Macroeconomics and ARCH*

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Abstract

Although ARCH-related models have proven quite popular in finance, they are less frequently used in macroeconomic applications. In part this may be because macroeconomists are usually more concerned about characterizing the conditional mean rather than the conditional variance of a time series. This paper argues that even if one's interest is in the conditional mean, correctly modeling the conditional variance can still be quite important, for two reasons. First, OLS standard errors can be quite misleading, with a "spurious regression" possibility in which a true null hypothesis is asymptotically rejected with probability one. Second, the inference about the conditional mean can be inappropriately influenced by outliers and high-variance episodes if one has not incorporated the conditional variance directly into the estimation of the mean, and infinite relative efficiency gains may be possible. The practical relevance of these concerns is illustrated with two empirical examples from the macroeconomics literature, the first looking at market expectations of future changes in Federal Reserve policy, and the second looking at changes over time in the Fed's adherence to a Taylor Rule.

1 Introduction.

One of the most influential econometric papers of the last generation was Engle's (1982a) introduction of autoregressive conditional heteroskedasticity (ARCH) as a tool for describing how the conditional variance of a time series evolves over time. The ISI Web of Science lists over 2000 academic studies that have cited this article, and simply reciting the acronyms for the various extensions of Engle's theme involves a not insignificant commitment of paper (see Table 1, or the more detailed glossary in Bollerslev, 2008).

The vast majority of empirical applications of ARCH models have studied financial time series such as stock prices, interest rates, or exchange rates. To be sure, there have also been a number of interesting applications of ARCH to macroeconomic questions. Lee, Ni, and Ratti (1995) noted that the conditional volatility of oil prices, as captured by a GARCH model, seems to matter for the magnitude of the effect on GDP of a given movement in oil prices, and Elder and Serletis (2006) use a vector autoregression with GARCH-in-mean elements to describe the direct consequences of oil-price volatility for GDP. Grier and Perry (2000) and Fountas and Karanasos (2007) use such models to conclude that inflation and output volatility also can depress real GDP growth, while Servén (2003) studied the effects of uncertainty on investment spending.

However, despite these interesting applications, studying volatility has traditionally been a much lower priority for macroeconomists than for researchers in financial markets because the former's interest is primarily in describing the first moments. There seems to be an assumption among many macroeconomists that, if your primary interest is in the first moment, ARCH has little relevance apart from possible GARCH-M effects.

The purpose of this paper is to suggest that even if our primary interest is in estimating the conditional mean, having a correct description of the conditional variance can still be quite important, for two reasons. First, hypothesis tests about the mean in a model in which the variance is misspecified will be invalid. Second, by incorporating the observed features of the heteroskedasticity into the estimation of the conditional mean, substantially more efficient estimates of the conditional mean can be obtained.

Section 2 develops the theoretical basis for these claims, illustrating the potential magnitude of the problem with a small Monte Carlo study and explaining why the popular White (1980) or Newey-West (1987) corrections may not fully correct for the inference problems introduced by ARCH. The subsequent sections illustrate the practical relevance of these concerns using two examples from the macroeconomics literature. The first application concerns measures of what the market expects the U.S. Federal Reserve's next move to be, and the second explores the extent to which U.S. monetary policy today is following a fundamentally different rule from that observed thirty years ago.

I recognize that it may require more than these limited examples to persuade macroeconomists to pay more attention to ARCH. Another thing I learned from Rob Engle is that, in addition to coming up with a great idea, it doesn't hurt if you also have a catchy acronym that people can use to describe what you're talking about. After all, where would we be today if we all had to pronounce "autoregressive conditional heteroskedasticity" every time we wanted to discuss these issues? However, Table 1 reveals that the acronyms one might logically use for "Macroeconomics and ARCH" seem already to be taken. "MARCH", for example, is already used (twice), as is "ARCH-M".

Fortunately, Engle and Manganelli (2004) have shown us that it's also OK to mix upper- and lower-case letters, picking and choosing handy vowels or consonants so as to come up with something catchy, as in "CAViaR" (Conditional Autoregressive Value at Risk). In that spirit, I propose to designate "Macroeconomics and ARCH" as "McARCH." Maybe not a new product so much as new packaging.

Herewith, then, discussion of the relevance of McARCH.

2 GARCH and inference about the mean.

We can illustrate some of the issues with the following simple model:

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t \tag{1}$$

$$u_t = \sqrt{h_t} v_t \tag{2}$$

$$h_t = \kappa + \alpha u_{t-1}^2 + \delta h_{t-1} \qquad \text{for } t = 1, 2, ..., T$$

$$h_0 = \kappa/(1 - \alpha - \delta)$$

$$v_t \sim \text{i.i.d. } N(0, 1). \tag{3}$$

Bollerslev (1986, pp. 312-313) showed that if

$$3\alpha^2 + 2\alpha\delta + \delta^2 < 1, \tag{4}$$

then the noncentral unconditional second and fourth moments of u_t exist and are given by

$$\mu_2 = E(u_t^2) = \frac{\kappa}{1-\alpha-\delta} \tag{5}$$

$$\mu_4 = E(u_t^4) = \frac{3\kappa^2(1+\alpha+\delta)}{(1-\alpha-\delta)(1-\delta^2-2\alpha\delta-3\alpha^2)}. \tag{6}$$

Consider the consequences if the mean parameters β_0 and β_1 are estimated by ordinary least squares,

$$\hat{\boldsymbol{\beta}} = \left(\sum \mathbf{x}_t\mathbf{x}_t^{'}\right)^{-1}\left(\sum \mathbf{x}_t y_t\right)$$

$$\boldsymbol{\beta} = (\beta_0, \beta_1)'$$

$$\mathbf{x}_t = (1, y_{t-1})',$$

and where all summations are for $t = 1, ..., T$. Suppose further that inference is based on the usual OLS formula for the variance, with no correction for heteroskedasticity:

$$\hat{\mathbf{V}} = s^2\left(\sum \mathbf{x}_t\mathbf{x}_t^{'}\right)^{-1} \tag{7}$$

$$s^2 = (T-2)^{-1}\sum \hat{u}_t^2$$

$$\hat{u}_t = y_t - \mathbf{x}_t^{'}\hat{\boldsymbol{\beta}}$$

Consider first the consequences of this inference when the fourth-moment condition (4) is satisfied. For simplicity of exposition, consider the case when the true value of $\boldsymbol{\beta} = \mathbf{0}$. Then from the standard consistency results (e.g., Lee and Hansen, 1994; Lumsdaine, 1996) we see that

$$
\begin{aligned}
T\hat{\mathbf{V}} &= s^2\left(T^{-1}\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1} && (8)\\
&\xrightarrow{p} E(u_t^2)\begin{bmatrix} 1 & E(y_{t-1}) \\ E(y_{t-1}) & E(y_{t-1}^2)\end{bmatrix}^{-1} \\
&= \begin{bmatrix} \mu_2 & 0 \\ 0 & 1 \end{bmatrix}^{-1}.
\end{aligned}
$$

