV	N.A.	Aug 1971
1973:IV	N.A.	May 1974	1975:I	N.A.	Feb 1976
1980:I	June 1980	Nov 1979	1980:III	July 1981	May 1981
1981:III	Jan 1982	Feb 1982	1982:IV	July 1983	Aug 1983
1990:III	April 1991	Feb 1991	1991:I	Dec 1992	Feb 1993
2001:I	Nov 2001	Feb 2002	2001:IV	July 2003	Aug 2002

Table 5

Parameter estimates and asymptotic standard errors based on values that maximize the observed sample log likelihood of the multivariate monthly model

Parameter	Estimate (std error)	Parameter	Estimate (std error)
$\alpha^{(1)}$	0.95 (0.18)	$\lambda_1$	0.33 (0.028)
$\alpha^{(2)}$	-0.4 (0.24)	$\lambda_2$	0.23 (0.015)
$\phi$	0.37 (0.07)	$\lambda_3$	0.12 (0.009)
$\theta_1$	-0.30 (0.04)	$\lambda_4$	0.37 (0.03)
$\theta_2$	-0.029 (0.06)	$\sigma_{\varepsilon_1}^2$	0.74 (0.05)
$\theta_3$	-0.021 (0.05)	$\sigma_{\varepsilon_2}^2$	0.08 (0.008)
$\theta_4$	-0.19 (0.05)	$\sigma_{\varepsilon_3}^2$	0.07 (0.005)
$p_{11}$	0.97 (0.01)	$\sigma_{\varepsilon_4}^2$	0.33 (0.027)
$p_{22}$	0.90 (0.05)	$\ln f(\mathbf{Y}_T)$	-1594.61

Table 6

Dates of recessions as determined by (1) NBER and (2) Multivariate Markov-Switching Model (MMS) based on full-sample smoothed probabilities with revised monthly data

Start of recession		End of Recession	
NBER	Smoothed probabilities MMS	NBER	Smoothed probabilities MMS
Apr 1960	Feb 1960	Feb 1961	Dec 1960
Dec 1969	Nov 1969	Nov 1970	Nov 1970
Nov 1973	Dec 1973	Mar 1975	Mar 1975
Jan 1980	Jan 1980	Jul 1980	Jun 1980
Jul 1981	Aug 1981	Nov 1982	Nov 1982
Jul 1990	May 1990	Mar 1991	Feb 1991
Mar 2001	Sept 2000	Nov 2001	Oct 2001

Table 7

Monthly real-time multiple-indicator recession probability index and proposed announcements. Note: preliminary index released 2 months after the indicated month; revised index and determination are not released until 3 months after the indicated month.

Month ( $t$ )	Preliminary Index ( $p_t^{(m)}$ )	Revised Index ( $q_t^{(m)}$ )	Determination ( $D_t^{(m)}$ )
Nov 1977	1	1	expansion
Dec 1977	1	20	expansion
Jan 1978	57	23	expansion
Feb 1978	8	1	expansion
Mar 1978	0	0	expansion
Apr 1978	0	0	expansion
May 1978	2	1	expansion
Jun 1978	1	2	expansion
Jul 1978	8	3	expansion
Aug 1978	3	2	expansion
Sep 1978	2	1	expansion
Oct 1978	1	0	expansion
Nov 1978	1	0	expansion
Dec 1978	1	3	expansion
Jan 1979	8	4	expansion
Feb 1979	3	1	expansion
Mar 1979	1	22	expansion
Apr 1979	77	48	expansion
May 1979	21	15	expansion
Jun 1979	13	16	expansion
Jul 1979	16	39	expansion
Aug 1979	41	31	expansion
Sep 1979	24	10	expansion
Oct 1979	6	5	expansion
Nov 1979	5	3	expansion
Dec 1979	3	2	expansion
Jan 1980	3	10	expansion
Feb 1980	17	79	expansion
Mar 1980	88	100	recession
Apr 1980	100	100	recession
May 1980	99	100	recession
Jun 1980	90	90	recession
Jul 1980	75	41	recession
Aug 1980	14	3	expansion
Sep 1980	0	0	expansion
Oct 1980	0	0	expansion
Nov 1980	0	0	expansion
Dec 1980	2	9	expansion
Jan 1981	19	34	expansion
Feb 1981	35	22	expansion
Mar 1981	16	24	expansion
Apr 1981	24	11	expansion
May 1981	7	8	expansion
Jun 1981	9	2	expansion
Jul 1981	1	1	expansion
Aug 1981	4	31	expansion
Sep 1981	49	85	expansion
Oct 1981	85	97	recession
Nov 1981	94	99	recession
Dec 1981	96	99	recession
Jan 1982	97	85	recession
Feb 1982	77	60	recession
Mar 1982	70	58	recession
Apr 1982	58	43	recession
May 1982	80	83	recession
Jun 1982	82	94	recession
Jul 1982	90	95	recession
Aug 1982	89	93	recession
Sep 1982	84	96	recession
Oct 1982	94	94	recession
Nov 1982	81	81	recession
Dec 1982	70	28	recession
Jan 1983	3	8	expansion
Feb 1983	13	3	expansion
Mar 1983	1	0	expansion
Apr 1983	1	0	expansion
May 1983	1	0	expansion
Jun 1983	0	0	expansion

Jul 1983	0	0	expansion
Aug 1983	3	1	expansion
Sep 1983	0	0	expansion
Oct 1983	0	0	expansion
Nov 1983	1	0	expansion
Dec 1983	0	0	expansion
Jan 1984	0	0	expansion
Feb 1984	2	1	expansion
Mar 1984	1	0	expansion
Apr 1984	1	0	expansion
May 1984	1	0	expansion
Jun 1984	1	1	expansion
Jul 1984	2	4	expansion
Aug 1984	7	5	expansion
Sep 1984	5	6	expansion
Oct 1984	8	3	expansion
Nov 1984	2	1	expansion
Dec 1984	2	4	expansion
Jan 1985	8	5	expansion
Feb 1985	4	2	expansion
Mar 1985	2	3	expansion
Apr 1985	7	5	expansion
May 1985	6	14	expansion
Jun 1985	18	49	expansion
Jul 1985	53	19	expansion
Aug 1985	5	2	expansion
Sep 1985	1	1	expansion
Oct 1985	3	4	expansion
Nov 1985	5	1	expansion
Dec 1985	1	1	expansion
Jan 1986	3	5	expansion
Feb 1986	9	6	expansion
Mar 1986	6	3	expansion
Apr 1986	3	3	expansion
May 1986	5	5	expansion
Jun 1986	7	3	expansion
Jul 1986	2	2	expansion
Aug 1986	3	2	expansion

