Econ 220B, Winter 2010

Answers to final exam

1.) a.) $\text{plim}(\mathbf{b}) = \mathbf{\Sigma}_{xx}^{-1}\mathbf{\Sigma}_{xy}$

b.) (1) if for the $\boldsymbol{\beta}$ we are interested in it is the case that $E(y_t - \mathbf{x}_t'\boldsymbol{\beta})\mathbf{x}_t = \mathbf{0}$, then OLS is allowing us to estimate this $\boldsymbol{\beta}$. (2) If our goal is to minimize the mean squared error of a forecast of y_t using a linear function of $\mathbf{x}_t$, $\hat{y}_t = \mathbf{x}_t'\boldsymbol{\beta}$, then the plim of the regression is the optimal $\boldsymbol{\beta}$ to use.

2a.) Let $\hat{\boldsymbol{\alpha}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$, $s^2 = (T-3)^{-1}\Sigma(y_t - \mathbf{x}_t'\hat{\boldsymbol{\alpha}})^2$, and $\mathbf{1}' = (1,1,1)$. Then reject H_0 if

$$\frac{\mathbf{1}'\hat{\boldsymbol{\alpha}} - 1}{\sqrt{s^2\mathbf{1}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{1}}} < z$$

where z is the number such that the probability that a Student t variable with $T-3$ degrees of freedom is less than z is equal to 0.05, so z is about -1.66. This is an exact small-sample test under the assumptions.

b.) Let $\mathbf{S} = \Sigma\hat{u}_t^2\mathbf{x}_t\mathbf{x}_t'$ and reject H_0 if

$$\frac{(\mathbf{1}'\hat{\boldsymbol{\alpha}} - 1)^2}{s^2\mathbf{1}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{S}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{1}} > z$$

where $z = 3.84$, the number such that the probability that at $\chi^2(1)$ variable exceeds z is 0.05. This test is only a large-sample approximation under the stated conditions.

c.) A constant term should have been included in the regression– could have tested with simple t test. One would also suspect serial correlation or even spurious regression. Could test for this by regressing $\hat{u}_t$ on $\mathbf{x}_t$ and $\hat{u}_{t-1}$ and calculating a t-test that the coefficient on $\hat{u}_{t-1}$ is zero. Would also worry about endogeneity, e.g., when productivity is unusually high ($\varepsilon_t > 0$), the firm probably wants to hire more labor (ℓ_t positively correlated with ε_t). To test for this we'd need an instrument (something correlated with ℓ_t but uncorrelated with ε_t).

3.) a.) no

b.) $\mathbf{g}(\boldsymbol{\theta}, \mathbf{Y}_T) = T^{-1}\Sigma\mathbf{x}_t(y_t - \mathbf{z}_t'\boldsymbol{\theta})$, $\mathbf{S} = \sigma^2\mathbf{\Sigma}_{xx}$, $\mathbf{D}' = -\mathbf{\Sigma}_{xz}$

c.) $\hat{\mathbf{S}} = \hat{\sigma}^2T^{-1}\Sigma\mathbf{x}_t\mathbf{x}_t'$, $\hat{\sigma}^2 = T^{-1}\Sigma(y_t - \mathbf{z}_t'\hat{\boldsymbol{\theta}})^2$, $\hat{\mathbf{D}} = -T^{-1}\Sigma\mathbf{z}_t\mathbf{x}_t'$

d.) GMM FOC gives

$$\left(-T^{-1}\Sigma\mathbf{x}_t\mathbf{z}_t'\right)'\hat{\sigma}^{-2}\left(T^{-1}\Sigma\mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(T^{-1}\Sigma\mathbf{x}_t(y_t - \mathbf{z}_t'\hat{\boldsymbol{\theta}}\right) = \mathbf{0}$$

which on rearranging is 2SLS.

e.)

$$\begin{aligned} J &= T\left(T^{-1}\Sigma \mathbf{x}_t \hat{u}_t\right)' \left(\hat{\sigma}^{-2} T^{-1} \Sigma \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \left(T^{-1}\Sigma \mathbf{x}_t \hat{u}_t\right) \\ &= \frac{\left(\Sigma \hat{u}_t \mathbf{x}_t'\right)\left(\Sigma \mathbf{x}_t \mathbf{x}_t'\right)^{-1}\left(\Sigma \mathbf{x}_t \hat{u}_t\right)}{\hat{\sigma}^2} \approx \chi^2(r-a) \end{aligned}$$

f.) Use

$$\frac{\left(\Sigma u_t \mathbf{x}_t'\right)\left(\Sigma \mathbf{x}_t \mathbf{x}_t'\right)^{-1}\left(\Sigma \mathbf{x}_t u_t\right)}{\hat{\sigma}^2}$$

for $u_t = y_t - \mathbf{z}_t' \boldsymbol{\theta}_0$. We have here conditions for m.d.s. CLT,

$$T^{-1/2} \Sigma \mathbf{x}_t u_t \xrightarrow{L} N(\mathbf{0}, \sigma^2 \mathbf{\Sigma}_{xx})$$

and so

$$\begin{aligned} &\frac{\left(T^{-1/2}\Sigma u_t \mathbf{x}_t'\right)\left(T^{-1}\Sigma \mathbf{x}_t \mathbf{x}_t'\right)^{-1}\left(T^{-1/2}\Sigma \mathbf{x}_t u_t\right)}{\hat{\sigma}^2} \\ &\xrightarrow{L} T^{-1/2}\Sigma u_t \mathbf{x}_t' (\sigma^2 \mathbf{\Sigma}_{xx})^{-1}\left(T^{-1/2}\Sigma \mathbf{x}_t u_t\right) \\ &\xrightarrow{L} \chi^2(r) \end{aligned}$$