In other words, the OLS formulas will lead us to act as if $\sqrt{T}\hat{\beta}_1$ is approximately $N(0,1)$ if the true value of β_1 is zero. But notice

$$
\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = \left(T^{-1}\sum \mathbf{x}_t\mathbf{x}_t'\right)^{-1}\left(T^{-1/2}\sum \mathbf{x}_t u_t\right). \tag{9}
$$

Under the null hypothesis, the term inside the second summation, $\mathbf{x}_t u_t$, is a martingale difference sequence with variance

$$
E(u_t^2\mathbf{x}_t\mathbf{x}_t') = \begin{bmatrix} E(u_t^2) & E(u_t^2 u_{t-1}) \\ E(u_{t-1}u_t^2) & E(u_t^2 u_{t-1}^2) \end{bmatrix}.
$$

When the (2,2) element of this matrix is finite, it then follows from the Central Limit Theorem (e.g., Hamilton, 1994, p. 173) that

$$
T^{-1/2}\sum y_{t-1}u_t \xrightarrow{L} N\left(0, E\left(u_t^2 u_{t-1}^2\right)\right). \tag{10}
$$

To calculate the value of this variance, recall (e.g., Hamilton, 1994, p. 666) that the GARCH(1,1) structure for u_t implies an ARMA(1,1) structure for u_t^2:

$$
u_t^2 = \kappa + (\delta + \alpha)u_{t-1}^2 + w_t - \delta w_{t-1}
$$

for w_{t-1} a white noise process. It follows from the first-order autocovariance for an ARMA(1,1) process (e.g., Box and Jenkins, 1976, p. 76) that

$$\begin{aligned} E(u_t^2 u_{t-1}^2) &= E(u_t^2 - \mu_2)(u_{t-1}^2 - \mu_2) + \mu_2^2 \\ &= \rho(\mu_4 - \mu_2^2) + \mu_2^2 \end{aligned} \tag{11}$$

for

$$\rho = \frac{[1 - (\alpha + \delta)\delta]\alpha}{1 + \delta^2 - 2(\alpha + \delta)\delta}. \tag{12}$$

Substituting (11), (10) and (8) into (9),

$$\sqrt{T}\hat{\beta}_1 \xrightarrow{L} N(0, V_{11})$$

$$\begin{aligned} V_{11} &= \frac{\rho\mu_4 + (1-\rho)\mu_2^2}{\mu_2^2} \\ &= \rho\frac{3(1+\alpha+\delta)(1-\alpha-\delta)}{(1-\delta^2-2\alpha\delta-3\alpha^2)} + (1-\rho). \end{aligned}$$

with the last equality following from (5) and (6).

Notice that $V_{11} \geq 1$, with equality if and only if $\alpha = 0$. Thus OLS treats $\sqrt{T}\hat{\beta}_1$ as approximately $N(0,1)$, whereas the true asymptotic distribution is Normal with a variance bigger than unity, meaning that the OLS t test will systematically reject more often than it should. The probability of rejecting the null hypothesis that $\beta_1 = 0$ (even though the null hypothesis is true) gets bigger and bigger as the parameters get closer to the region at which the fourth moment becomes infinite, at which point the asymptotic rejection probability becomes unity. Figure 1 plots the rejection probability as a function of a and δ. If these parameters are in the range typically found in estimates of GARCH processes, an OLS t

test with no correction for heteroskedasticity would spuriously reject with arbitrarily high probability for a sufficiently large sample.

The good news is that the rate of divergence is pretty slow– it may take a lot of observations before the accumulated excess kurtosis overwhelms the other factors. I simulated 10,000 samples from the above Gaussian GARCH process for samples of size $T = 100$, 200, and 1000 and 10,000, (and 1,000 samples of size 100,000), where the true values were specified as follows:

$$\beta_0 = \beta_1 = 0$$

$$\kappa = 2$$

$$\alpha = 0.35$$

$$\delta = 0.6.$$

The solid line in Figure 2 plots the fraction of samples for which an OLS t test of $\beta_1 = 0$ exceeds two in absolute value. Thinking we're only rejecting a true null hypothesis 5% of the time, we would in fact do so 15% of the time in a sample of size $T = 100$ and 33% of the time when $T = 1,000$.

As one might imagine, for a given sample size, the OLS t-statistic is more poorly behaved if the true innovations v_t in (2) are Student's t with 5 degrees of freedom (the dashed line in Figure 2) rather than Normal.

What happens if instead of the OLS formula (7) for the variance of $\hat{\boldsymbol{\beta}}$ we use White's

(1980) heteroskedasticity-consistent estimate,

$$\tilde{\mathbf{V}} = \left(\sum \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1} \left(\sum \hat{u}_t^2 \mathbf{x}_t \mathbf{x}_t^{'}\right) \left(\sum \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1} ? \tag{13}$$

ARCH is not a special case of the class of heteroskedasticity for which $\tilde{\mathbf{V}}$ is intended to be robust, and indeed, unlike typical cases, $T\tilde{\mathbf{V}}$ is not a consistent estimate of a given matrix:

$$T\tilde{\mathbf{V}} = \left(T^{-1}\sum \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1} \left(T^{-1}\sum \hat{u}_t^2 \mathbf{x}_t \mathbf{x}_t^{'}\right) \left(T^{-1}\sum \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1}.$$