Sep 1986	2	3	expansion
Oct 1986	5	4	expansion
Nov 1986	4	1	expansion
Dec 1986	1	6	expansion
Jan 1987	19	4	expansion
Feb 1987	1	0	expansion
Mar 1987	2	2	expansion
Apr 1987	3	1	expansion
May 1987	1	1	expansion
Jun 1987	3	1	expansion
Jul 1987	1	0	expansion
Aug 1987	1	1	expansion
Sep 1987	4	1	expansion
Oct 1987	1	0	expansion
Nov 1987	2	1	expansion
Dec 1987	1	3	expansion
Jan 1988	12	4	expansion
Feb 1988	2	2	expansion
Mar 1988	3	2	expansion
Apr 1988	2	4	expansion
May 1988	8	3	expansion
Jun 1988	2	1	expansion
Jul 1988	2	2	expansion
Aug 1988	3	6	expansion
Sep 1988	9	3	expansion
Oct 1988	2	1	expansion
Nov 1988	2	1	expansion
Dec 1988	1	0	expansion
Jan 1989	1	3	expansion
Feb 1989	11	10	expansion
Mar 1989	10	13	expansion
Apr 1989	15	54	expansion
May 1989	60	43	expansion
Jun 1989	30	70	expansion
Jul 1989	72	34	expansion
Aug 1989	7	22	expansion
Sep 1989	27	33	expansion
Oct 1989	31	10	expansion



Nov 1989	4	2	expansion
Dec 1989	3	3	expansion
Jan 1990	6	2	expansion
Feb 1990	1	1	expansion
Mar 1990	3	4	expansion
Apr 1990	8	11	expansion
May 1990	13	9	expansion
Jun 1990	9	13	expansion
Jul 1990	14	25	expansion
Aug 1990	26	61	expansion
Sep 1990	63	96	expansion
Oct 1990	96	99	recession
Nov 1990	94	96	recession
Dec 1990	88	98	recession
Jan 1991	96	97	recession
Feb 1991	85	87	recession
Mar 1991	77	43	recession
Apr 1991	15	8	expansion
May 1991	7	2	expansion
Jun 1991	1	3	expansion
Jul 1991	7	11	expansion
Aug 1991	13	4	expansion
Sep 1991	2	8	expansion
Oct 1991	16	19	expansion
Nov 1991	19	36	expansion
Dec 1991	37	30	expansion
Jan 1992	25	7	expansion
Feb 1992	2	1	expansion
Mar 1992	2	1	expansion
Apr 1992	1	2	expansion
May 1992	4	4	expansion
Jun 1992	6	2	expansion
Jul 1992	2	12	expansion
Aug 1992	27	12	expansion
Sep 1992	7	2	expansion
Oct 1992	2	1	expansion
Nov 1992	2	2	expansion
Dec 1992	5	2	expansion

Jan 1993	2	2	expansion
Feb 1993	4	12	expansion
Mar 1993	17	4	expansion
Apr 1993	0	0	expansion
May 1993	3	3	expansion
Jun 1993	5	5	expansion
Jul 1993	7	4	expansion
Aug 1993	4	4	expansion
Sep 1993	6	2	expansion
Oct 1993	2	1	expansion
Nov 1993	2	1	expansion
Dec 1993	1	2	expansion
Jan 1994	5	2	expansion
Feb 1994	1	0	expansion
Mar 1994	1	0	expansion
Apr 1994	1	0	expansion
May 1994	1	1	expansion
Jun 1994	3	5	expansion
Jul 1994	7	2	expansion
Aug 1994	1	1	expansion
Sep 1994	2	0	expansion
Oct 1994	1	0	expansion
Nov 1994	2	1	expansion
Dec 1994	1	1	expansion
Jan 1995	2	4	expansion
Feb 1995	8	12	expansion
Mar 1995	14	24	expansion
Apr 1995	26	36	expansion
May 1995	35	16	expansion
Jun 1995	9	17	expansion
Jul 1995	19	5	expansion
Aug 1995	1	1	expansion
Sep 1995	1	2	expansion
Oct 1995	6	3	expansion
Nov 1995	3	2	expansion
Dec 1995	3	14	expansion
Jan 1996	25	5	expansion
Feb 1996	1	1	expansion

Mar 1996	3	1	expansion
Apr 1996	2	1	expansion
May 1996	1	0	expansion
Jun 1996	1	1	expansion
Jul 1996	5	2	expansion
Aug 1996	1	1	expansion
Sep 1996	1	1	expansion
Oct 1996	3	1	expansion
Nov 1996	1	1	expansion
Dec 1996	3	5	expansion
Jan 1997	9	2	expansion
Feb 1997	1	1	expansion
Mar 1997	1	1	expansion
Apr 1997	2	1	expansion
May 1997	2	1	expansion
Jun 1997	2	1	expansion
Jul 1997	1	0	expansion
Aug 1997	1	1	expansion
Sep 1997	2	1	expansion
Oct 1997	1	0	expansion
Nov 1997	1	1	expansion
Dec 1997	2	1	expansion
Jan 1998	1	0	expansion
Feb 1998	1	1	expansion
Mar 1998	1	1	expansion
Apr 1998	2	1	expansion
May 1998	2	3	expansion
Jun 1998	6	10	expansion
Jul 1998	13	3	expansion
Aug 1998	1	0	expansion
Sep 1998	2	1	expansion
Oct 1998	2	1	expansion
Nov 1998	2	2	expansion
Dec 1998	3	2	expansion
Jan 1999	2	1	expansion
Feb 1999	2	2	expansion
Mar 1999	4	8	expansion
Apr 1999	12	4	expansion

May 1999	2	2	expansion
Jun 1999	3	2	expansion
Jul 1999	3	1	expansion
Aug 1999	1	4	expansion
Sep 1999	10	2	expansion
Oct 1999	1	0	expansion
Nov 1999	1	0	expansion
Dec 1999	1	0	expansion
Jan 2000	0	0	expansion
Feb 2000	2	2	expansion
Mar 2000	3	1	expansion
Apr 2000	2	3	expansion
May 2000	8	5	expansion
Jun 2000	5	5	expansion
Jul 2000	7	9	expansion
Aug 2000	10	7	expansion
Sep 2000	7	11	expansion
Oct 2000	13	26	expansion
Nov 2000	29	38	expansion
Dec 2000	36	60	expansion
Jan 2001	60	66	expansion
Feb 2001	59	75	expansion
Mar 2001	71	92	recession
Apr 2001	90	96	recession
May 2001	90	94	recession
Jun 2001	87	81	recession
Jul 2001	60	76	recession
Aug 2001	72	91	recession
Sep 2001	89	78	recession
Oct 2001	52	61	recession
Nov 2001	56	58	recession
Dec 2001	52	53	recession
Jan 2002	46	26	recession
Feb 2002	17	27	expansion
Mar 2002	28	14	expansion
Apr 2002	10	5	expansion
May 2002	4	3	expansion
Jun 2002	3	7	expansion

Jul 2002	12	12	expansion
Aug 2002	12	20	expansion
Sep 2002	22	43	expansion
Oct 2002	44	48	expansion
Nov 2002	43	75	expansion
Dec 2002	75	39	expansion
Jan 2003	11	17	expansion
Feb 2003	18	35	expansion
Mar 2003	37	30	expansion
Apr 2003	24	14	expansion
May 2003	11	9	expansion

Jun 2003	9	3	expansion
Jul 2003	2	2	expansion
Aug 2003	5	3	expansion
Sep 2003	3	2	expansion
Oct 2003	2	1	expansion
Nov 2003	1	1	expansion
Dec 2003	4	3	expansion
Jan 2004	4	4	expansion

