Answer key for the final exam in 2016

1.) Calculate OLS residuals $\hat{u}_t = y_t - x_t b$ for $b = (\sum x_t^2)^{-1}(\sum x_t y_t)$ and regress $\hat{u}_t^2 = \gamma_0 + \gamma_1 x_t^2 + v_t$. If $T \bullet R^2$ from this second regression is bigger than critical value for $\chi^2(1)$ (namely, if $TR^2 > 3.84$), then reject null hypothesis of homoskedastic.

2.) It is true enough that $E(x_t u_t) = E(u_t) = 0$ (so instrument is valid) and that $E(x_t z_t) = E(z_t) \neq 0$ (so instrument is relevant). However, we would almost always have included a constant in the original regression (e.g., as first element of the vector $\mathbf{z}_t$), and the instrument $x_t = 1$ has already been used to estimate the constant term. We need an additional instrument besides the constant to estimate the effects of any variables.

3a.) $\hat{\beta}_{OLS} = (\sum z_t^2)^{-1} \sum (z_t y_t) = \beta + (\sum z_t^2)^{-1} \sum (z_t y_t) \xrightarrow{p} \beta + \sigma_{zz}^{-1}\sigma_{zu}$. Since this is greater than β, OLS will overestimate the effect of z_t on y_t.

b.) valid: $E(x_t u_t) = 0$; relevant: $E(x_t z_t) \neq 0$.

$$\begin{aligned} \hat{\beta}_{2SLS} &= \left[\left(\sum z_t x_t\right)\left(\sum x_t^2\right)^2\left(\sum x_t z_t\right) \right]^{-1} \left[\left(\sum z_t x_t\right)\left(\sum x_t^2\right)^2\left(\sum x_t y_t\right) \right] \\ &= \left(\sum z_t x_t\right)^{-1}\left(\sum x_t y_t\right). \end{aligned}$$

c.) Let $\hat{V}_{2SLS} = \hat{\sigma}^2 \left[(\sum z_t x_t)(\sum x_t^2)^2(\sum x_t z_t) \right]^{-1}$ for $\hat{\sigma}^2 = (T-1)^{-1}\sum (y_t - z_t \hat{\beta}_{2SLS})^2$. Then t-stat is $\hat{\beta}_{2SLS}/\sqrt{\hat{V}_{2SLS}}$; reject if this is greater than 2 in absolute value.

d.) Do OLS t-test for coefficient in regression of y_t on x_t, namely $\hat{\pi}/\sqrt{\hat{V}_\pi}$ for $\hat{\pi} = (\sum x_t^2)^{-1}\sum (x_t y_t)$ and $\hat{V}_\pi = s^2 (\sum x_t^2)^{-1}$ with $s^2 = (T-1)^{-1}\sum (y_t - x_t \hat{\pi})^2$. Reject if this is greater than ± 2. AR has reasonable small-sample size when instrument is weak.

e.) Instead of OLS t-test of $\pi = 0$ use White test, namely $\hat{\pi}/\sqrt{\tilde{V}_\pi}$ for $\tilde{V}_\pi = (\sum x_t^2)^{-2}(\sum x_t^2 \hat{v}_t^2)$ for $\hat{v}_t = y_t - x_t \hat{\pi}$.

4ai.) $\hat{\mu}$ solves $T^{-1}\sum h(\hat{\mu}, w_t) = 0$ or

$$\begin{aligned} T^{-1}\sum [\exp(\hat{\mu} + 0.5) - x_t] &= 0 \\ \exp(\hat{\mu} + 0.5) &= \bar{x} \\ \hat{\mu} &= \log(\bar{x}) - 0.5 \end{aligned}$$

aii.)

$$S = E[\exp(\hat{\mu} + 0.5) - x_t]^2 = Var(x_t) = (e-1)\exp(2\mu + 1)$$

$$D = E\frac{\partial h(\theta, w_t)}{\partial \theta^2} = \exp(\mu + 0.5)$$

aiii.)

$$DS^{-1}D = (e-1)^{-1}$$

$$\sqrt{T}(\hat{\mu}_{GMM} - \mu) \xrightarrow{L} N(0, e-1)$$

bi.)

$$\frac{\partial \log f(x_t)}{\partial \mu} = \log x_t - \mu$$

which has expectation zero since $\log x_t \sim N(\mu,1)$ and $\hat{\mu}_{MLE} = T^{-1}\sum \log x_t$

bii.)

$$S = E(\log x_t - \mu)^2 = 1 \quad \text{since } \log x_t \sim N(\mu,1)$$

$$D = \frac{\partial(\log x_t - \mu)}{\partial\mu} = -1$$

biii.) $DS^{-1}D = 1$ so $\sqrt{T}\,(\hat{\mu}_{MLE} - \mu) \xrightarrow{L} N(0,1)$

c.) Since $e = 2.781828$, $e - 1 > 1$ and $\mathrm{Var}(\hat{\mu}_{GMM}) > \mathrm{Var}(\hat{\mu}_{MLE})$. This is to be expected since MLE should be asymptotically efficient whereas in general this need not be true of *GMM*.