The first and last matrices will converge as before,

$$T^{-1}\sum \mathbf{x}_t \mathbf{x}_t^{'} \xrightarrow{p} \begin{bmatrix} 1 & 0 \\ 0 & \mu_2 \end{bmatrix},$$

but $T^{-1}\sum \hat{u}_t^2 \mathbf{x}_t \mathbf{x}_t^{'}$ will diverge if the fourth moment μ_4 is infinite. Figure 3 plots the simulated value for the square root of the lower-right element of $T\tilde{\mathbf{V}}$ for the Gaussian simulations above. However, this growth in the estimated variance of $\sqrt{T}\hat{\beta}_1$ is exactly right, given the growth of the actual variance of $\sqrt{T}\hat{\beta}_1$ implied by the GARCH specification. And a t test based on (13) seems to perform reasonably well for all sample sizes (see the second row of Table 2). The small-sample size distortion for the White test is a little worse for Student's t compared with Normal errors, though still acceptable. Table 2 also explores the consequences of using the Newey-West (1987) generalization of the White formula to allow for serial correlation, using a lag window of $q = 5$:

$$\tilde{\mathbf{V}}^{*} = \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1} \left[(1 - \frac{v}{q+1}) \sum_{t=v+1}^{T} \hat{u}_t \hat{u}_{t-v} \left(\mathbf{x}_t \mathbf{x}_{t-v}^{'} + \mathbf{x}_{t-v} \mathbf{x}_t^{'}\right)\right] \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t^{'}\right)^{-1}.$$

These results (reported in the third row of the two panels of Table 2) illustrate one potential pitfall of relying too much on "robust" statistics to solve the small-sample problems, in that it has more serious size distortions than does the simple White statistic for all specifications investigated.

Another reason one might not want to assume that White or Newey-West standard errors can solve all the problems is that these formulas only correct the standard error for $\hat{\boldsymbol{\beta}}$, but are still using the OLS estimate itself, which from Figure 3 was seen not to be $\sqrt{T}$ convergent. By contrast, even if the fourth moment does not exist, maximum likelihood estimation as an alternative to OLS is still $\sqrt{T}$ convergent. Hence the relative efficiency gains of MLE relative to OLS become infinite as the sample size grows for typical values of GARCH parameters. Engle (1982, p. 999) observed that it is also possible to have an infinite relative efficiency gain for some parameter values even with exogenous explanatory variables and ARCH as opposed to GARCH errors.

Results here are also related to the well-known result that ARCH will render inaccurate traditional tests for serial correlation in the mean. That fact has previously been noted, for example, by Milhøj (1985, 1987), Diebold (1988), Stambaugh (1993), and Bollerslev and Mikkelsen (1996). However, none of the above seems to have commented on the fact (though it is implied by the formulas they use) that the test size goes to unity as the fourth moment approaches infinity, or noted the implications as here for OLS regression.

Finally, I observe that just checking for a difference between the OLS and the White standard errors will sometimes not be sufficient to detect these problems. The difference between $\hat{\mathbf{V}}$ and $\tilde{\mathbf{V}}$ will be governed by the size of

$$\sum (s^2 - \hat{u}_t^2)\mathbf{x}_t\mathbf{x}_t'.$$

White (1980) suggested a formal test of whether this magnitude is sufficiently small on the basis of an OLS regression of $\hat{u}_t^2$ on the vector $\boldsymbol{\psi}_t$ consisting of the unique elements of $\mathbf{x}_t\mathbf{x}_t'$. In the present case, $\boldsymbol{\psi}_t = (1, y_{t-1}, y_{t-1}^2)'$. White showed that, under the null hypothesis that the OLS standard errors are correct, TR^2 from a regression of $\hat{u}_t^2$ on $\boldsymbol{\psi}_t$ would have a $\chi^2(2)$ distribution. The next-to-last row of each panel of Table 2 reports the fraction of samples for which this test would (correctly) reject the null hypothesis. It would miss about half the time in a sample as small as 100 observations but is more reliable for larger sample sizes.

Alternatively, one can look at Engle's (1982) analogous test for the null of homoskedasticity against the alternative of qth-order ARCH by looking at TR^2 from a regression of $\hat{u}_t^2$ on $(1, \hat{u}_{t-1}^2, \hat{u}_{t-2}^2, ..., \hat{u}_{t-q}^2)'$, which asymptotically has a $\chi^2(q)$ distribution under the null. The last rows in Table 2 report the rejection frequency for this test using $q = 3$ lags. Not surprisingly, since this test is designed specifically for the ARCH class of alternatives whereas the White test is not, this test has a little more power. Its advantage over the White test for homoskedasticity is presumably greater in many macro applications in which $\mathbf{x}_t$ includes a number of variables and their lags, in which case the vector $\boldsymbol{\psi}_t$ can become unwieldy, whereas the Engle test remains a simple $\chi^2(q)$ regardless of the size of $\mathbf{x}_t$.

The philosophy of McARCH, then, is quite simple. The Engle TR^2 diagnostic should be calculated routinely in any macroeconomic analysis. If a violation of homoskedasticity is found, one should compare the OLS estimates with maximum likelihood to make sure that the inference is robust. The following sections illustrate the potential importance of doing so with two examples from applied macroeconomics.

3 Application 1: Measuring market expectations of what the Federal Reserve is going to do next.

My first example is adapted from Hamilton (forthcoming). The fed funds rate is a market-determined interest rates at which banks lend reserves to one another overnight. This interest rate is extremely sensitive to the supply of reserves created by the Fed, and in recent years monetary policy has been implemented in terms of a clearly announced target for the fed funds rate that the Fed intends to achieve.

A critical factor that determines how Fed actions affect the economy is expectations by the public as to what the Fed is going to do next, as discussed, for example, in my (2008) paper. One natural place to look for an indication of what those expectations might be is the fed funds futures market.

Let $t = 1, 2, .., T$ index monthly observations. In the empirical results reported here, $t = 1$ corresponds to October, 1988 and the last observation ($T = 213$) is June 2006. For each month, we're interested in what the market expects for the average effective fed funds rate over that month, denoted r_t. For the empirical estimates reported in this section, r_t is measured in basis points, so that for example $r_t = 525$ corresponds to an annual interest rate of 5.25%.

On any business day, one can enter into a futures contract through the Chicago Board of Trade whose settlement is based on what the value of r_{t+j} actually turns out to be for some future month. The terms of a j-month-ahead contract traded on the last day of month t can be translated[1] into an interest rate $f_t^{(j)}$ such that, if r_{t+j} turns out to be less than $f_t^{(j)}$, then the seller of the contract has to compensate the buyer a certain amount (specifically, \$41.67 on a standard contract) for every basis point by which $f_t^{(j)}$ exceeds r_{t+j}. If $f_t^{(j)} < r_{t+j}$, the buyer pays the seller. Since $f_t^{(j)}$ is known as of the end of month t but r_{t+j} will not be known until the end of month $t+j$, the buyer of the contract is basically making a bet that r_{t+j} will be less than $f_t^{(j)}$. If the marginal market participant were risk neutral, it would be the case that

$$f_t^{(j)} = E_t(r_{t+j}) \tag{14}$$

where $E_t(.)$ denotes the mathematical expectation on the basis of any information publicly available as of the last day of month t. If (14) holds, we could just look at the value of $f_t^{(j)}$ to infer what market participants expect the Federal Reserve to do in the coming months.