Table 8

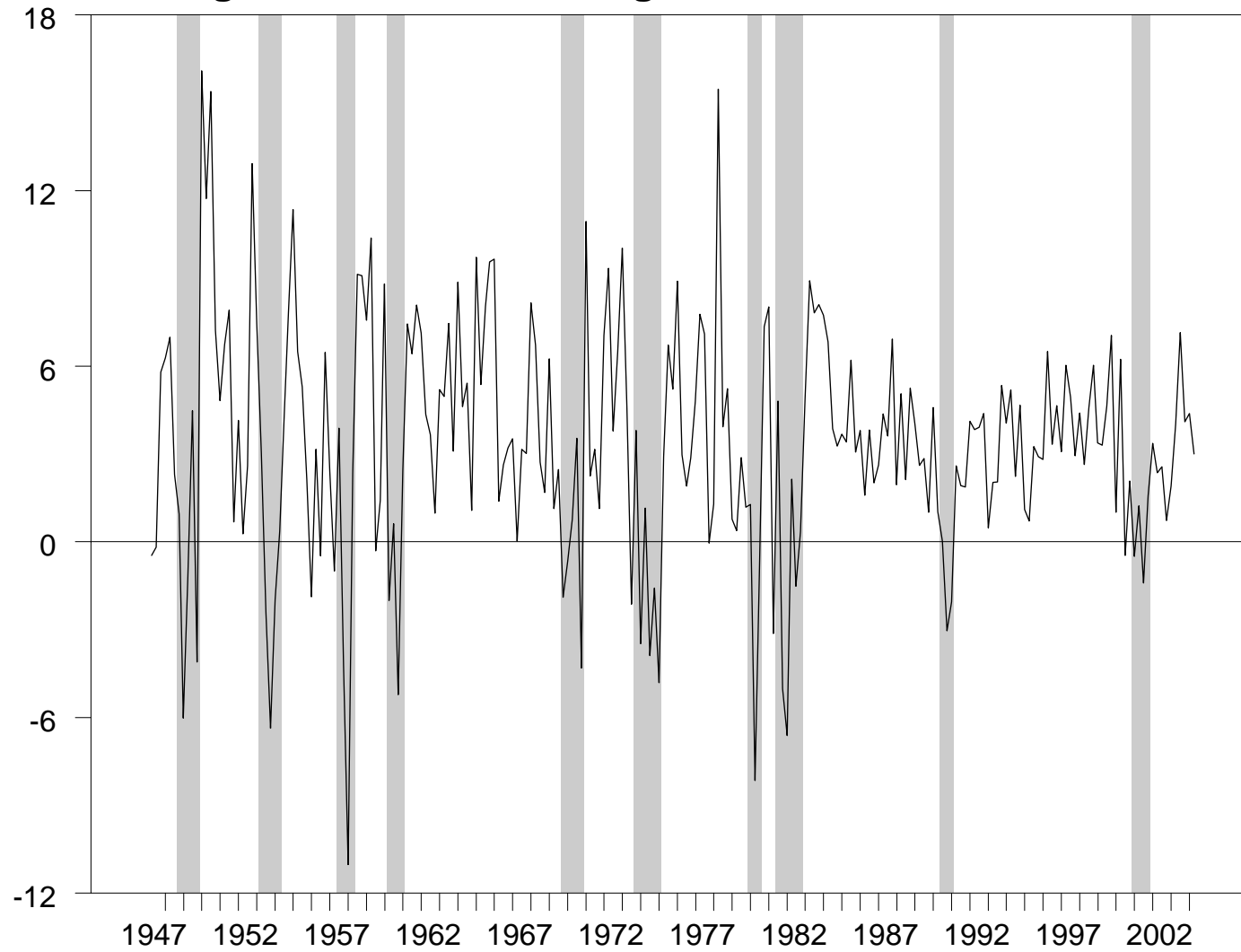
Dates of recession as determined by (1) NBER and in real time by (2) multivariate Markov-switching model estimated over recursive samples

Start Date: NBER	Start Date: MMS Model	Lead (+)/ Lag (-) Discrepancy	Start Date Announced: NBER	Start Date Available: MMS Model	Lead Announcement Date of MMS Model
Jan 1980	Mar 1980	2 months	Jun 3, 1980	June 1980	0 months
Jul 1981	Oct 1981	3 months	Jan 6, 1982	Jan 1982	0 months
Jul 1990	Oct 1990	3 months	Apr 25, 1991	Jan 1991	3 months
Mar 2001	Mar 2001	0 months	Nov 26, 2001	June 2001	5 months

End Date: NBER	End Date: MMS Model	Lead (+)/ Lag (-) Discrepancy	End Date Announced: NBER	End Date Available: MMS Model	Lead Announcement Date of MMS Model
Jul 1980	Jul 1980	0 months	Jul 8, 1981	Nov 1980	8 months
Nov 1982	Dec 1982	1 month	Jul 8, 1983	Apr 1983	3 months
Mar 1991	Mar 1991	0 months	Dec 22, 1992	Jul 1991	17 months
Nov 2001	Jan 2002	2 months	Jul 17, 2003	May 2002	14 months

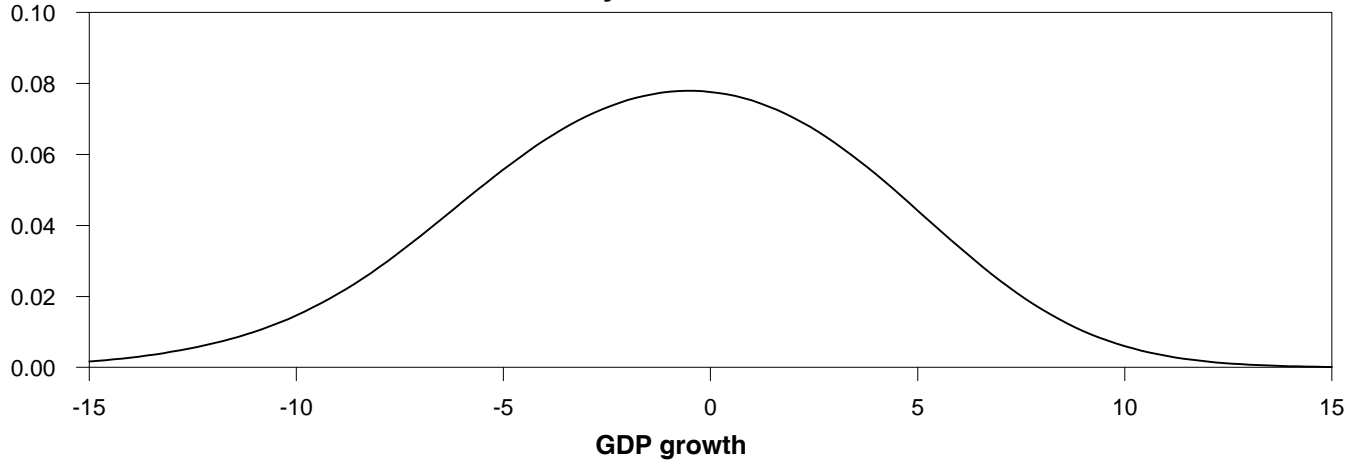
Note: Leads or lags are represented by + or -, respectively, and indicate how many months the Markov switching model anticipates or lags the NBER dating, whereas 0 indicates that the two dating systems coincide. A business cycle downturn is announced when there are two succeeding months all with probabilities greater than 0.65. An upturn is announced when there are two successive months with probabilities below 0.35.

