Answer key for the midterm in 2011

1a.)

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix}
= \begin{bmatrix}
  n_M & 0 \\
  0 & n_F
\end{bmatrix}^{-1}
\begin{bmatrix}
  \sum y_i x_{1i} \\
  \sum y_i x_{2i}
\end{bmatrix}
\]

\(b_1\) is exactly equal to the average wage earned by male workers in the sample.

b.) \(s^2 = (T - 2)^{-1} \sum (y_i - x_{1i} b_1 - x_{2i} b_2)^2\)

c.)

\[
\frac{(b_1 - b_2)^2}{s^2 \begin{bmatrix}
  1 & -1 \\
  -1 & 1
\end{bmatrix} \begin{bmatrix}
  n^{-1}_M & 0 \\
  0 & n^{-1}_F
\end{bmatrix} \begin{bmatrix}
  1 \\
  -1
\end{bmatrix}} = \frac{(b_1 - b_2)^2}{s^2(n^{-1}_M + n^{-1}_F)}
\]

d.) One could not add a constant term because \(x_3 = x_{1r} + x_{2r}\) and \((X'X)^{-1}\) would not exist.

2a.) \(F(m, n)\)

b.) \((1/m)\) times a \(\chi^2(m)\)

3a.) \(\beta = (X'X)^{-1}X'y\)

b.) \(\hat{\beta} = \beta_0\) (also correct is \(\hat{\beta} = [E(x, x')]^{-1}E(x, y)\))

c.) \(\hat{\beta} = \{X'[V(X)]^{-1}X'y\}^{-1}X'[V(X)]^{-1}y\) (also correct is \(\hat{\beta} = (\hat{X}'\hat{X})^{-1}\hat{X}'\hat{y}\) where row \(t\) of \(\hat{X}\) is \(x_t/\sqrt{x_{1t}}\) and element \(t\) of \(\hat{y}\) is \(y_t/\sqrt{x_{1t}}\)

4a.)

\[
b = (X'X)^{-1}X'y
\]

\[
s^2 = (T - k)^{-1} \sum (y_i - x'_i b)^2
\]

\[
G = \left. \frac{\partial g(\beta)}{\partial \beta} \right|_{\beta = b}
\]

\[
[g(b)]' \left\{s^2G(X'X)^{-1}G'\right\}^{-1} [g(b)]
\]

\(m\) degrees of freedom

b.)

\[
S = \sum e_i^2 x_i x'_i
\]

\[
[g(b)]' \left\{s^2G(X'X)^{-1}S(X'X)^{-1}G'\right\}^{-1} [g(b)]
\]

\(m\) degrees of freedom