However, previous investigators such as Sack (2004) and Piazzesi and Swanson (forthcoming) have concluded that (14) does not hold. The simplest way to investigate this claim is to construct the forecast error implied by the 1-month-ahead contract,

$$u_t^{(1)} = r_t - f_{t-1}^{(1)}$$

and test whether this error indeed has mean zero, as it should if (14) were correct. For contracts at longer horizons $j > 1$, one can look at the monthly change in contract terms,

$$u_t^{(j)} = f_t^{(j-1)} - f_{t-1}^{(j)}.$$

[1] Specifically, if P_t is the price of the contract agreed to by the buyer and seller on day t, then $f_t = 100 \times (100 - P_t)$.

If (14) holds, then $u_t^{(j)}$ would also be a martingale difference sequence:

$$u_t^{(j)} = E_t(r_{t+j-1}) - E_{t-1}(r_{t+j-1}).$$

One simple test is then to perform the regression

$$u_t^{(j)} = \mu^{(j)} + \varepsilon_t^{(j)}$$

and test the null hypothesis that $\mu^{(j)} = 0$; this is of course just the usual t-test for a sample mean. Table 3 reports the results of this test using 1-, 2-, and 3-month-ahead futures contracts. For the historical sample, the 1-month-ahead futures contract $f_t^{(1)}$ overestimated the value of r_{t+1} by an average of 2.66 basis points and $f_t^{(j)}$ overestimated the value of $f_{t+1}^{(j-1)}$ by almost 4 basis points. One interpretation is that there is a risk premium built into these contracts. Another possibility is that the market participants failed to recognize fully the chronic decline in interest rates over this period.

Before putting too much credence in such interpretations, however, recall that the theory (14) implies that $u_t^{(j)}$ should be a martingale difference sequence but makes no claims about predictability of its variance. Figure 4 reveals that each of the series $u_t^{(j)}$ exhibits some clustering of volatility and a significant decline in variability over time, in addition to occasional very large outliers. Engle's TR^2 test for omitted 4th-order ARCH finds very strong evidence of conditional heteroskedasticity at least for $u_t^{(1)}$ and $u_t^{(3)}$; see Table 3. Hence if we are interested in a more accurate estimate of the bias and statistical test of its significance, we might want to model these features of the data.

Hamilton (forthcoming) calculated maximum likelihood estimates for parameters of the following EGARCH specification (with (j) superscripts on all variables and parameters suppressed for ease of readability):

$$u_t = \mu + \sqrt{h_t}\varepsilon_t \tag{15}$$

$$\log h_t - \boldsymbol{\gamma}'\mathbf{z}_t = \alpha(|\varepsilon_{t-1}| - k_2) + \delta(\log h_{t-1} - \boldsymbol{\gamma}'\mathbf{z}_{t-1}) \tag{16}$$

$$\mathbf{z}_t = (1, t/1000)'$$

$$k_2 = E|\varepsilon_t| = \frac{2\sqrt{\nu}\Gamma[(\nu+1)/2]}{(\nu-1)\sqrt{\pi}\Gamma(\nu/2)}$$

for ε_t a Student's t variable with ν degrees of freedom and $\Gamma(.)$ the gamma function:

$$\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}\,dx.$$

The log likelihood is then found from

$$\sum_{t=1}^{T} \log f(u_t|\mathbf{U}_{t-1};\boldsymbol{\theta}) \tag{17}$$

$$f(u_t|\mathbf{U}_{t-1},\boldsymbol{\theta}) = \left(k_1/\sqrt{h_t}\right)[1 + (\varepsilon_t^2/\nu)]^{-(\nu+1)/2}$$

$$k_1 = \Gamma[(\nu+1)/2]/[\Gamma(\nu/2)\sqrt{\nu\pi}].$$

Given numerical values for the parameter vector $\boldsymbol{\theta} = (\mu, \boldsymbol{\gamma}', \alpha, \delta, \nu)'$ and observed data $\mathbf{U}_T = (u_1, u_2, ..., u_T)'$ we can then begin the iteration (16) for $t = 1$ by setting $h_1 = \exp(\boldsymbol{\gamma}'\mathbf{z}_0)$. Plugging this into (15) gives us a value for ε_1, which from (16) gives us the number for h_2. Iterating in this fashion gives the sequence $\{h_t, \varepsilon_t\}_{t=1}^{T}$ from which the log likelihood (17) can be evaluated for the specified numerical value of $\boldsymbol{\theta}$. One then tries another guess for $\boldsymbol{\theta}$ in order to numerically maximize the likelihood function. Asymptotic standard errors can be obtained from numerical second derivatives of the log likelihood as in Hamilton (1994, equation [5.8.3]).

Maximum likelihood parameter estimates are reported in Table 4. Adding these features provides an overwhelming improvement in fit, with a likelihood ratio test statistic well in excess of 100 when adding just 4 parameters to a simple Gaussian specification with constant variance. The very low estimated degrees of freedom results from the big outliers in the data, and both the serial dependence (δ) and trend parameter (γ_2) for the variance are extremely significant.

A very remarkable result is that the estimates for the mean of the forecast error μ actually switch signs, shrink by an order of magnitude, and become far from statistically significant. Evidently the sample means of $u_t^{(j)}$ are more influenced by negative outliers and observations early in the sample than they should be.

Note that for this example, the problem is not adequately addressed by simply replacing OLS standard errors with White standard errors, since when the regressors consist only of a constant term, the two would be identical. Moreover, whenever, as here, there is an affirmative objective of obtaining accurate estimates of a parameter (the possible risk premium incorporated in these prices) as opposed solely to testing a hypothesis, the concern is with the quality of the coefficient estimate itself rather than the correct size of a hypothesis test.