**Figure 1. U.S. real GDP growth rates, 1947-2004**

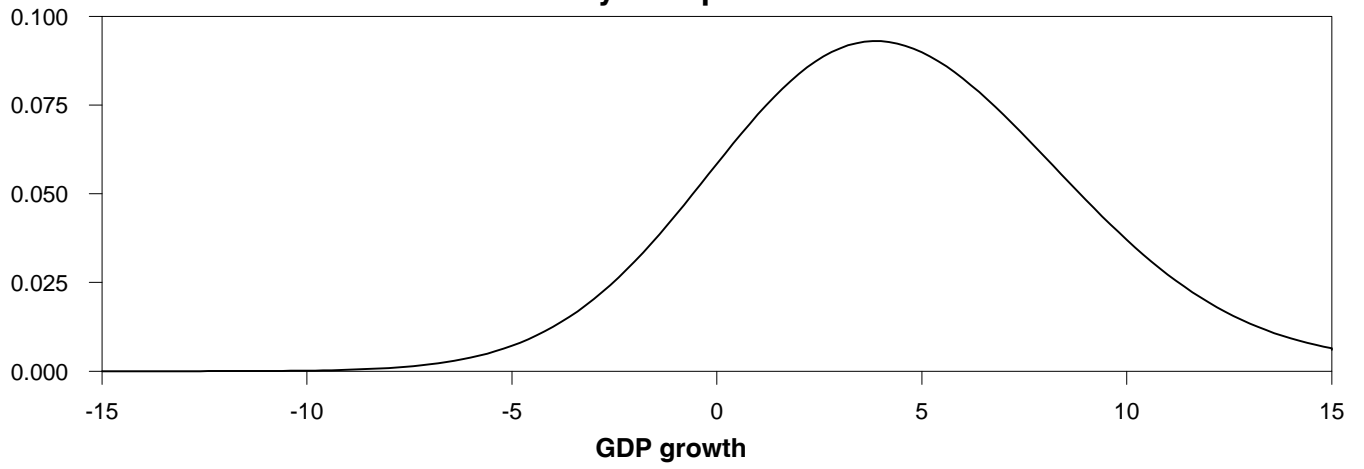


# Figure 2

## Density of recessions

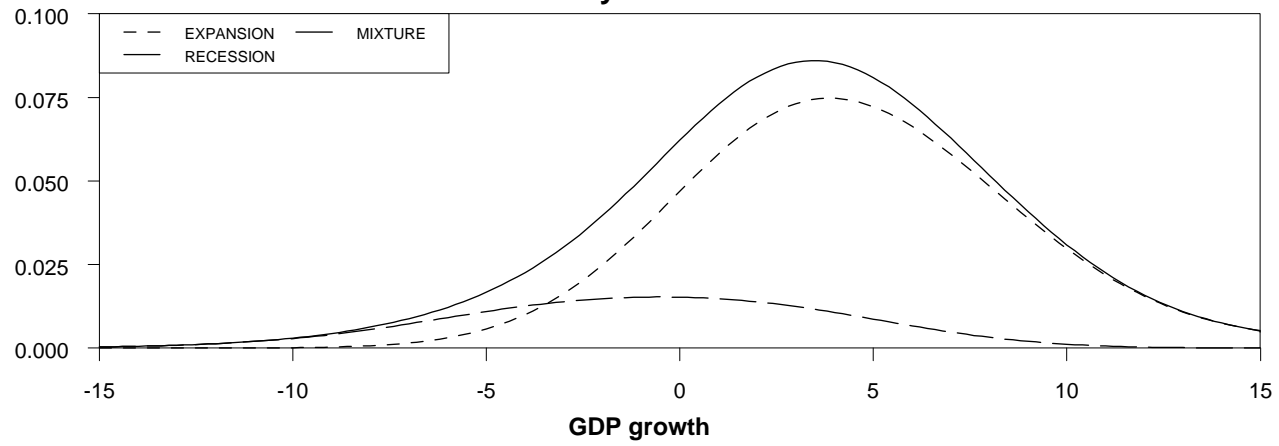


## Density of expansions

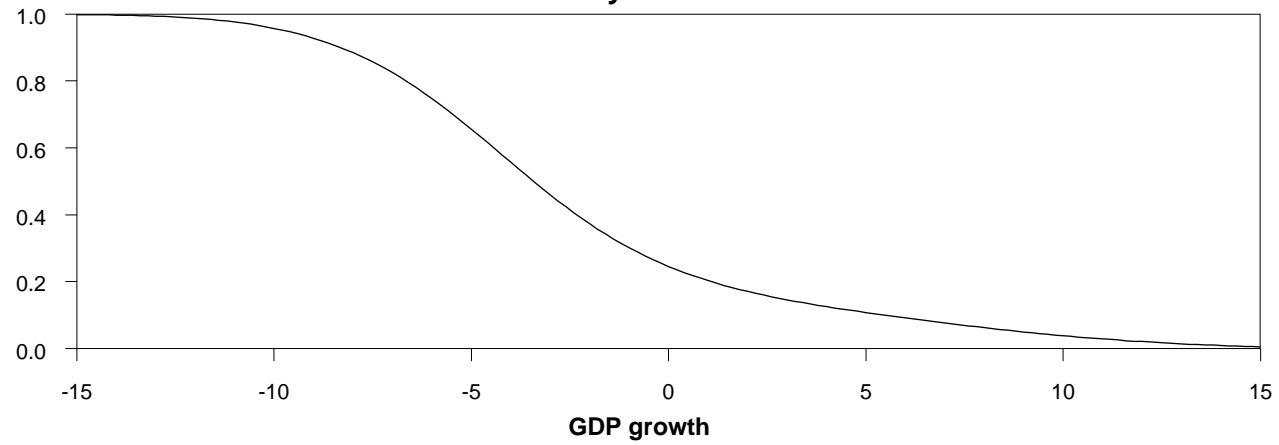


# Figure 3

## Density of mixture

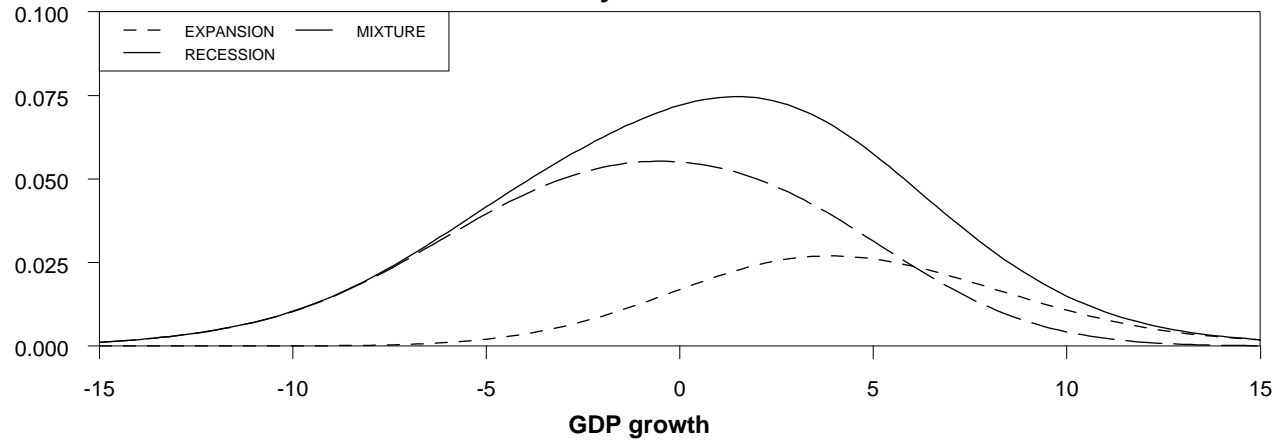


## Probability of recession

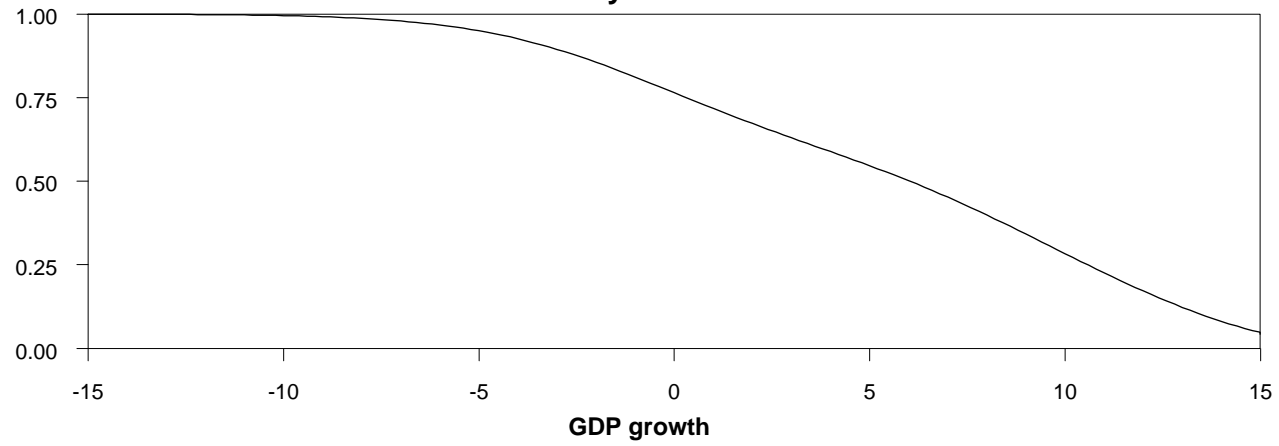


# Figure 4

## Density of mixture



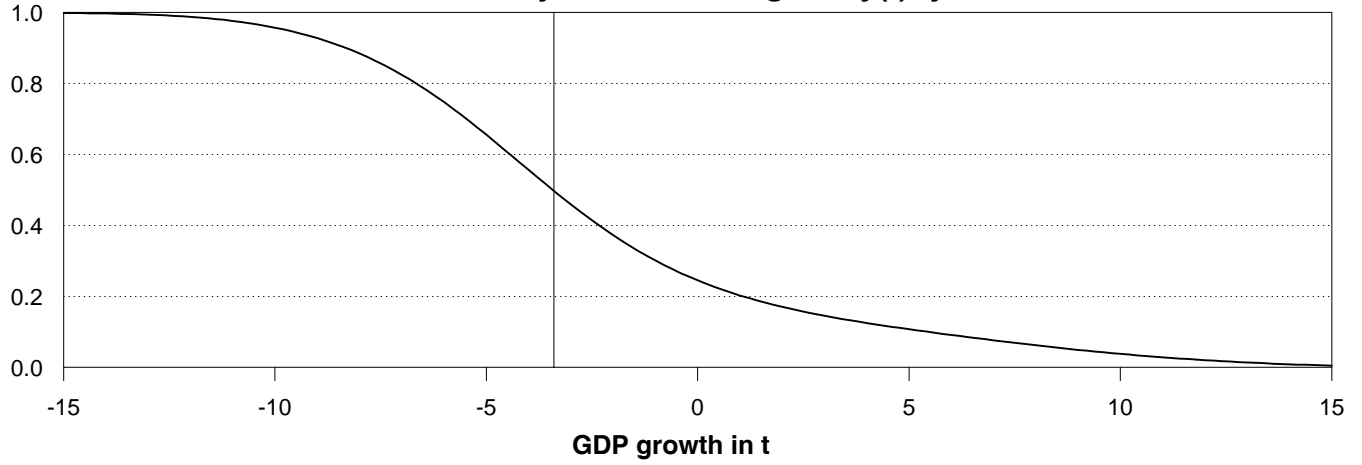
