Answers to Econ 220B Final Exam, Winter 2013

1.)

a.) Reject $\beta = 0$ since $t > 2$. Note $t = \hat{\beta}/\hat{\sigma}_{\hat{\beta}}$ so $\hat{\sigma}_{\hat{\beta}} = 0.8/4 = 0.2$ and t stat for $\beta = 1$ is $(0.8 - 1.0)/0.2 = -1$ so do not reject $\beta = 1$.

b.) $\hat{\beta}_0 = 0$, $\hat{\beta}_1 = 0.80$ and t-statistic for β_1 is numerically identical to that from (1).

c.) $\hat{\beta}_1 = \left(T^{-1}\sum_{t=1}^{T}(x_t - \overline{x})^2\right)^{-1}\left(T^{-1}\sum_{t=1}^{T}(x_t - \overline{x})(y_t - \overline{y})\right) \xrightarrow{p} \sigma_{xx}^{-1}\sigma_{xy} = \pi_1$. Note $\pi_1$is the value of β_1 for which the following expression is minimized,

$$E(y_t - \beta_0 - \beta_1 x_t)^2,$$

that is, π_1 is the coefficient to we would multiply father's income by if we wanted to obtain the optimal linear forecast of the child's income.

d.) Regress $\hat{e}_t^2 = \alpha_0 + \alpha_1\tilde{x}_t^2 + v_t$ and compare T times the R^2 of this regression with a $\chi^2(1)$ distribution, that is, if $66R^2 > 4$, reject the null of homoskedasticity.

e.) (i) Use White standard errors instead of OLS standard errors. This would result in identical estimate of β_1 (namely 0.80) but a different t statistic.

(ii) Use feasible GLS, e.g., regress $\hat{e}_t^2 = \alpha_0 + \alpha_1\tilde{x}_t^2 + v_t$ and let $\hat{q}_t = \hat{\alpha}_0 + \hat{\alpha}_1\tilde{x}_t^2$ denote fitted values. Then regress $y_t^* = \tilde{y}_t/\hat{q}_t^{1/2}$ on $x_t^* = \tilde{x}_t/\hat{q}_t^{1/2}$. This will change both the coefficient estimate $\hat{\beta}_1$ and the t statistic.

f.)

$$1 - R^2 = \sum_{t=1}^{T} e_t^2 / \sum_{t=1}^{T}\tilde{y}_t^2 = SSR_1/SSR_0$$

$$\begin{aligned}
F &= (T-2)(SSR_0 - SSR_1)/SSR_1 \\
&= (T-2)\left(\frac{SSR_0}{SSR_1} - 1\right) \\
&= (T-2)\left(\frac{1}{1-R^2} - 1\right) \\
&= (T-2)\left(\frac{R^2}{1-R^2}\right)
\end{aligned}$$

But F is the square of the t statistic and $T = 66$, so

$$\begin{aligned}
4^2 &= 64\left(\frac{R^2}{1-R^2}\right) \\
1 - R^2 &= 4R^2 \\
R^2 &= 0.2.
\end{aligned}$$

2.) a.) yes. $d_{t1} + d_{t2} + d_{t3} = 1$, but since there is no separate constant term in the regression, this is not a problem.

b.) if person t has high ability and motivation, then ε_t is likely high and that person likely chose to get more schooling. Hence ε_t and s_t are positively correlated, biasing β upwards.

c.) Need instrument correlated with s_t but uncorrelated with ε_t, such as person $t's$ date of birth within the calendar year.

3.)
a.) yes
b.) no, need $E(\mathbf{x}_t\mathbf{z}_t')$ has full rank k
c.)
i.) $\boldsymbol{\theta} = \boldsymbol{\beta}$
ii.) $\mathbf{w}_t = (y_t, \mathbf{z}_t', \mathbf{x}_t')'$
iii.) $\mathbf{h}(\boldsymbol{\theta}, \mathbf{w}_t) = \mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\beta})$
iv.) $\mathbf{Y}_T = (\mathbf{w}_1', \mathbf{w}_2', \ldots, \mathbf{w}_T')'$
v.) $\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T) = T^{-1}\sum_{t=1}^T \mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\beta})$
vi.) $\hat{\mathbf{D}}' = -T^{-1}\sum_{t=1}^T \mathbf{x}_t\mathbf{z}_t'$
vii.) $\mathbf{S} = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_1'$
viii.) $\hat{\boldsymbol{\theta}}_{GMM} = \left(\sum_{t=1}^T \mathbf{x}_t\mathbf{z}_t'\right)^{-1}\left(\sum_{t=1}^T \mathbf{x}_t y_t\right)$
ix.) $\mathbf{V} = \{E(\mathbf{z}_t\mathbf{x}_t')(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_1')^{-1}E(\mathbf{x}_t\mathbf{z}_t')\}^{-1}$
x.) $\hat{\mathbf{V}} = \left\{(T^{-1}\sum_{t=1}^T \mathbf{z}_t\mathbf{x}_t')(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_1')^{-1}(T^{-1}\sum_{t=1}^T \mathbf{x}_t\mathbf{z}_t')\right\}^{-1}$
d.)

$$T(\mathbf{R}\hat{\boldsymbol{\theta}} - \mathbf{r})'[\mathbf{R}\hat{\mathbf{V}}\mathbf{R}']^{-1}(\mathbf{R}\hat{\boldsymbol{\theta}} - \mathbf{r}) \xrightarrow{L} \chi^2(m)$$

e.)
i.) Smallest eigenvalue of $\boldsymbol{\Sigma}_{xz}\boldsymbol{\Sigma}_{zx}$ is positive but near zero for $\boldsymbol{\Sigma}_{xz} = E(\mathbf{x}_t\mathbf{z}_t')$
ii.) Estimate of $\boldsymbol{\beta}$ may be badly behaved in small samples and distribution of statistic proposed in (d) may be far from $\chi^2(m)$ in small samples
iii.) Let $\tilde{y}_t = y_t - \mathbf{z}_t'\boldsymbol{\beta}_0$ and do OLS regression of $\tilde{y}_t = \mathbf{x}_t'\boldsymbol{\alpha} + v_t$:

$$\hat{\boldsymbol{\alpha}} = \left(\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(\sum_{t=1}^T \mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\beta}_0\right)$$

and test $\boldsymbol{\alpha} = \mathbf{0}$ using appropriate variance matrix:

$$T\hat{\boldsymbol{\alpha}}'\left[\left(T^{-1}\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_1')^{-1}\left(T^{-1}\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t'\right)\right]\hat{\boldsymbol{\alpha}} \xrightarrow{L} \chi^2(k).$$