4 Application 2: Using the Taylor Rule to summarize changes in Federal Reserve policy.

One of the most influential papers for both macroeconomic research and policy over the last decade has been John Taylor's (1993) proposal of a simple rule that the central bank should follow in setting an interest rate like the fed funds rate r_t. Taylor's proposal called for the Fed to raise the interest rate by an amount governed by a parameter ψ_1 when the observed inflation rate π_t is higher than it wishes (so as to bring inflation back down), and to raise the interest rate by an amount governed by ψ_2 when y_t, the gap between real GDP and its potential value, is positive:

$$r_t = \psi_0 + \psi_1 \pi_t + \psi_2 y_t$$

In this equation, the value of ψ_0 reflects factors such as the Fed's long-run inflation target and the equilibrium real interest rate. There are a variety of ways such an expression has been formulated in practice, such as "forward-looking" specifications, in which the Fed is responding to what it expects to happen next to inflation and output, and "backward-looking" specifications, in which lags are included to capture expectations formation and adjustment dynamics.

A number of studies have looked at the way that the coefficients in such a relation may have changed over time, including Judd and Rudebusch (1998), Clarida, Galí and Gertler (2000), Jalil (2004), and Boivin and Giannoni (2006). Of particular interest has been that the claim that the coefficient on inflation ψ_1 has increased relative to the 1970s, and that this increased willingness on the part of the Fed to fight inflation has been a factor helping to make the U.S. economy become more stable. In this paper, I will explore the variant investigated by Judd and Rudebusch, whose reduced-form representation is

$$\Delta r_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 y_t + \gamma_3 y_{t-1} + \gamma_4 r_{t-1} + \gamma_5 \Delta r_{t-1} + v_t. \qquad (18)$$

Here $t = 1, 2, ..., T$ now will index quarterly data, with $t = 1$ in my sample corresponding to 1956:Q1 and $T = 205$ corresponding to 2007:Q1. The value of r_t for a given quarter is the average of the three monthly series for the effective fed funds rate, with $\Delta r_t = r_t - r_{t-1}$, and for empirical results here is reported as percent rather than basis points, e.g., $r_t = 5.25$ when the average fed funds rate over the three months of the quarter is 5.25%. Inflation π_t is measured as 100 times the natural logarithm of the difference between the level of the implicit GDP deflator for quarter t and its value for the corresponding quarter of the preceding year, with data taken from Bureau of Economic Analysis Table 1.1.9. As in Judd and Rudebusch, the output gap y_t was calculated as

$$y_t = \frac{100(Y_t - Y_t^*)}{Y_t^*}$$

for Y_t the level of real GDP (in billions of chained 2000 dollars, from BEA Table 1.1.6) and Y_t^* the series for potential GDP from the Congressional Budget Office (obtained from the St. Louis FRED database). Judd and Rudebusch focused on certain rearrangements of the parameters in (18), though here I will simply report results in terms of the reduced-form estimates themselves. The term v_t in (18) is the regression error.

Table 5 presents results from OLS estimation of (18) using the full sample of data. Of particular interest are γ_1 and γ_2, the contemporary responses to inflation and output, respectively. Table 6 then re-estimates the relation, allowing for separate coefficients since

1979:Q3, when Paul Volcker became Chair of the Federal Reserve. The OLS results reproduce the findings of the many researchers noted above that monetary policy seems to have responded much more vigorously to disturbances since 1979, with the inflation coefficient γ_1 increasing by 0.26 and the output coefficient γ_2 increasing by 0.64.

However, the White standard errors for the coefficients on $d_t\pi_t$ and $d_t y_t$ are almost twice as large as the OLS standard errors, and suggest that the increased response to inflation is in fact not statistically significant and the increased response to output is measured very imprecisely. Moreover, Engle's LM test for the null of Gaussian errors with no heteroskedasticity against the alternative of 4th-order ARCH leads to overwhelming rejection of the null hypothesis.[2] All of which suggests that, if we are indeed interested in measuring the magnitudes by which these coefficients have changed, it is preferable to adjust not just the standard errors but the parameter estimates themselves in light of the dramatic ARCH displayed in the data.

I therefore estimated the following GARCH-t generalization of (18):

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + v_t$$

$$v_t = \sqrt{h_t}\varepsilon_t$$

$$h_t = \kappa + \tilde{h}_t$$

$$\tilde{h}_t = \alpha(v_{t-1}^2 - \kappa) + \delta\tilde{h}_{t-1} \tag{19}$$

with ε_t a Student's t random variable with ν degrees of freedom. Iteration on (19) is

[2] Siklos and Wohar (2005) also make this point.

initialized with $\tilde{h}_1 = 0$. The log likelihood is then evaluated exactly as in (17). Maximum likelihood estimates are reported in Table 7.

Once again generalizing a homoskedastic Gaussian specification is overwhelmingly favored by the data, with a comparison of the specifications in Tables 6 and 7 producing a likelihood ratio $\chi(4)$ statistic of 183.34. The degrees of freedom for the Student's t distribution are only 2.29, and the implied GARCH process is highly persistent ($\hat{\alpha} + \hat{\delta} = 0.82$). Of particular interest is the fact that the changes in the Fed's response to inflation and output are now considerably smaller than suggested by the OLS estimates. The change in γ_1 is now estimated to be only 0.09 and the change in γ_2 has dropped to 0.05 and no longer appears to be statistically significant.

Figure 5 offers some insight into what produces these results. The top panel illustrates the tendency for interest rates to exhibit much more volatility at some times than others, with the 1979:Q2 to 1982:Q3 episode particularly dramatic. The bottom panel plots observations on the pairs $(y_t, \Delta r_t)$ in the second half of the sample. The apparent positive slope in that scatter plot is strongly influenced by the observations in the 1979-82 period. If one allowed the possibility of serial dependence in the squared residuals, one would give less weight to the 1979-82 observations, resulting in a flatter slope estimate over 1979-2007 relative to OLS.

This is not to attempt to overturn the conclusion of earlier researchers that there has been a change in Fed policy in the direction of a more active policy. A comparison of the changing-parameter specification of Table 7 with a fixed-parameter GARCH specification produces a $\chi(4)$ likelihood ratio statistic of 18.22, which is statistically significant with a

p-value of 0.001. Nevertheless, the magnitude of this change appears to be substantially smaller than one would infer on the basis of OLS estimates of the parameters.

Nor is this discussion meant to displace the large and thoughtful literature on possible changes in the Taylor Rule, which has raised a number of other substantive issues not explored here. These include whether one wants to use real-time or subsequent revised data (Orphanides (2001)), the distinction between the "backward-looking" Taylor Rule explored here and "forward-looking" specifications (Clarida, Galí, and Gertler, 2000), and continuous evolution of parameters rather than a sudden break (Jalil, 2004; Boivin, 2006). The simple exercise undertaken nevertheless does in my mind establish the potential importance for macroeconomists to check for the presence of ARCH even when their primary interest is in the conditional mean.