## Probability of recession





# Figure 5

## Probability of recession given $y(t)=y$



## Probability of recession given $y(t+1)=y(t)=y$

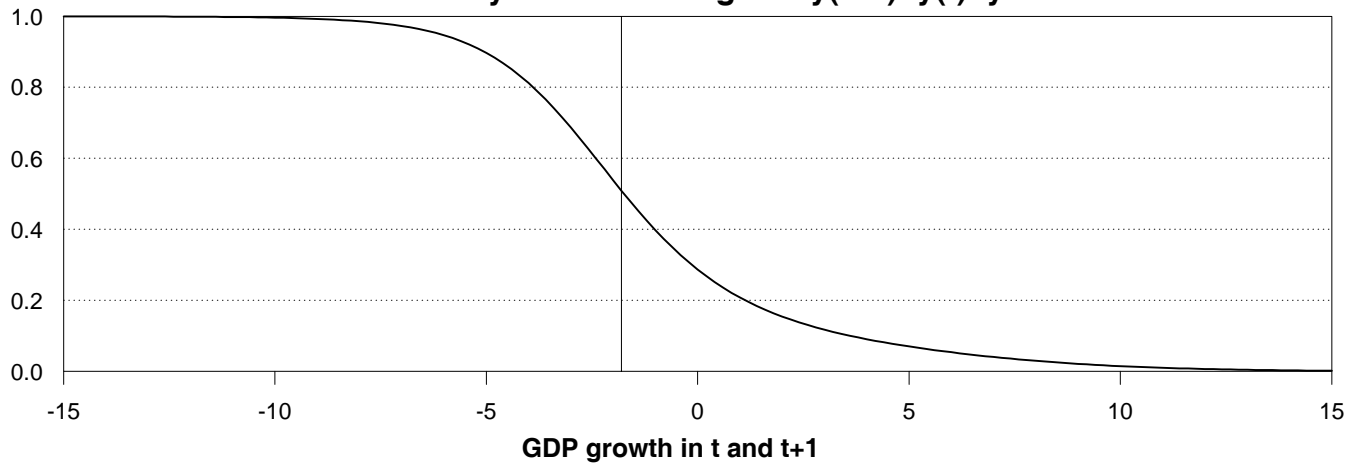


Figure 6

Probability that the U.S. economy is in recession based on revised quarterly data. Top panel: filter probabilities (inference using only GDP growth rates available at the time). Bottom panel: smoothed probabilities (inference using the full historical record). Shaded regions: dates of recessions as determined by NBER up to two years after the fact.

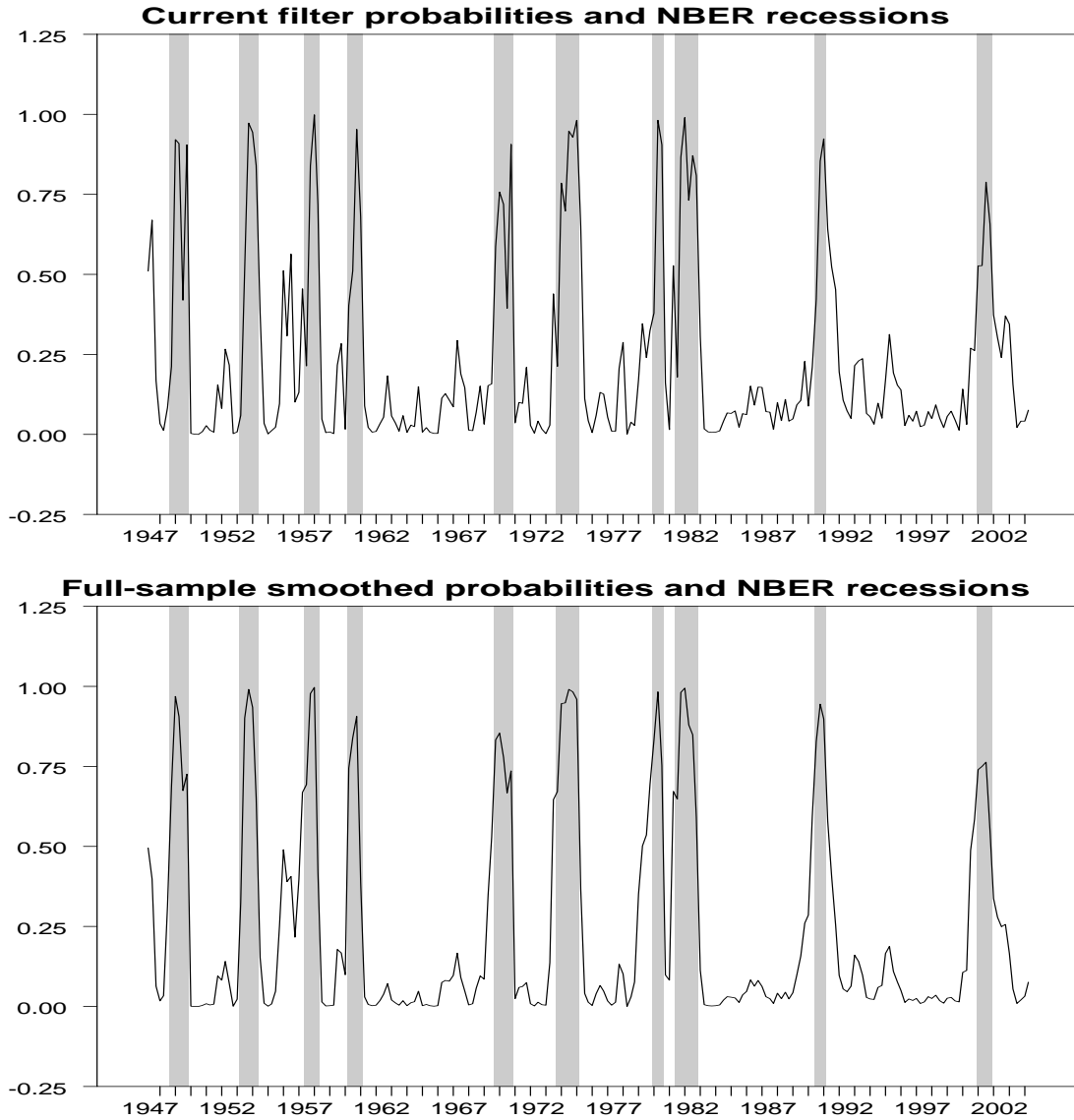


Figure 7

Probability that the U.S. economy is in recession. Top panel: one-quarter smoothed probabilities based on data as released historically as of two quarters after the indicated date (e.g., the entry for 2003:I is based on GDP growth rates for 2003:II as reported in August 2003). Bottom panel: one-quarter smoothed probabilities based on historically revised data as reported for all quarters in August 2004.