5 Conclusions.

The reader may note that both of the examples I have used to illustrate the potential relevance of McARCH use the fed funds rate as the dependent variable. This is not entirely an accident. Although Kilian and Gonçalves (2004) concluded that most macro series exhibit some ARCH, the fed funds rate may be the macro series for which one is most likely to observe wild outliers and persistent volatility clustering, regardless of the data frequency or subsample. It is nevertheless, as the examples used here illustrate, a series that features very importantly for some of the most fundamental questions in macroeconomics.

The rather dramatic way in which accounting for outliers and ARCH can change one's inference that was seen in these examples presumably would not be repeated for every macroeconomic relation estimated. However, routinely checking something like a TR^2 statistic, or the difference between OLS and White standard errors, seems a relatively costless and potentially quite beneficial habit. And the assumption by many practitioners that we can avoid all these problems simply by always relying on the White standard errors may not represent best possible practice.

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Table 1. How many ways can you spell "ARCH"? (A partial lexicography).

AARCH	Augmented ARCH	Bera, Higgins and Lee (1992)
APARCH	Asymmetric power ARCH	Ding, Engle, and Granger (1993)
ARCH-M	ARCH in mean	Engle, Lilien and Robins (1987)
FIGARCH	Fractionally integrated GARCH	Baillie, Bollerslev, Mikkelsen (1996)
GARCH	Generalized ARCH	Bollerslev (1986)
GARCH-t	Student's *t* GARCH	Bollerslev (1987)
GJR-ARCH	Glosten-Jagannathan-Runkle ARCH	Glosten, Jagannathan, and Runkle (1993)
EGARCH	Exponential generalized ARCH	Nelson (1991)
HGARCH	Hentschel GARCH	Hentschel (1995)
IGARCH	Integrated GARCH	Bollerslev and Engle (1986)
MARCH	Modified ARCH	Friedman, Laibson, and Minsky (1989)
MARCH	Multiplicative ARCH	Milhøj (1987)
NARCH	Nonlinear ARCH	Higgins and Bera (1992)
PNP-ARCH	Partially Non-parametric ARCH	Engle and Ng (1993)
QARCH	Quadratic ARCH	Sentana (1995)
QTARCH	Qualitative Threshold ARCH	Gourieroux and Monfort (1992)
SPARCH	Semiparametric ARCH	Engle and Gonzalez-Rivera (1991)
STARCH	Structural ARCH	Harvey, Ruiz, and Sentana (1992)
SWARCH	Switching ARCH	Hamilton and Susmel (1994)
TARCH	Threshold ARCH	Zakoian (1994)
VGARCH	Vector GARCH	Bollerslev, Engle, and Wooldrige (1988)

Table 2. Fraction of samples for which indicated hypothesis is rejected by test of nominal size 0.05.

Errors Normally distributed

H_0	Test based on	$T = 100$	$T = 200$	$T = 1000$
$\beta_1 = 0$ (H_0 is true)	OLS standard error	0.152	0.200	0.327
$\beta_1 = 0$ (H_0 is true)	White standard error	0.072	0.063	0.054
$\beta_1 = 0$ (H_0 is true)	Newey-West standard error	0.119	0.092	0.062
ε_t homoskedastic (H_0 is false)	White TR^2	0.570	0.874	1.000
ε_t homoskedastic (H_0 is false)	Engle TR^2	0.692	0.958	1.000

Errors Student's *t* with 5 degrees of freedom

H_0	Test based on	$T = 100$	$T = 200$	$T = 1000$
$\beta_1 = 0$ (H_0 is true)	OLS standard error	0.174	0.229	0.389
$\beta_1 = 0$ (H_0 is true)	White standard error	0.081	0.070	0.065
$\beta_1 = 0$ (H_0 is true)	Newey-West standard error	0.137	0.106	0.079
ε_t homoskedastic (H_0 is false)	White TR^2	0.427	0.691	0.991
ε_t homoskedastic (H_0 is false)	Engle TR^2	0.536	0.822	0.998

Table 3. OLS estimates of bias in monthly fed funds futures forecast errors.

dependent variable ($u_t^{(j)}$)	estimated mean ($\hat{\mu}^{(j)}$)	standard error	OLS *p*-value	ARCH(4) LM *p*-value	Log like-lihood
j = 1 month	-2.66	0.75	0.001	0.006	-812.61
j = 2 months	-3.17	1.06	0.003	0.204	-884.70
j = 3 months	-3.74	1.27	0.003	0.001	-922.80

Table 4. Maximum likelihood estimates (asymptotic standard errors in parentheses) for EGARCH model of fed funds futures forecast errors.

horizon (j)	$u_t^{(1)}$	$u_t^{(2)}$	$u_t^{(3)}$
mean (μ)	0.12	0.43	0.27
	(0.24)	(0.34)	(0.67)
log average variance (γ_1)	5.73	6.47	7.01
	(0.42)	(0.51)	(0.54)
trend in variance (γ_2)	-22.7	-23.6	-17.1
	(3.1)	(3.3)	(3.8)
$\| u_{t-1}\|$ (α)	0.18	0.15	0.30
	(0.07)	(0.07)	(0.12)
log h_{t-1} (δ)	0.63	0.74	0.84
	(0.16)	(0.22)	(0.11)
Student t degrees of freedom (υ)	2.1	2.2	4.1
	(0.4)	(0.4)	(1.2)
log likelihood	-731.08	-793.38	-860.16

Table 5. Fixed-coefficient Taylor Rule as estimated from full sample OLS regression.

Regressor	Coefficient	Std error (OLS)	Std error (White)
constant	0.06	0.13	0.18
π_t	0.13	0.04	0.06
y_t	0.37	0.07	0.11
y_{t-1}	-0.27	0.07	0.10
r_{t-1}	-0.08	0.03	0.03
Δr_{t-1}	0.14	0.07	0.15
TR^2 for ARCH(4) (p-value)	23.94	(0.000)	
Log likelihood	-252.26		

Table 6. Taylor Rule with separate pre- and post-Volcker parameters as estimated by OLS regression ($d_t = 1$ for $t > 1979$:Q2).

Regressor	Coefficient	Std error (OLS)	Std error (White)
constant	0.37	0.19	0.19
π_t	0.17	0.07	0.04
y_t	0.18	0.08	0.07
y_{t-1}	-0.07	0.08	0.07
r_{t-1}	-0.21	0.07	0.06
Δr_{t-1}	0.42	0.11	0.13
d_t	-0.50	0.24	0.30
$d_t\pi_t$	0.26	0.09	0.16
$d_t y_t$	0.64	0.14	0.24
$d_t y_{t-1}$	-0.55	0.14	0.21
$d_t r_{t-1}$	0.05	0.08	0.08
$d_t \Delta r_{t-1}$	-0.53	0.13	0.24
TR^2 for ARCH(4) (p-value)	45.45	(0.000)	
Log likelihood	-226.80		

Table 7. Taylor Rule with separate pre- and post-Volcker parameters as estimated by GARCH-t maximum likelihood ($d_t = 1$ for $t > 1979$:Q2).

Regressor	Coefficient	Asymptotic std error
constant	0.13	0.08
π_t	0.06	0.03
y_t	0.14	0.03
y_{t-1}	-0.12	0.03
r_{t-1}	-0.07	0.03
Δr_{t-1}	0.47	0.09
d_t	-0.03	0.12
$d_t\pi_t$	0.09	0.04
$d_t y_t$	0.05	0.07
$d_t y_{t-1}$	0.02	0.07
$d_t r_{t-1}$	-0.01	0.03
$d_t \Delta r_{t-1}$	-0.01	0.11
GARCH parameters		
constant	0.015	0.010
α	0.11	0.05
δ	0.71	0.07
ν	2.29	0.48
Log likelihood	-135.13	

Figure 1. Asymptotic rejection probability for OLS *t*-test that autoregressive coefficient is zero as a function of GARCH(1,1) parameters α and δ. Note: null hypothesis is actually true and test has nominal size of 5%.

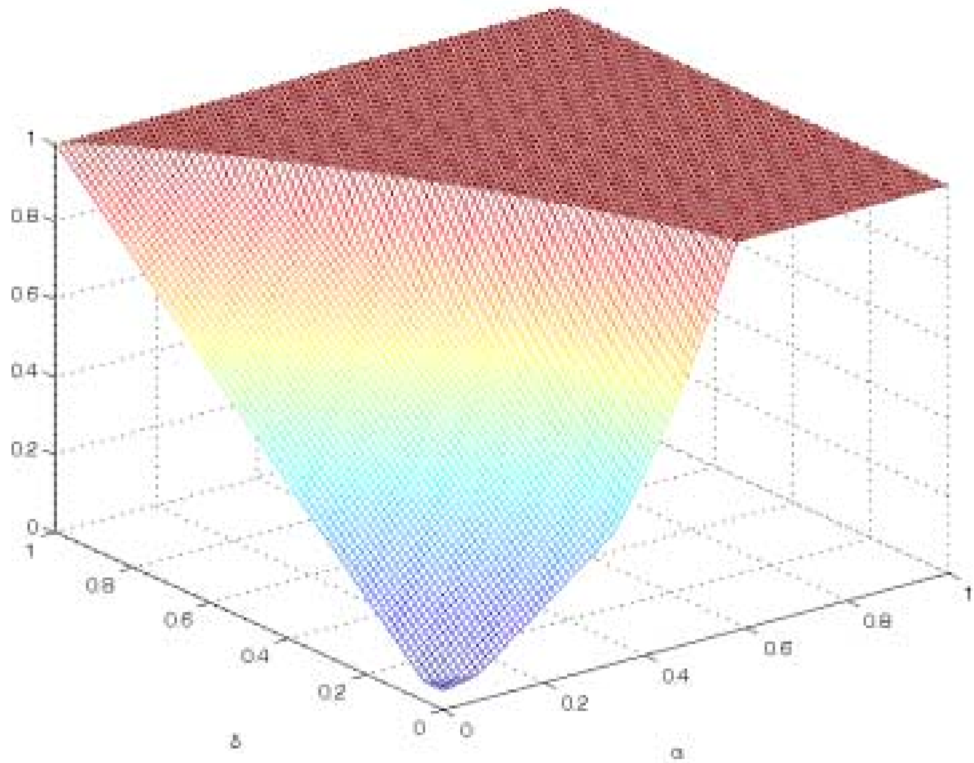

Figure 2. Fraction of samples in which OLS *t*-test leads to rejection of the null hypothesis that autoregressive coefficient is zero as a function of the sample size for regression with Gaussian errors (solid line) and Student's *t* errors (dashed line). Note: null hypothesis is actually true and test has nominal size of 5%.

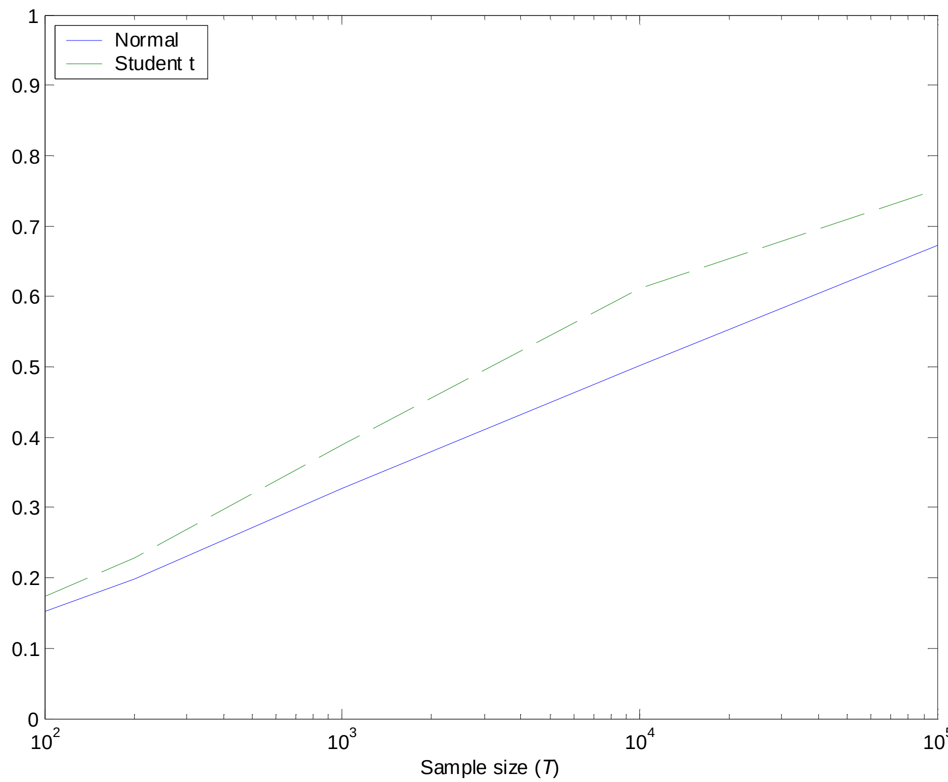

Figure 3. Average value of $\sqrt{T}$ times estimated standard error of estimated autoregressive coefficient as a function of the sample size for White standard error (solid line) and OLS standard error (dashed line).