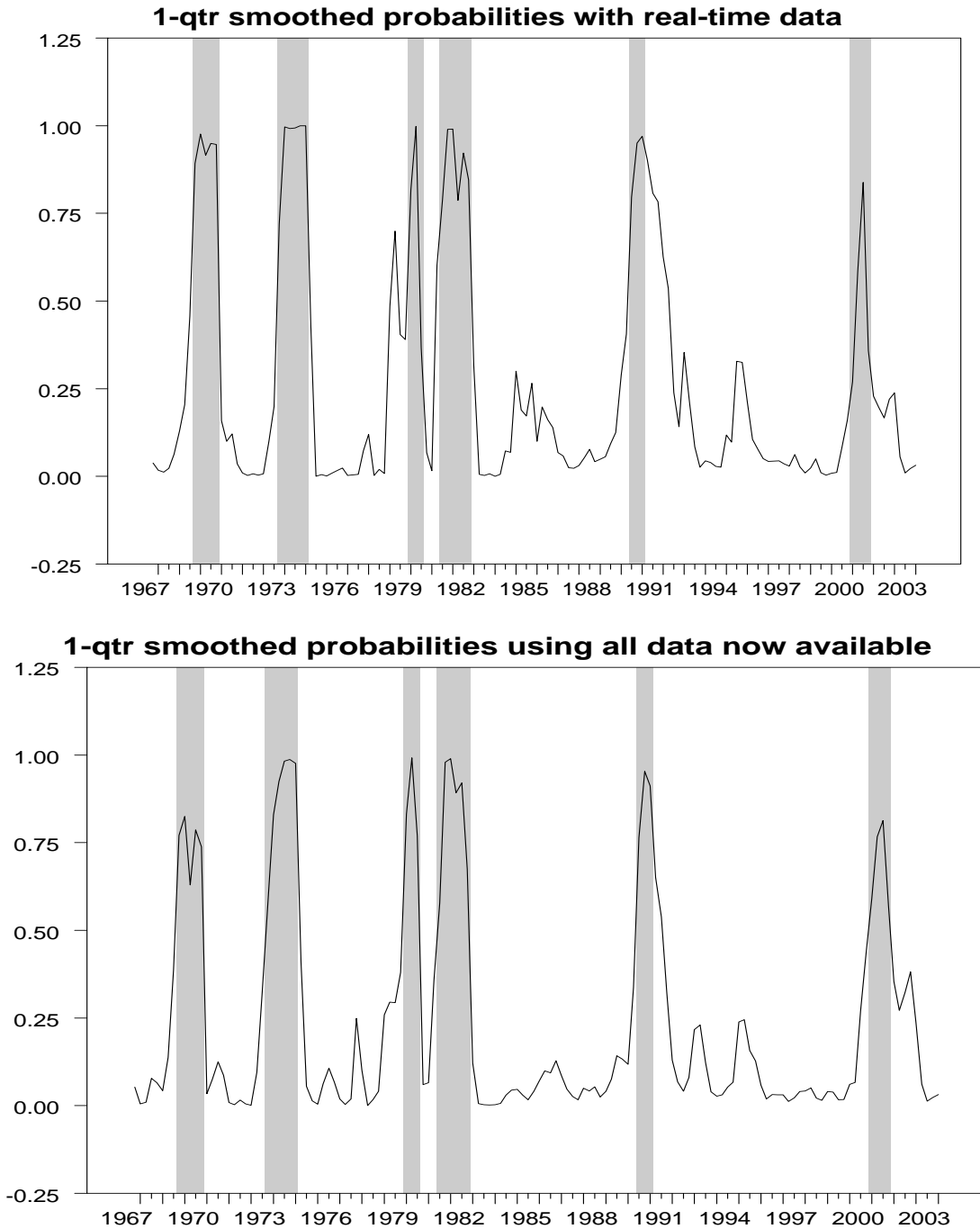


Figure 8

Full-sample smoothed probabilities and NBER recessions based on revised monthly data

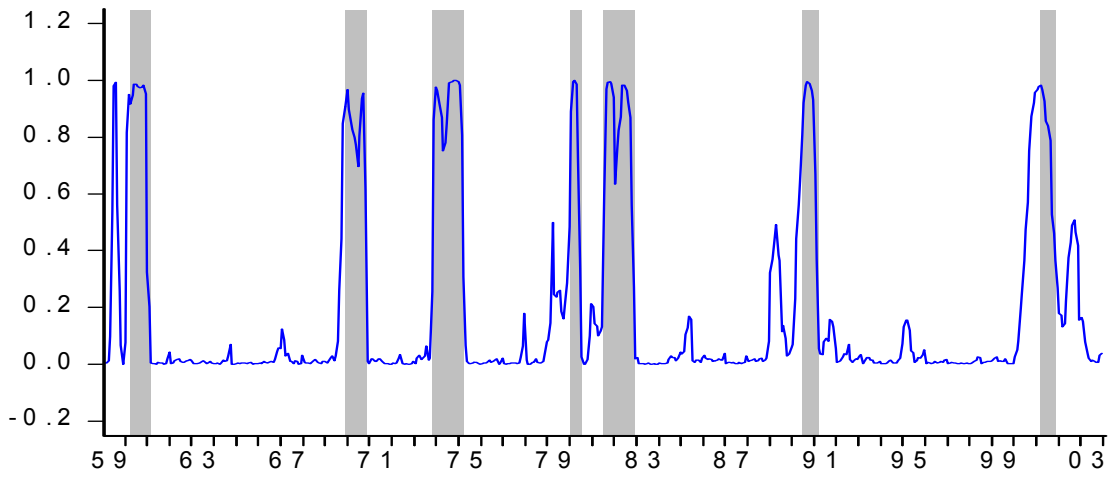


Figure 9  
Current filter probabilities and NBER recessions based on real-time monthly data after 1977:12 and revised monthly data before 1977:12

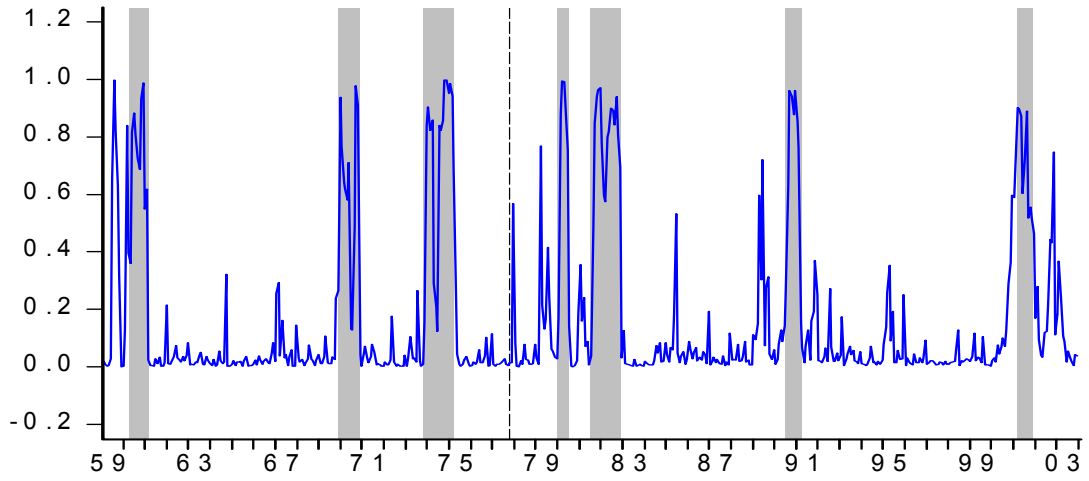


Figure 10

Current filtered probabilities and h-month-smoothed probabilities of recession (h=1, 2, 3) based on real-time monthly data