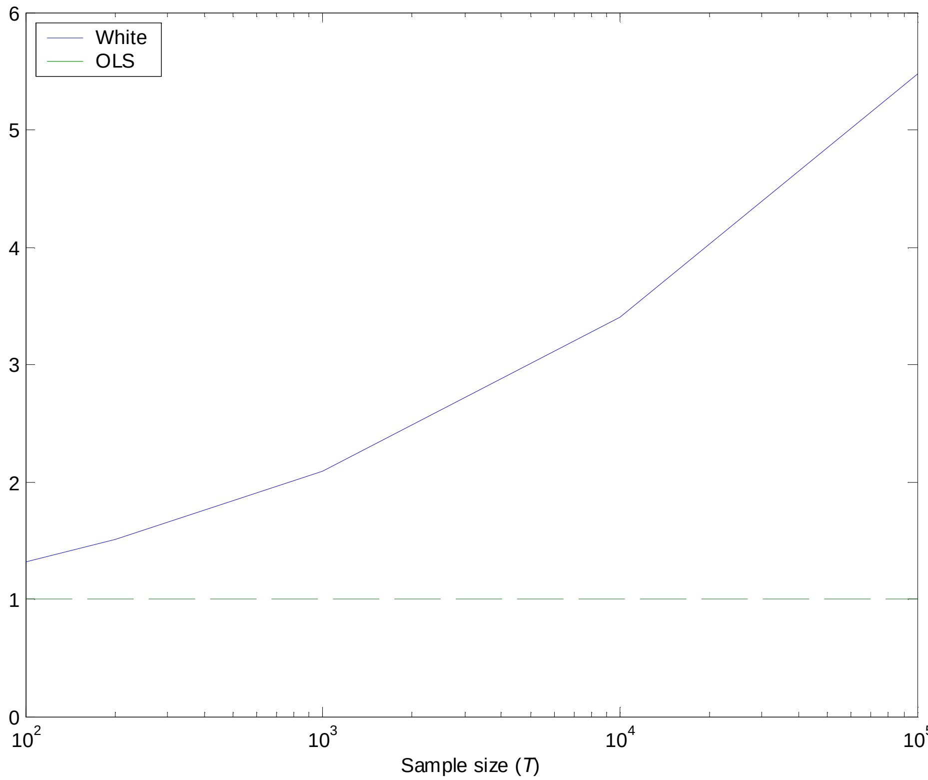

Figure 4. Plots of 1-month-ahead forecast errors $(u_t^{(j)})$ as a function of month t based on j = 1-, 2-, or 3-month ahead futures contracts.

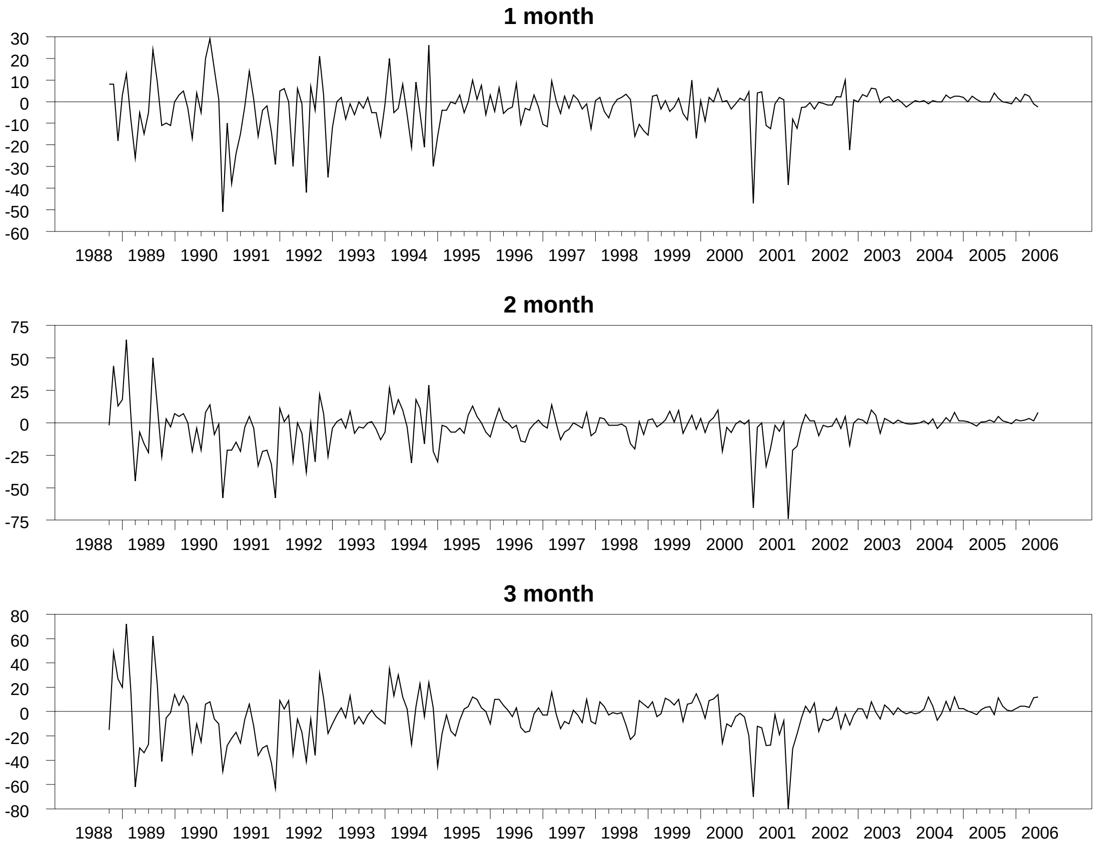

Figure 5. Change in fed funds rate for the full sample (1956:Q2-2007:Q1), and scatter plot for later subsample (1979:Q2-2007:Q1) of change in fed funds rate against deviation of GDP from potential.