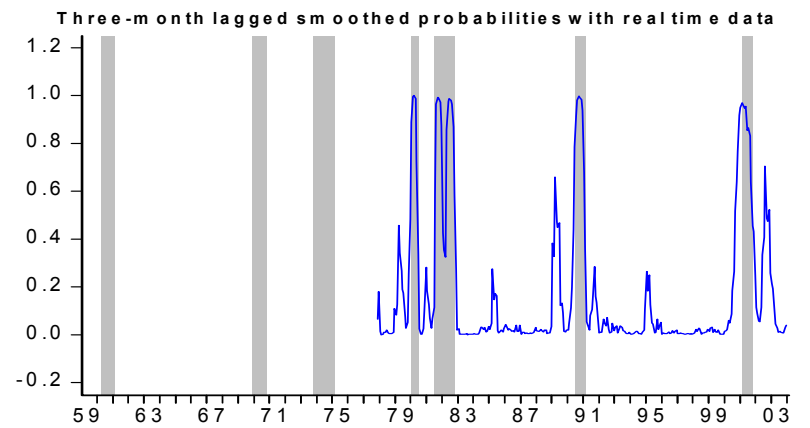
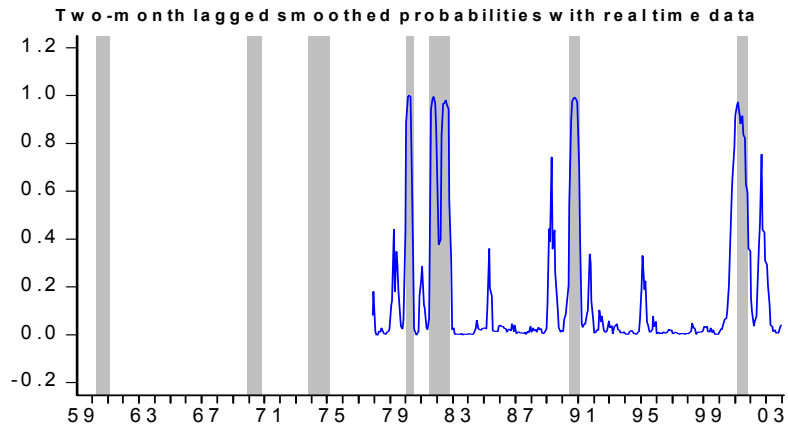
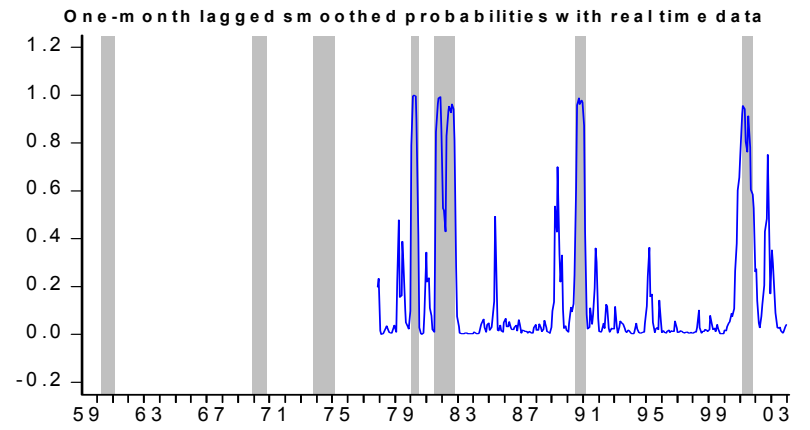
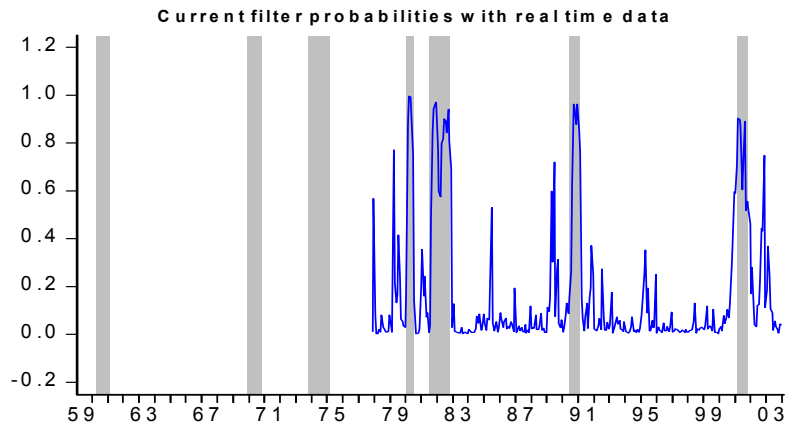
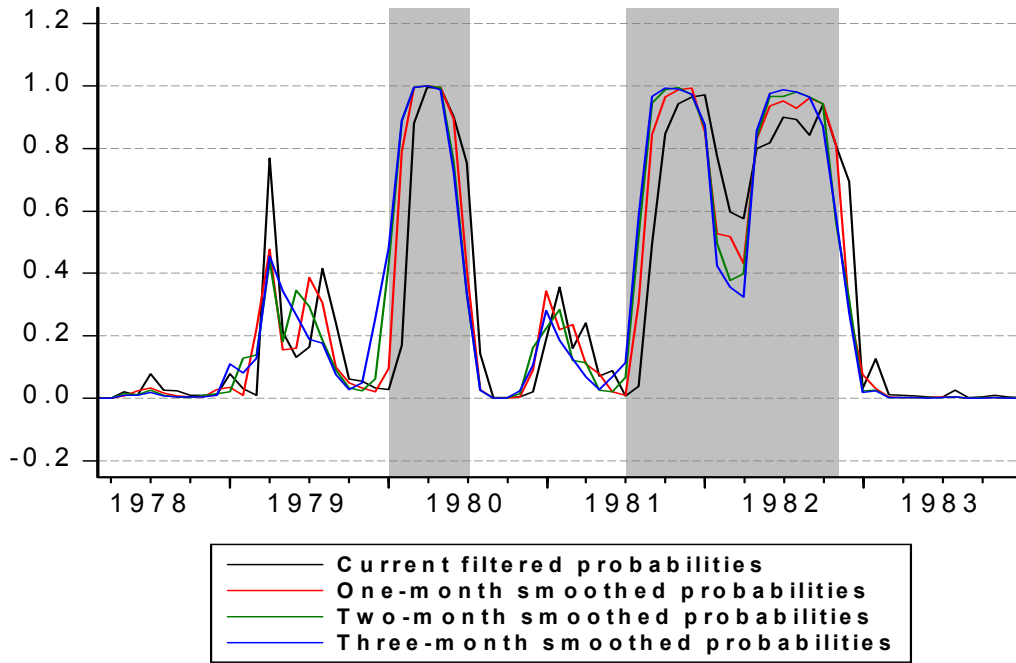


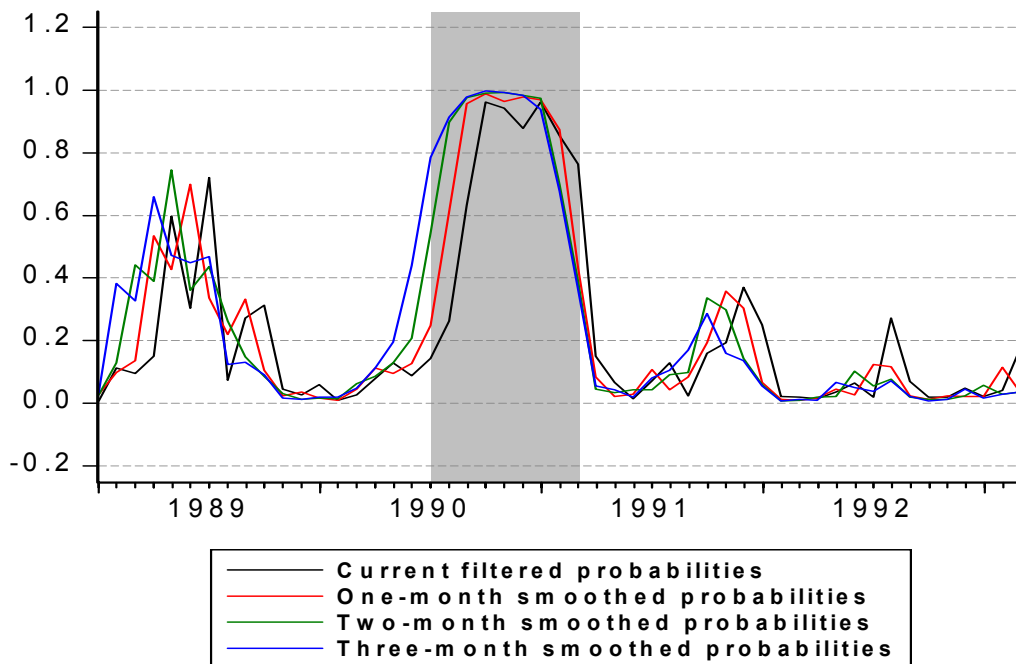
Figure 11

Filtered and smoothed probabilities with real-time monthly data over recursive samples

1980 and 1981-82 Recessions



1990-1991 Recession



# 2001 Recession

